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Stationary points

Mark schemes for the Stationary points question pack

WJEC Level 2 Additional Mathematics (9550) · Calculus

Official WJEC mark schemes for the 11 questions in the matching revise.wales question pack (77 marks total), from the 2011–2024 papers. Pack layout © revise.wales.

15	$(dy/dx=) 6x^2 - 12x$	B1	
	$dy/dx = 0$ or $6x^2 - 12x = 0$	M1	FT their dy/dx form $ax^2 + bx$ throughout
	$x = 0$ and $y = -7$	A1	
	$x = 2$ and $y = -15$	A1	If A0, A0 here, award A1 for $x = 0$ with $x = 2$ *Answer only, no working shown M0 A0 A0*
	$d^2y/dx^2 = 12x - 12$	M1	*Method for determining min or max MUST be shown, final answer only is M0 here, then A0,A0* Or first derivative test, interpretation of first derivative test. Or alternative.
	At (0, -7): $d^2y/dx^2 < 0$, point is a maximum	A1	FT for their x value (ignore y-values)
	At (2, -15): $d^2y/dx^2 > 0$, point is a minimum	A1	FT for their other x value provided this does not have the same interpretation as the first x value (ignore y-values) *SC1 for correct FT from $d^2y/dx^2 = ax + b$, $a>0$, including allowed FT from $d^2y/dx^2 = 12x$, with $x=0$ as maximum and $x=2$ as a minimum (despite $d^2y/dx^2 = 0$)* *Do not accept trial & improvement methods unless both stationary points are found correctly and confirmed as stated in the mark scheme*
		7	

13	(dy/dx=) $\frac{3x^2}{3} + 2x - 15$ or $x^2 + 2x - 15$	B1	Must not be from sight of an incorrect equation, i.e. finding dy/dx from $y = x^3 + 3x^2 - 45x$ is awarded B0
	$dy/dx = 0$ or $x^2 + 2x - 15 = 0$	M1	FT their dy/dx from $ax^2 + bx$ throughout
	$x = \{-(2) \pm \sqrt{(2^2 - 4\times1\times-15)}\}/2$ or $(x - 3)(x + 5) = 0$	m1	If the quadratic formula used, working must be shown, allow 1 slip in substitution
	$x = 3$ and $x = -5$	A1	*y-coordinates not required*
	$d^2y/dx^2 = 2x + 2$	M1	*Method for determining min or max MUST be shown, final answer only is M0 here, then A0, A0* Or first derivative test, interpretation of first derivative test. Or alternative. FT 'their dy/dx' provided equivalent difficulty
	At $x = -5$, $d^2y/dx^2 < 0$, point is a maximum At $x = 3$, $d^2y/dx^2 > 0$, point is a minimum	A1 A1	FT for 'their x value' FT for 'their other x value' provided this does not have the same interpretation as the first x value *If M0A0A0, award SC1 for correct FT from 'their $d^2y/dx^2 = ax + b$, $a>0$' applied correctly provided it leads to 1 maximum and 1 minimum* *Do not accept trial & improvement methods unless both stationary points are found correctly and confirmed as stated in the mark scheme*
		7	

		10	
13	(dy/dx=) $\frac{3x^2}{3} - \frac{14x}{2} + 10$ or $x^2 - 7x + 10$	B1	
	$dy/dx = 0$ or $x^2 - 7x + 10 = 0$	M1	FT their $dy/dx = ax^2 + bx$ (+ c) throughout for equivalent level of difficulty
	$x = \{-(-7) \pm \sqrt{((-7)^2 - 4\times1\times10)}\} / 2$ or $(x \ldots2)(x\ldots5) = 0$	m1	Allow 1 slip in substitution. Working must be shown
	$x = 2$ with $x = 5$	A1	Both solutions are required
	$d^2y/dx^2 = 2x - 7$	M1	*Method for determining min or max MUST be shown, final answer only is M0 here, then A0,A0* Or first derivative test, interpretation of first derivative test. Or alternative. FT 'their dy/dx' for M1 provided equivalent difficulty
	At $x = 2$, $d^2y/dx^2 < 0$, point is a maximum	A1	FT for 'their x value'
	At $x = 5$, $d^2y/dx^2 > 0$, point is a minimum	A1	FT for 'their other x value' provided this does not have the same interpretation as the first x value *If M0A0A0, award SC1 for correct FT from at least 'their $d^2y/dx^2 = ax + b$, $a>0$' applied correctly provided it leads to 1 maximum and 1 minimum* *Do not accept trial & improvement methods unless both stationary points are found correctly and confirmed as stated in the mark scheme*
		7	

7	(a) $(3)^3 + 8(3)^2 - 2(3) + 6$ (= 27 + 72 - 6 + 6) = 99	M1 A1	Or division method giving $x^2 + 11x$ …
	(b)(i) Substitute x = -3 Showing f(-3) = 0	M1 A1	Or division method giving $x^2 - 2x$ … Convincing from working shown (not if incorrect working seen), allow $(-3)^3 + (-3)^2 - 41(-3) - 105 = 0$, also allow for sight of $-3^3 + -3^2 - 41\times-3 - 105 = 0$ provided no incorrect calculation is given such as -3^2 as -9
	(ii) $(x + 3)(x^2 + bx + c)$ or intention to divide by (x+ 3) with x^2 shown ((x + 3)) (x^2 - 2x - 35) ((x + 3))(x + 5)(x - 7)	M1 A2 A1 8	A1 for -2x or -35. Or use of factor theorem A1 (x+5), A1 (x- 7) CAO. Mark final answer, *but ignore attempts to 'solve'*
8	(dy/dx=) $12x^2 - 6x$ dy/dx = 0 or $12x^2 - 6x = 0$ or $12x^2 = 6x$ x = 0 and y = 20 x = ½ and y = 19¾	B1 M1 A1 A1	FT their dy/dx form $ax^2 \pm bx$ If A0, A0 here, award A1 for x = 0 with x = ½ *Answer only, no working shown M0 A0 A0*
	$d^2y/dx^2 = 24x - 6$ (0, (20)): $d^2y/dx^2 < 0$, point is a maximum (½, (19¾)): $d^2y/dx^2 > 0$, point is a minimum	M1 A1 A1 7	Or first derivative test, interpretation of first derivative test. Or alternative (e.g. full graphical method with explanation) FT for their x value FT for their other x value provided this does not have the same interpretation as the first x value *Answer only, no working shown M0 A0 A0* *If $d^2y/dx^2 = cx + d$ where $c \neq 0$ and test applied correctly then SC2 instead of final A1, A1 (as M1 has not been awarded))provided one minimum and one maximum*
9	$\frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4}$	B1 1	**Working must be shown**
10	(a) $FG^2 = (-4 - 8)^2 + (10 - 28)^2$ ($= 12^2 + 18^2 = 468$) FG = $6\sqrt{13}$	M1 A1	Or equivalent. Allow 1 slip or error M1, A0 for answers $\sqrt{468}$ or 21.6(3...) CAO
	(b) Gradient FG (28-10)/(8 - -4) = 18/12 (= 9/6 = 3/2)	M1 A1	Do not ignore incorrect cancelling, mark final answer
	(c) (-4 + 8)/2 or (10 + 28)/2 Mid point (2, 19)	M1 A1	Sight of (2, …) or (… , 19) implies M1 provided no incorrect working is seen
	Perpendicular gradient -2/3 (or -6/9 or -12/18)	B1	FT -1/ 'their answer in (b)'
	$\frac{y - 19}{x - 2} = \frac{-2}{3}$ or 19 = -2/3 × 2 + c	M1	OR for an alternative correct method of finding the equation of a straight line, for the idea of how an equation of a straight line can be found. FT 'their perpendicular gradient' or 'their answer in (b)' AND 'their mid point' or for 'points F or G' used
	y – 19 = -2/3(x - 2) or 3(y – 19) = -2(x – 2) or 3y = -2x + 61 or c = 20⅓ or c = 61/3	m1	**Do not allow** use gradient from their answer in (b), and/or points F or G as the mid-point of FG. Only FT for 'their perpendicular gradient' (not 'their answer' from (b)) AND 'their mid point'
	2x + 3y – 61 = 0 or -2x – 3y + 61 = 0	A1 10	CAO. Must be in this form with '=0' with terms in any order

13	$(dy/dx =) 6x^2 + 24x$	B1	
	$dy/dx = 0$ or $6x^2 + 24x = 0$	M1	FT their dy/dx from $ax^2 + bx$ throughout
	$x = 0$ and $y = 11$	A1	
	$x = -4$ and $y = 75$	A1	If A0, A0 here, award A1 for $x = 0$ with $x = -4$ *Answer only, no working shown M0 A0 A0*
			Method for determining min or max MUST be shown, final answer only is M0 here, then A0,A0
	$d^2y/dx^2 = 12x + 24$	M1	Or first derivative test, interpretation of first derivative test. Or alternative. FT 'their dy/dx' for M1 provided equivalent difficulty
	At (0, 11) $d^2y/dx^2 > 0$, point is a minimum	A1	FT for 'their x value'
	At (-4, 75): $d^2y/dx^2 < 0$, point is a maximum	A1	FT for 'their other x value' provided this does not have the same interpretation as the first x value *If M0A0A0, award SC1 for correct FT from 'their $d^2y/dx^2 = ax + b$, $a>0$' applied correctly provided it leads to 1 maximum and 1 minimum*
			Do not accept trial & improvement methods unless both stationary points are found correctly and confirmed as stated in the mark scheme
		7	

11	$(dy/dx=) 9x^2 + 18x$	B1	
	$dy/dx = 0$ or $9x^2 + 18x = 0$	M1	FT their dy/dx form $ax^2 + bx$ throughout
	$x = 0$ and $y = 4$	A1	
	$x = -2$ and $y = 16$	A1	*Answer only, no working shown M0 A0 A0* *Method for determining min or max MUST be shown, final answer only is M0 here, then A0,A0*
	$d^2y/dx^2 = 18x + 18$	M1	Or first derivative test, interpretation of first derivative test. Or alternative.
	At $(0, 4)$ $d^2y/dx^2 > 0$, point is a minimum	A1	FT for their x value (ignore y-values)
	At $(-2, 16)$: $d^2y/dx^2 < 0$, point is a maximum	A1	FT for their other x value provided this does not have the same interpretation as the first x value (ignore y-values) *SC1 for correct FT from $d^2y/dx^2 = ax + b$, $a>0$, including allowed FT from $d^2y/dx^2 = 18x$, with $x=-2$ as maximum and $x=0$ as a minimum (despite $d^2y/dx^2 = 0$)* *Do not accept trial & improvement methods unless both stationary points are found correctly and confirmed as stated in the mark scheme*
		7	

8	$(dy/dx=) 3x^2 - 6x$	B1	
	$dy/dx = 0$ or $3x^2 - 6x = 0$ or $3x^2 = 6x$	M1	FT their dy/dx form $ax^2 \pm bx$
	$x = 0$ and $y =$ 11	A1	
		A1	*Answer only, no working shown M0 A0 A0*
	$x = 2$ and $y =$ 7	M1	Or first derivative test, interpretation of first derivative test. Or alternative (e.g. full graphical method with explanation)
	$d^2y/dx^2 = 6x - 6$	A1	FT for their x value
		A1	FT for their other x value provided this does not have the same interpretation as the first x value
	(0, (11)): $d^2y/dx^2 < 0$, point is a maximum (2, (7)): $d^2y/dx^2 > 0$, point is a minimum		*Answer only, no working shown M0 A0 A0* *If $d^2y/dx^2 = cx + d$ where $c \neq 0$ and test applied correctly then SC2 instead of final A1, A1 (as M1 has not been awarded))*
		7	

	Summer 2015		
14	$(dy/dx=)\ 6x^2 - 24$	B1	
	$dy/dx = 0$ or $6x^2 - 24 = 0$	M1	FT their dy/dx form $ax^2 + b$ throughout
	$x = 2$ and $y = -$	A1	
	19	A1	*Answer only, no working shown M0 A0 A0*
	$x = -2$ and $y =$		
	45		*Method for determining min or max MUST be shown, final answer only is M0 here, then A0,A0*
		M1	Or first derivative test, interpretation of first derivative test. Or alternative.
	$d^2y/dx^2 = 12x$	A1	FT for their x value
		A1	FT for their other x value provided this does not have the same interpretation as the first x value
	At (2, -19) $d^2y/dx^2 > 0$, point is a minimum At (-2, 45): $d^2y/dx^2 < 0$, point is a maximum		*SC1 for correct FT from $d^2y/dx^2 = ax$, $a>0$*
			Do not accept trial & improvement methods unless both stationary points are found correctly and confirmed as stated in the mark scheme
		7	

	Summer 2014		
8	$(dy/dx=)\ 12x^2 - 12$	B1	
	$dy/dx = 0$ or $12x^2 - 12 = 0$	M1	FT their dy/dx form $ax^2 + b$
	$x = 1$ and $y = -1$	A1	
		A1	*Answer only, no working shown M0 A0 A0*
	$x = -1$ and $y = 15$	M1	Or first derivative test, interpretation of first derivative test. Or alternative.
	$d^2y/dx^2 = 24x$	A1	FT for their x value
		A1	FT for their other x value provided this does not have the same interpretation as the first x value
	$(-1, (15))$: $d^2y/dx^2 < 0$, point is a maximum $(1, (-1))$: $d^2y/dx^2 > 0$, point is a minimum		*Answer only, no working shown M0 A0 A0* *If $d^2y/dx^2 = nx$ where $n \neq 0$ and test applied correctly then SC2 instead of final A1, A1 (as M1 has not been awarded)*
		7	

	Summer 2013		
12	$(dy/dx=)\ 9x^2 - 36$	B1	
	$dy/dx = 0$ or $9x^2 - 36 = 0$	M1	FT their dy/dx form $ax^2 + b$
	$x = 2$ and $y = -37$	A1	
	$x = -2$ and $y = 59$	A1	*Answer only, no working shown M0 A0 A0*
			Method for determining min or max MUST be shown, final answer only is M0 here, then A0,A0
	$d^2y/dx^2 = 18x$	M1	Or first derivative test, interpretation of first derivative test. Or alternative.
	At (2, -37) $d^2y/dx^2 > 0$, point is a minimum	A1	FT for their x value
	At (-2, 59): $d^2y/dx^2 < 0$, point is a maximum	A1	FT for their other x value provided this does not have the same interpretation as the first x value
		7	*SC1 for correct FT from $d^2y/dx^2 = ax$, $a>0$*

		11	
10	$(dy/dx=) 6x^2 - 6$	B1	
	$dy/dx = 0$ or $6x^2 - 6 = 0$	M1	FT their dy/dx form $ax^2 + b$
	$x = 1$ and $y = 1$	A1	
	$x = -1$ and $y = 9$	A1	*Answer only, no working shown M0A0A0*
	$d^2y/dx^2 = 12x$	M1	Or first derivative test, interpretation of first derivative test. Or alternative.
	$(-1, (9))$: $d^2y/dx^2 < 0$, point is a maximum	A1	FT for their x value
	$(1, (1))$: $d^2y/dx^2 > 0$, point is a minimum	A1	FT for their other x value provided this does not have the same interpretation as the first x value *Answer only, no working shown M0A0A0* *If $d^2y/dx^2 = nx$ where $n \neq 0$ and test applied correctly then SC2*
		7	

	WJEC Level 2 Certificate in Additional Mathematics Summer 2011	Mark	Comments (Final)

End of solutions