



Integration

Mark schemes for the Integration question pack

WJEC Level 2 Additional Mathematics (9550) · Calculus

Official WJEC mark schemes for the 15 questions in the matching revise.wales question pack (132 marks total), from the 2011–2024 papers. Pack layout © revise.wales.

7	$55x^{11}/11 - 6x - 10x^{-5}/-5$ $5x^{11} - 6x + 2x^{-5}$ or $5x^{11} - 6x + 2/x^5$ + c (constant)	 B3 B1 B1 5	B1 for each term ISW from correct unsimplified form. CAO simplified form Awarded only if at least B 1 is awarded for integration
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11	<p>(a) $14x^7/7 - 5x + 4x^2/-2$ $2x^7 - 5x - 2x^2$ or $2x^7 - 5x - 2/x^2$ $+ c$ (constant)</p> <p>(b) $8x^4/4 + 4x^2/2$ or $2x^4 + 2x^2$</p> <p>Use of correct limits 3 & 2 in correct order and intention to subtract</p> <p>$((2 \times 81 + 2 \times 9) - (2 \times 16 + 2 \times 4) =)$</p> <p>140</p>	<p>B3</p> <p>B1</p> <p>B1</p> <p>M2</p> <p>m1</p> <p>A1</p> <p>9</p>	<p>B1 for each term ISW from correct unsimplified form.</p> <p>CAO simplified form. Mark final answer</p> <p>Awarded only if at least B 1 is awarded for integration</p> <p><i>No workings, no marks</i> Ignore sight of '+c' for M marks only M1 one term correct.</p> <p>CAO. Must be from correct working</p>
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12	(a) $16x^8/8 + 15x^5/5 - 4x + 6x^{-3}/-3$	B4	B1 for each term ISW from correct unsimplified form.
	$2x^8 + 3x^5 - 4x - 2x^{-3}$ or $2x^8 + 3x^5 - 4x - 2/x^3$ + c (constant)	B1 B1	CAO simplified form. Mark final answer Awarded only if at least B1 is awarded for integration
	(b)		<i>No workings, no marks</i>
	$3x^3/3 + 2x^2/2$ or $x^3 + x^2$	M2	Ignore sight of '+c. M1 one term correct.
	Use of correct limits 3 & 2 in correct order and intention to subtract	m1	FT from M2 or M1
	24	A1 10	CAO. Must be from correct working

<p>11</p>	<p>(a) Correct shaped graph with (0°,) 180° & 360° labelled on the x-axis AND 2, 7 & 12 labelled on the y-axis</p> <p>(b) Maximum value 12 AND Minimum value 2</p>	<p>B3</p> <p>B1</p> <p>4</p>	<p>Ignore outside the required range</p> <p><i>Intention for approximately (0°, 7), (90°, 2), (180°, 7), (270°, 12) and (360°, 7)</i></p> <p>B2 awarded a for correct shape graph with conditions:</p> <ul style="list-style-type: none"> • $\sin x$ reflected • with one complete period, labelled 0° to 360° • with difference in y values between maximum and minimum of 10, for their labels <p>OR</p> <p>B1 for a correct shape graph with any 2 of the 3 bullet points above met, OR</p> <p>B1 for a graph with all 3 bullet points above met but joined by straight lines (even if turning points curved), OR</p> <p>B1 for a curved graph through intended points: (0°, 7), (90°, 2), (180°, 7), (270°, 12) and (360°, 7)</p> <p>Accept Maximum (270°, 12) and Minimum (90°, 2)</p> <p>Allow unsupported correct responses</p> <p>FT provided at least B2 previously awarded in (a)</p>
<p>12</p>	<p>(a) $(\frac{dy}{dx}=) 16x^7 + 8x$ $(\frac{d^2y}{dx^2}=) 112x^6 + 8$</p> <p>(b) $(\frac{5}{5}) x^5 + (\frac{3}{-1}) x^1 + (-2/-2)x^2$ $(= x^5 - 3x^1 + x^2)$ + c (constant)</p> <p>(c) $6x^2/2 + 10x$ [$6x^2/2 + 10x$]² and with intention to substitute and subtract</p> <p>$= (6 \times 3^2/2 + 10 \times 3) - (6 \times 2^2/2 + 10 \times 2)$ $(= 57 - 32)$</p> <p style="text-align: right;">= 25</p>	<p>B1</p> <p>B1</p> <p>B3</p> <p>B1</p> <p>B2</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>11</p>	<p>FT to 2nd B1 from $\frac{dy}{dx} = kx^n (+ \dots)$</p> <p>B1 for each term. Accept unsimplified. ISW</p> <p>Award if at least B1 given for integration</p> <p>B1 for $6x^2/2$ or $10x$</p> <p>Intention to use 3, 2 (in either order) and subtract FT their integration, not the same terms as given or differentiated, this includes if there is only 1 term seen.</p> <p>FT for correct use of limits provided working with 2 terms from 'their integration'</p> <p>CAO, not FT.</p> <p><i>Answer only, no working shown, MOAOAO</i></p>
<p>13</p>	<p>(When $x = 2$) $y = 27$ (Gradient when $x = 2$, $\frac{dy}{dx} =) 5 \times 2x$ 20</p> <p>Equation $\frac{y - 27}{x - 2} = 20$ or $27 = 20 \times 2 + c$ $y - 27 = 20(x - 2)$ or $c = -13$ $y = 20x - 13$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>m1</p> <p>A1</p> <p>6</p>	<p>For differentiation, before substitution of $x = 2$</p> <p>FT values for 'their 27' and 'their 20' provided at least one of these is correct.</p> <p>Implies previous M1</p> <p>CAO. Mark final answer</p>
<p>14</p>	<p>Method to solve simultaneously, e.g. use of $y = 2x + 1$ or $x = (y - 1)/2$ into the first equation</p> <p>$x^2 - 7x + 12 = 0$ or $y^2 - 16y + 63 = 0$</p> <p>$(x - 3)(x - 4) (=0)$ or $(y - 9)(y - 7) (=0)$</p> <p>(3, 7) and (4, 9)</p>	<p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>4</p>	<p>$2x + 1 = x^2 - 5x + 13$ or $y = \frac{(y - 1)^2}{2} - 5\frac{(y - 1)}{2} + 13$</p> <p>Or equivalent but must '$=0$' or implied in further working</p> <p>OR $x = (7 \pm \sqrt{1})/2$ or $y = (16 \pm \sqrt{4})/2$</p> <p>FT from their quadratic</p> <p>CAO</p> <p>Need not be in this form, accept $x=3, y=7$ with $x=4, y=9$</p> <p>x & y values must be given</p> <p>Do not accept unsupported responses</p> <p>Do not accept trial & improvement</p>

10	<p>(a) $480x^{14}$</p> <p>(b) For sight of $(dy/dx \Rightarrow) 3ax^2 + 2bx + c$ OR $(y \Rightarrow) \frac{12x^3}{3} + \frac{4x^2}{2} + x$ (+ constant)</p> <p style="text-align: right;">a = 4 b = 2 c = 1 d = 3</p>	<p style="text-align: center;">3</p> <p>B2</p> <p>B1</p> <p>B3</p> <p style="text-align: center;">6</p>	<p>B1 for sight of $32x^{15}$. FT to 2nd B1 from $dy/dx = kx^n$ Ignore incorrect notation</p> <p>May be implied by 2 or 3 correct values</p> <p>B2 for any 2 or 3 values correct, or B1 for 1 value correct However, do not award for $c = 1$ if $a = 12$ and $b = 4$</p> <p>For 'd' FT from 'their 10 – a – b – c' <i>Accept sight of correct answers from 'uncorrected' working</i> <i>Only accept embedded answers if clearly stated unambiguously</i></p>
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12	$12x^6/6 + 24x^4/4 - 2x + 4x^{-2}/-4$ $2x^6 + 6x^4 - 2x - x^{-2}$ or $2x^6 + 6x^4 - 2x - 1/x^2$ + c (constant)	11 B4 B1 B1 6	B1 for each term ISW from correct unsimplified form. CAO simplified form Awarded only if at least B1 is awarded for integration
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8	<p>(a) $1140x^{18}$</p> <p>(b) $a = 3$ $b = 2$ $c = -6$</p>	<p>10</p> <p>B2</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>5</p>	<p>B1 for sight of $60x^{19}$. FT to 2nd B1 from $dy/dx = kx^n$</p> <p>B0 for 60^{19} or 1140^{18}</p> <p>Ignore incorrect notation</p> <p><i>Accept sight of correct answers from 'uncorrected' working</i></p> <p><i>Do not accept embedded answers, candidates need to identify values for a, b and c, not accept as left in working without clearly stating.</i></p>
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Summer 2017			
10	(a) $10x^5/5 + 24x^3/3 - 2x + 3x^{-3}/-3$	B4	B1 for each term ISW from correct unsimplified form. Simplified form is $2x^5 + 8x^3 - 2x - x^{-3}$
	$+ c$ (constant)	B1	Awarded only if at least B1 is awarded for integration
	(b) $12x^4/4 + 6x^3/3$	B2	Mark final answer, then FT. B1 for sight of $12x^4/4$ or $6x^3/3$ Ignore inclusion of '+c' throughout' (except final A mark)
	$[12x^4/4 + 6x^3/3]^2_1$	M1	FT their integration, not original.
	$=(3 \times 2^4 + 2 \times 2^3) - (3 \times 1^4 + 2 \times 1^3)$	A1	Intention to use 2, 1 and subtract FT for correct use of limits
	$= 59$	A1	Accept unsimplified fractions included CAO, not FT. Do not accept '59 + c'
		A1	<i>Answer only, no working shown MO A0 A0</i>
		10	

		3	
12	(a) $(dy/dx=) 21x^6+4$ $(d^2y/dx^2=) 126x^5$	B1 B1	Accept sight of $21x^6+4$ FT to 2 nd B1 from $dy/dx = kx^n (+ m)$
	(b) $(4/4) x^4 + (2/2)x^2 + (4/-1) x^{-1}$ + c (constant)	B3 B1	B1 for each term. Accept unsimplified. ISW Award if at least B1 given for integration
	(c) $8x^2/2 + 2x$ [$8x^2/2 + 2x$] ₂ ³ and with intention to substitute and subtract	B2 M1	B1 for $8x^2/2$ or $2x$ Intention to use 3, 2 (in either order) and subtract FT their integration, not the same terms as given or differentiated, this includes if there is only 1 term seen.
	$= (8 \times 3^2/2 + 2 \times 3) - (8 \times 2^2/2 + 2 \times 2) (= 42 - 20)$	A1	FT for correct use of limits provided working with 2 terms from 'their integration'
	$= 22$	A1 11	CAO, not FT. <i>Answer only, no working shown, M0 A0 A0</i>

	Additional Mathematics Summer 2016		Final
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	$(x-2)(x+3)(x+1)$	\circ	
8	(a) $180x^8$ (b) $a = 1$ $c = 5$ $b = 3$	B2 B1 B1 B1	B1 for sight of $20x^9$. FT to 2^{nd} B1 from $dy/dx = kx^n$ Ignore incorrect notation FT $b = 8 - \text{'their c'}$ or $b = 4 - \text{'their a'}$ <i>Accept sight of correct answers from 'uncorrected' working</i> <i>Do not accept embedded answers, candidates need to identify values for a, b and c, not accept as left in working without clearly stating.</i>
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WJEC Additional Mathematics
Summer 2015

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Final

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Summer 2015			
9	(a) $21x^7/7 - 3x^3/3 - x^{-1}/-1 + 6x$ + c (constant)	B4 B1	B1 for each term. Accept unsimplified $-x^{-1}/-1$. ISW Awarded if at least B1 for integration
	(b) $6x^3/3 + 4x^2/2$	B2	Mark final answer, then FT. B1 for sight of $6x^3/3$ or $4x^2/2$
	$[6x^3/3 + 4x^2/2]_2^5$	M1	FT their integration, not original. Intention to use 5, 2 and subtract
	$=(2 \times 5^3 + 2 \times 5^2) - (2 \times 2^3 + 2 \times 2^2)$	A1	FT for correct use of limits Accept unsimplified fractions included
	$= 276$	A1	CAO, not FT. Do not accept '276 + c' Answer only, no working shown M0 A0 A0
		10	

	$(x \Rightarrow) \text{D.O (CIII) and } (x \Rightarrow) \text{I.O (CIII)}$	<i>i</i>	<i>Final and improvement methods are not accepted</i>
12	(a) $12x^3 + 3$ $60x^4$	B1	
	(b) $(3/5)x^5 + (6/2)x^2 + (8/-1)x^{-1}$ $+ c$	B1 B3 B1 B2	FT to 2 nd B1 from $dy/dx = kx^n (+ m)$ B1 for each term. Accept unsimplified. ISW Award if at least B1 given for integration B1 for $4x^2/2$ or x
	(constant)	M1	FT their <u>integration</u> . Intention to use 5, 2 and
	(c) $4x^2/2 + x$	A1	subtract
	$[4x^2/2 + x]_2^5$	A1	FT for correct use of limits
	$= (4 \times 5^2/2 + 5) - (4 \times 2^2/2 + 2) (= 55 - 10)$ $= 45$	I1	CAO, not FT.
			<i>Answer only, no working shown, M0 A0 A0</i>

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~~EXAMINER USE ONLY~~
Summer 2014

| Marks |

Final

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7	(a) $432x^7$	0	A1 $x = 1.50, y = 2.50$ and $x = -2.50, y = -1.50$
	(b) $\frac{3}{5}x^5 - 1/(2x^2) + 4x$ + c (constant)	B2	B1 for sight of $54x^8$. FT to 2 nd B1 from $dy/dx = kx^n$
	(c) $6x^6/6 + 5x$ $[6x^6/6 + 5x]^3_2$ $= (3^6 + 15) - (2^6 + 10)$ $= 670$	B3 B1	B1 for each term. Accept unsimplified $+x^{-2}/-2$ ISW Awarded if at least B1 for integration
		B2 M1	B1 for $6x^6/6$ or $5x$ FT their integration, not original. Intention to use 3, 2 and subtract
		A1 A1 11	FT for correct use of limits CAO, not FT. <i>Answer only, no working shown M0 A0 A0</i>

	Additional Mathematics Summer 2012	Final Mark Scheme
8	(a) $\frac{24x^3+4}{72x^2}$	B1 B1 FT to 2 nd B1 from $dy/dx = kx^n + a$, equivalent level of difficulty
	(b) $(3/3)x^3 + 4/(-2x^2) + (8/2)x^2 + c$ (constant)	B3 B1 for each term. Accept unsimplified. ISW B1 Award if at least B1 given for integration
	(c) $\frac{6x^2}{2} + x$ $[6x^2/2 + x]_2^4$ $= (3 \times 4^2 + 4) - (3 \times 2^2 + 2)$ $= 38$	B2 B1 for $6x^2/2$ or x M1 FT their integration not use of $6x + 1$. Intention to use 4, 2 and subtract A1 FT for correct use of limits A1 CAO, not FT. 11 <i>Answer only no working shown M0 A0 A0</i>

9	<p>(a) $280x^6$</p> <p>(b) $4/7 x^7 - 1/x + 9x$ + c (constant)</p> <p>(c) $3x^3/3 + x$ $[3x^3/3 + x]_1^2$ $= (2^3 + 2) - (1^3 + 1)$ $= 8$</p>	<p>B2 B1 for sight of $40x^7$. FT to 2^{nd} B1 from $dy/dx = kx^n$</p> <p>B3 B1 for each term. Accept unsimplified $(-x^{-1}$ or $+x^{-1}/-1)$ ISW</p> <p>B1 Awarded if at least B1 for integration</p> <p>B2 B1 for $3x^3/3$ or x. Mark final answer</p> <p>M1 FT their <u>integration</u>. Intention to use 2, 1 and subtract</p> <p>A1 FT for correct use of limits</p> <p>A1 CAO, not FT. <i>Answer only, no working shown MOAOAO</i></p>
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End of solutions