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Trigonometric graph sketching

Mark schemes for the Trigonometric graph sketching question pack

WJEC Level 2 Additional Mathematics (9550) · Trigonometry

Official WJEC mark schemes for the 12 questions in the matching revise.wales question pack (47 marks total), from the 2011–2024 papers.

19	(a) General y = tan2x form • passing through (0°,0), (90°,0) and (180°,0) as unique intersections with the x-axis **with** • asymptotes uniquely at 45° and 135°	B2	For B2, sketch must not cross or touch asymptotes, maximum of B1 if cross or touch of asymptotes seen, or 'curve away' from the asymptotes If asymptotes are not shown it must be clear there is no overlap of the curves (mark intention) B1 for general y = tan2x form • passing through (0°,0), (90°,0) and (180°,0) as unique intersections with the x-axis **or** • asymptotes uniquely at 45° and 135°
	(b) 42.(1447...°) **and** 132.(1447...°)	B2 4	These values need to be selected, not amongst others unless unambiguously indicated as the response. B1 for sight of any one of the following: • 42.(1447...°) • 132.(1447...°) • a pair of positive angles, a and b, such that $0 < a < b < 180$ and $b - a = 90$.

15	(a) Sine curve that • intersects x-axis at (0°, 0) (90°, 0) (180°, 0) (270°, 0) and (360°, 0), • has maxima at ≈ 45° and 225° and minima at ≈ 135° and 315° • has 3 and -3 indicated on the y-axis	B2	Must show a clear curve, not straight at turning points B1 for one of the following, allowing straight rather than curves at turning points: • a sine curve intersecting x-axis at (0°, 0) (90°, 0) (180°, 0) (270°, 0) and (360°, 0) • a sine curve (which may have incorrect period) with 3 and -3 indicated on the y-axis
	(b) 7.5°, 82.5°, 187.5° and 262.5° alone	B2 4	Must be correct to 1 decimal place and not from incorrect working B1 for sight of 7.5(°) or for the correct 4 angles but not given to 1 decimal place Note: sin2x = 0.777/3 = 0.259, followed by incorrect working x = 0.259/sin2 = 7.42… is B0

15	(a) General sine curve intersecting x-axis at (0°,0) , (90°,0) (180°,0) (270°,0) and (360°,0)	M1	Maxima should be between 0° to 90° and 180° to 270° Minima should be between 90° to 180° and 270 to 360° Allow general shape as the joining of key values, but straight rather than clearly curving towards a turn
	Correct curve with 1 and -1 on y-axis	A1	Must show a clear curve, not straight at turning points
	(b) 14°, 76°, 194° and 256° alone	B2	Do not award B2 or B1 for answers from incorrect working: $2x = 28.034\ldots$ leading to $x = 14(.017\ldots)°$ correct, but $x = \sin^{-1} 0.47/2 = \sin^{-1}0.235 = 13.59(1\ldots)°$ is incorrect B1 for sight of 14° or for the correct 4 angles but not given to nearest degree

7	(a) $(3)^3 + 8(3)^2 - 2(3) + 6$ (= 27 + 72 - 6 + 6) = 99	M1 A1	Or division method giving $x^2 + 11x$ …
	(b)(i) Substitute x = -3 Showing f(-3) = 0	M1 A1	Or division method giving $x^2 - 2x$ … Convincing from working shown (not if incorrect working seen), allow $(-3)^3 + (-3)^2 - 41(-3) - 105 = 0$, also allow for sight of $-3^3 + -3^2 - 41\times-3 - 105 = 0$ provided no incorrect calculation is given such as -3^2 as -9
	(ii) $(x + 3)(x^2 + bx + c)$ or intention to divide by (x+ 3) with x^2 shown ((x + 3)) (x^2 - 2x - 35) ((x + 3))(x + 5)(x - 7)	M1 A2 A1 8	A1 for -2x or -35. Or use of factor theorem A1 (x+5), A1 (x- 7) CAO. Mark final answer, *but ignore attempts to 'solve'*
8	(dy/dx=) $12x^2 - 6x$ dy/dx = 0 or $12x^2 - 6x = 0$ or $12x^2 = 6x$ x = 0 and y = 20 x = ½ and y = 19¾	B1 M1 A1 A1	FT their dy/dx form $ax^2 \pm bx$ If A0, A0 here, award A1 for x = 0 with x = ½ *Answer only, no working shown M0 A0 A0*
	$d^2y/dx^2 = 24x - 6$ (0, (20)): d^2y/dx^2 <0, point is a maximum (½, (19¾)): d^2y/dx^2 >0, point is a minimum	M1 A1 A1 7	Or first derivative test, interpretation of first derivative test. Or alternative (e.g. full graphical method with explanation) FT for their x value FT for their other x value provided this does not have the same interpretation as the first x value *Answer only, no working shown M0 A0 A0* *If $d^2y/dx^2 = cx + d$ where $c \neq 0$ and test applied correctly then SC2 instead of final A1, A1 (as M1 has not been awarded))provided one minimum and one maximum*
9	$\frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4}$	B1 1	**Working must be shown**
10	(a) $FG^2 = (-4 - 8)^2 + (10 - 28)^2$ (= $12^2 + 18^2$ = 468) FG = 6√13	M1 A1	Or equivalent. Allow 1 slip or error M1, A0 for answers √468 or 21.6(3...) CAO
	(b) Gradient FG (28-10)/(8 - -4) = 18/12 (= 9/6 = 3/2)	M1 A1	Do not ignore incorrect cancelling, mark final answer
	(c) (-4 + 8)/2 or (10 + 28)/2 Mid point (2, 19)	M1 A1	Sight of (2, …) or (… , 19) implies M1 provided no incorrect working is seen
	Perpendicular gradient -2/3 (or -6/9 or -12/18)	B1	FT -1/ 'their answer in (b)'
	$\frac{y - 19}{x - 2} = \frac{-2}{3}$ or 19 = -2/3 × 2 + c	M1	OR for an alternative correct method of finding the equation of a straight line, for the idea of how an equation of a straight line can be found. FT 'their perpendicular gradient' or 'their answer in (b)' AND 'their mid point' or for 'points F or G' used
	y – 19 = -2/3(x - 2) or 3(y – 19) = -2(x – 2) or 3y = -2x + 61 or c = 20⅓ or c = 61/3	m1	**Do not allow** use gradient from their answer in (b), and/or points F or G as the mid-point of FG. Only FT for 'their perpendicular gradient' (not 'their answer' from (b)) AND 'their mid point'
	2x + 3y – 61 = 0 or -2x – 3y + 61 = 0	A1 10	CAO. Must be in this form with '=0' with terms in any order

11	(a) Correct shaped graph with (0°,) 180° & 360° labelled on the x-axis AND 2, 7 & 12 labelled on the y-axis	B3	Ignore outside the required range *Intention for approximately (0°, 7), (90°, 2), (180°, 7), (270°, 12) and (360°, 7)* B2 awarded a for correct shape graph with conditions: • sinx reflected • with one complete period, labelled 0° to 360° • with difference in y values between maximum and minimum of 10, for their labels OR B1 for a correct shape graph with any 2 of the 3 bullet points above met, OR B1 for a graph with all 3 bullet points above met but joined by straight lines (even if turning points curved), OR B1 for a curved graph through intended points: (0°, 7), (90°, 2), (180°, 7), (270°, 12) and (360°, 7)
	(b) Maximum value 12 AND Minimum value 2	B1	Accept Maximum (270(°), 12) and Minimum (90(°), 2) Allow unsupported correct responses
		4	FT provided at least B2 previously awarded in (a)
12	(a) $(dy/dx=)\ 16x^7 + 8x$ $(d^2y/dx^2=)\ 112x^6 + 8$	B1 B1	 FT to 2nd B1 from $dy/dx = kx^n$ (+ …)
	(b) $(5/5)\ x^5 + (3/-1)\ x^{-1} + (-2/-2)x^{-2}$ $(= x^5 - 3x^{-1} + x^{-2})$	B3	B1 for each term. Accept unsimplified. ISW
	+ c (constant)	B1	Award if at least B1 given for integration
	(c) $6x^2/2 + 10x$ $[\ 6x^2/2 + 10x\ ]^3_2$ and with intention to substitute and subtract	B2 M1	B1 for $6x^2/2$ or $10x$ Intention to use 3, 2 (in either order) **and** subtract FT their integration, not the same terms as given or differentiated, this includes if there is only1 term seen.
	$=(6\times3^2/2+10\times3) - (6\times2^2/2+10\times2)$ $(=\ 57 - 32)$	A1	FT for correct use of limits provided working with 2 terms from 'their integration'
	$=\ 25$	A1 11	CAO, not FT. *Answer only, no working shown, M0 A0 A0*
13	(When x = 2) y = 27 (Gradient when x = 2, dy/dx =) $5\times2x$ 20	B1 M1 A1	 For differentiation, before substitution of x = 2
	Equation $\frac{y-27}{x-2} = 20$ or $27 = 20\times2 + c$	M1	FT values for 'their 27' and 'their 20' provided at least one of these is correct.
	$y - 27 = 20(x - 2)$ or $c = -13$	m1	Implies previous M1
	$y = 20x - 13$	A1 6	CAO. Mark final answer
14	Method to solve simultaneously, e.g. use of $y = 2x + 1$ or $x = (y - 1)/2$ into the first equation	M1	$2x + 1 = x^2 - 5x + 13$ or $y = \frac{(y-1)^2}{2^2} - \frac{5(y-1)}{2} + 13$
	$x^2 - 7x + 12 = 0$ or $y^2 - 16y + 63 = 0$	A1	Or equivalent but must '=0' or implied in further working
	$(x - 3)(x - 4)\ (=0)$ or $(y - 9)(y - 7)\ (=0)$	m1	OR $x = (7 \pm \sqrt{1})/2$ or $y = (16 \pm \sqrt{4})/2$
	(3, 7) and (4, 9)	A1	FT from their quadratic CAO Need not be in this form, accept x=3, y=7 with x=4, y=9 x & y values must be given
		4	Do not accept unsupported responses Do not accept trial & improvement

		3	*then possible maximum is only 4 marks, final A0*
15	(a) General cosine curve intersecting x-axis only at (90°,0) and (270°,0)	M1	Allow general shape as the joining of key values, but straight rather than clearly curving towards a turn at 0°, 180° and 360° in particular
	Correct curve with 6 and -6 on y-axis	A1	Must show a clear curve, not straight at turning points
	(b) 99.59…(°) and 260.4059…(°) only	B2	Accept rounded or truncated These values need to be selected, not amongst others unless unambiguously indicated as the response. B1 for sight of 99.59…(°) or 260.4059…(°)
		4	

15	(a) General sin curve intersecting x-axis only at (0°,0), (180°,0) and (360°,0)	M1	Allow general shape as the joining of key values, but straight rather than clearly curving towards a turn at 90° and 270° in particular
	Correct curve with 4 and -4 on y-axis	A1	Must show a clear curve, not straight at turning points
	(b) 14.477…(°) and 165.522…(°) only	B2	Accept rounded or truncated, e.g. 14(°) with 166(°) or 15(°) with 165(°) These values need to be selected, not amongst others unless unambiguously indicated as the response. B1 for sight of 14.477…(°) or 165.522…(°) B0 for a pair of angles with sum 180°
		4	

11	(a) Correct shaped graph with (0°,) 180° & 360° labelled on the x-axis AND 2 & 8 labelled on the y-axis	B3	*Intention for approximately (90°, 5) and (270°, 5)* B2 awarded a for correct shape graph with conditions: • cosx reflected • with one complete period, labelled 0° to 360° • with difference in y values between maximum and minimum of 6, for their labels OR B1 for a correct shape graph with any two of the 3 bullet points above met *If no marks, award SC1 for a curve through at least 5 correct points across the full range with all other conditions met. Do not accept a parabola or straight lines*
	(b) Maximum value 8 AND Minimum value 2	B2	Accept Maximum (180(°), 8) and Minimum (360(°), 2) Allow unsupported correct responses FT provided at least B2 previously awarded in (a) B1 for either value correct, or for a difference between their max and min of 6, or if their answers are reversed (including Maximum (180(°), 2) and Minimum (360(°), 8))
		5	

			stated in the mark scheme
15	(a) General cos curve intersecting x-axis only at (90°,0) and (270°,0)	M1	Allow general shape as the joining of key values, but straight rather than clearly curving towards a turn at 0° and 360° in particular
	Correct curve with 5 and -5 on y-axis	A1	Must show a clear curve, not a straight a 0° and 360° in particular
	(b) 90° and 270° only	B1	This values need to be selected, not amongst others unless unambiguous indicated as the response.
		3	

		3	
15	(a) General sine curve through (0,0), (180,0) and (360,0) only Correct, sketch with 4 and -4 on y-axis (b) 0°, 180° and 360° only	B1 B1 B1 3	

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16	(a) (y =) 4sin3x selected (b)(i) -1 (ii) 18(°) and 90(°) with no other angles given	B1 B1 B2 4	 CAO B1 for either 18(°) or 90(°). Accept embedded answers

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14	(a) General sine curve through (0,0), y values ±3 Sine curve with period clearly 180 (b) 9.7° and 80.3° only	B1 B1 B3	Must have clear the intention of ±3 B2 for any 1 correct **or** 9.8 with 80.2, **or** 9.7.. with 80.2.. (un)rounded or truncated to 2 or more decimal places

End of solutions