

**REVISE**

.wales

## **Differentiation from first principles**

*Mark schemes for the Differentiation from first principles question pack*

*WJEC Level 2 Additional Mathematics (9550) · Calculus*

Official WJEC mark schemes for the 12 questions in the matching revise.wales question pack (64 marks total), from the 2011–2024 papers. Pack layout © revise.wales.

6	$y + \delta y = 11(x + \delta x)^2 + 2(x + \delta x)$ <p>Intention to subtract <math>(y =) 11x^2 + 2x</math> to find <math>\delta y</math></p> $22x\delta x + 11(\delta x)^2 + 2\delta x$ <p>Dividing by <math>\delta x</math> <b>and</b> <math>(\lim) \delta x \rightarrow 0</math></p> $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = 22x + 2$	<p>B1 Or alternative notation</p> <p>M1</p> <p>A1 Accept <math>\delta x^2</math> as meaning <math>(\delta x)^2</math></p> <p>M1 FT equivalent level of difficulty</p> <p>A1 CAO. Must follow from correct working</p> <p><i>Use of <math>dy/dx</math> throughout or incorrect notation then possible maximum is only 4 marks, final A0</i></p> <p>5</p>
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14	$y + \delta y = 3(x + \delta x)^2 + 4$ Intention to subtract $(y =) 3x^2 + 4$ to find $\delta y$ $\delta y = 6x\delta x + 3(\delta x)^2$ Dividing by $\delta x$ <b>and</b> $(\lim) \delta x \rightarrow 0$ $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = 6x$	B1 M1 A1 M1 A1 5	Or alternative notation. Accept $\delta x^2$ as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working <i>Use of <math>dy/dx</math> throughout or incorrect notation then possible maximum is only 4 marks, final A0</i>
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<p>3</p>	$\{ 55(x) - 22(x+3) + 10(x+5) \} \quad (/110)$ $\{ 55x - 22x - 66 + 10x + 50 \} \quad (/110)$ $(43x - 16)/110 \text{ or showing LHS} \equiv \text{RHS}$	<p>M1 B1 B1 A1</p>	<p>Attempt to use common denominator, may be implied by sight of <math>55(x) - 22(x+3) + 10(x+5)</math> without sight of /110 May be seen in stages</p> <p>Or equivalent. May be seen in stages, as intention of method</p> <p>B1 for 1 slip (e.g. +66). Must be as a sum of 5 terms. Convincing <b>must</b> follow from fully correct working at each stage Allow following sight of 3 separate correct fractions with denominator 110 seen</p> <p><i>If no denominator then possible M1 (see note above), B1 B1 A0, however if denominator replaced later all marks are allowable</i></p>
<p>4</p>	<p>(a) <math>(y+\delta y =) \quad (x+\delta x)^2 + 7(x+\delta x) + 2</math> Intention to subtract <math>(y =) x^2 + 7x + 2</math> to find <math>\delta y</math> <math>(\delta y =) \quad 2x\delta x + (\delta x)^2 + 7\delta x</math> Dividing by <math>\delta x</math> <b>and</b> <math>(\lim) \delta x \rightarrow 0</math> <math>dy/dx = \lim_{\delta x \rightarrow 0} \delta y/\delta x = 2x + 7</math></p>	<p>B1 M1 A1 M1 A1</p>	<p>Or alternative notation. Allow if final bracket omitted</p> <p>Accept <math>\delta x^2</math> as meaning <math>(\delta x)^2</math> FT equivalent level of difficulty CAO. Must follow from correct working and notation All notation throughout the working must be correct in order to award the final A1 Do not accept <math>dy/dx = \lim_{x \rightarrow 0} 2x + 7</math> as a final answer <i>Use of <math>dy/dx</math> throughout max 4 marks only, final A0</i></p>

<p>5</p>	<p>Overall strategy that could lead to finding <math>\hat{E}CB</math>, e.g. length of 3 sides and then cosine rule  <math>EC^2 = 6.2^2 + 3.7^2</math>  <math>BC^2 = 2.5^2 + (8.4 - 6.2)^2</math>  <math>EB^2 = 8.4^2 + 3.7^2 + 2.5^2</math></p> <p>With substituted values:  <math>\cos \hat{E}CB = \frac{EC^2 + BC^2 - EB^2}{2 \times EC \times BC}</math>  i.e.  <math>\cos \hat{E}CB = \frac{52.13 + 11.09 - 90.5}{2 \times 7.22... \times 3.33...}</math> (= - 0.567...)</p> <p><math>\hat{E}CB = 124.56(...^\circ)</math> or <math>124.6^\circ</math> or <math>125^\circ</math></p> <p>QWC2: Candidates will be expected to</p> <ul style="list-style-type: none"> <li>present work clearly, with words explaining process or steps</li> </ul> <p>AND</p> <ul style="list-style-type: none"> <li>make few if any mistakes in mathematical form, spelling, punctuation and grammar in their answer</li> </ul> <p>QWC1: Candidates will be expected to</p> <ul style="list-style-type: none"> <li>present work clearly, with words explaining process or steps</li> </ul> <p>OR</p> <ul style="list-style-type: none"> <li>make few if any mistakes in mathematical form, spelling, punctuation and grammar in their final answer</li> </ul>	<p>S1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>M2</p> <p>A1</p> <p>QWC 2</p> <p>9</p>	<p>Or full alternative strategy</p> <p>(<math>EC^2 = 52.13</math>, <math>EC = 7.22...cm</math>)  (<math>BC^2 = 11.09</math>, <math>BC = 3.33...cm</math>)  May be shown in stages  (e.g. <math>BF^2 = 8.4^2 + 3.7^2</math> then <math>EB^2 = 2.5^2 + BF^2</math>)  (<math>EB^2 = 90.5</math>, <math>EB = 9.513...cm</math>)</p> <p>OR alternative full method, e.g. finding angles BEC or EBC using cosine rule followed by use of sine rule with <math>\sin \hat{E}CB</math> isolated  FT 'their derived lengths' provided at least 2 M marks previously awarded</p> <p>M1 for substituted values:  <math>EB^2 = EC^2 + BC^2 - 2 \times EC \times BC \times \cos \hat{E}CB</math>  OR for alternative full method without <math>\sin \hat{E}CB</math> isolated</p> <p>CAO, must be from correct working  Allow <math>124.4(...^\circ)</math>, <math>124.48^\circ</math>, <math>124.5^\circ</math> or <math>125.39(...^\circ)</math> or <math>125.4^\circ</math> from premature approximation</p> <p>QWC2 Presents relevant material in a coherent and logical manner, using acceptable mathematical form, and with few if any errors in spelling, punctuation and grammar.</p> <p>QWC1 Presents relevant material in a coherent and logical manner but with some errors in use of mathematical form, spelling, punctuation or grammar OR  evident weaknesses in organisation of material but using acceptable mathematical form, with few if any errors in spelling, punctuation and grammar.</p> <p>QWC0 Evident weaknesses in organisation of material, and errors in use of mathematical form, spelling, punctuation or grammar.</p>
<p>6</p>	<p>(a) Multiplier <math>(6-\sqrt{3}) / (6-\sqrt{3})</math></p> <p>Denominator  <math>36 + 6\sqrt{3} - 6\sqrt{3} - 3</math> OR <math>36 - 3</math> OR <math>33</math>  <math>\frac{12 - 2\sqrt{3}}{33}</math></p> <p>(b)(i) <math>y^{1/5}/y^{3/4}</math> or alternative correct 1<sup>st</sup> step  <math>y^{-11/20}</math> or <math>1/y^{11/20}</math></p> <p>(ii) Correctly extracting <math>x^{2\eta}</math> as a factor, or  <math>\frac{x^{2\eta}}{2x^{2\eta}} + \frac{6x^{3\eta}}{2x^{2\eta}}</math>  <math>\frac{1}{2} + 3x^{1\eta}</math> or <math>\frac{1 + 6x^{1\eta}}{2}</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>7</p>	<p>Allow if the multiplier is stated as <math>(6-\sqrt{3})</math> provided it is used as <math>(6-\sqrt{3}) / (6-\sqrt{3})</math></p> <p>CAO. Mark final answer  <i>Unsupported answer is awarded no marks.</i></p> <p>Or equivalent first stage of working with indices  CAO. Mark final answer</p> <p>CAO. Mark final answer</p>

14	$y + \delta y = (x + \delta x)^2 + 13(x + \delta x)$ <b>Intention to subtract</b> $(y =) x^2 + 13x$ to find $\delta y$ $\delta y = 2x\delta x + (\delta x)^2 + 13\delta x$ Dividing by $\delta x$ <b>and</b> $(\lim_{\delta x \rightarrow 0})$ $dy/dx = \lim_{\delta x \rightarrow 0} \delta y / \delta x = 2x + 13$	B1 M1 A1 M1 A1  5	Or alternative notation. Allow if final bracket omitted Accept $\delta x^2$ as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working <i>Use of <math>dy/dx</math> throughout or incorrect notation then possible maximum is only 4 marks, final A0</i>
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12	$y + \delta y = (x + \delta x)^2 + 10(x + \delta x)$ <b>Intention to subtract</b> $(y =) x^2 + 10x$ to find $\delta y$ $\delta y = 2x\delta x + (\delta x)^2 + 10\delta x$ Dividing by $\delta x$ <b>and</b> $(\lim) \delta x \rightarrow 0$ $dy/dx = \lim_{\delta x \rightarrow 0} \delta y / \delta x = 2x + 10$	/ B1 M1 A1 M1 A1  5	Or alternative notation. Allow if final bracket omitted  Accept $\delta x^2$ as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working <i>Use of <math>dy/dx</math> throughout or incorrect notation then possible maximum is only 4 marks, final A0</i>
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4	<p>(a) <math>(y+\delta y =) \quad (x+\delta x)^2 + 3(x+\delta x)</math>                  Intention to subtract <math>(y =) x^2 + 3x</math> to find <math>\delta y</math>  <math>(\delta y =) \quad 2x\delta x + (\delta x)^2 + 3\delta x</math>                  Dividing by <math>\delta x</math> <b>and</b> <math>(\lim) \delta x \rightarrow 0</math>  <math>dy/dx = \lim_{\delta x \rightarrow 0} \delta y/\delta x = 2x + 3</math></p>	<p>4                  B1                  M1                  A1                  M1                  A1                  5</p>	<p><i>marks are allowable</i>                  Or alternative notation. Allow if final bracket omitted                  Accept <math>\delta x^2</math> as meaning <math>(\delta x)^2</math>                  FT equivalent level of difficulty                  CAO. Must follow from correct working and notation                  All notation throughout the working must be correct in order to award the final A1                  Do not accept <math>dy/dx = \lim_{x \rightarrow 0} 2x + 3</math> as a final answer                  Use of <math>dy/dx</math> throughout max 4 marks only, final A0</p>
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	Additional Mathematics Summer 2016		Final
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12	$y + \delta y = (x + \delta x)^2 - 3(x + \delta x)$ <b>Intention to subtract</b> $(y =) x^2 - 3x$ to find $\delta y$ $\delta y = 2x\delta x + (\delta x)^2 - 3\delta x$ Dividing by $\delta x$ <b>and</b> $(\lim) \delta x \rightarrow 0$ $dy/dx = \lim_{\delta x \rightarrow 0} \delta y/\delta x = 2x - 3$	/ B1 M1 A1 M1 A1 /	Or alternative notation. Allow if final bracket omitted Accept $\delta x^2$ as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working <i>Use of <math>dy/dx</math> throughout or incorrect notation then possible maximum is only 4 marks, final A0</i>
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5	<p>(a) <math>(y + \delta y =) \quad (x + \delta x)^2 + 8(x + \delta x)</math>  Intention to subtract <math>(y =) x^2 + 8x</math> to find <math>\delta y</math>  <math>(\delta y =) \quad 2x\delta x + (\delta x)^2 + 8\delta x</math>  Dividing by <math>\delta x</math> and <math>\lim_{\delta x \rightarrow 0}</math>  <math>\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = 2x + 8</math></p>	<p>4  B1  M1  A1  M1  A1  5</p>	<p><i>denominator replaced with all marks are allowable</i>  Or alternative notation. Allow if final bracket omitted  Accept <math>\delta x^2</math> as meaning <math>(\delta x)^2</math>  FT equivalent level of difficulty  CAO. Must follow from correct working and notation  All notation throughout the working must be correct in order to award the final A1  <i>Use of <math>dy/dx</math> throughout max 4 marks only final A1</i></p>
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		<i>mark</i>	<i>strategy.</i>
10	$y + \delta y = (x + \delta x)^2 - 4(x + \delta x)$ Intention to subtract $(y =) x^2 - 4x$ to find $\delta y$ $\delta y = 2x\delta x + (\delta x)^2 - 4\delta x$ Dividing by $\delta x$ and $(\lim_{\delta x \rightarrow 0}) \delta x \rightarrow 0$ $dy/dx = \lim_{\delta x \rightarrow 0} \delta y / \delta x = 2x - 4$	B1 M1 A1 M1 A1 5	Or alternative notation. Allow if final bracket omitted Accept $\delta x^2$ as meaning $(\delta x)^2$ FT equivalent level of difficulty CAO. Must follow from correct working Use of $dy/dx$ throughout max 4 marks only, final A0

15	(a) $(y+\delta y =) \quad (x+\delta x)^2 - 5(x+\delta x)$	B1	<i>answers, use of <math>\pi</math> is incorrect</i> Or alternative notation. Allow if final bracket omitted
	<p>Intention to subtract <math>(y =) x^2 - 5x</math> to find <math>\delta y</math></p> <p><math>(\delta y =) \quad 2x\delta x + (\delta x)^2 - 5\delta x</math></p> <p>Dividing by <math>\delta x</math> and letting <math>\delta x \rightarrow 0</math></p> $dy/dx = \lim_{\delta x \rightarrow 0} \delta y/\delta x = 2x - 5$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Accept <math>\delta x^2</math> as meaning <math>(\delta x)^2</math></p> <p>CAO. Notation needs to be accurate <i>Use of <math>dy/dx</math> throughout max 4 marks only, final A0</i></p>
	(b) $2x - 5 = 15$ $x = 10$	<p>M1</p> <p>A1</p> <p>?</p>	FT from their response in (a) into (b)

	their answer		
8	<p>(a) <math>y + \delta y = (x + \delta x)^2 + 2(x + \delta x)</math>            Intention to subtract <math>(y =) x^2 + 2x</math> to find <math>\delta y</math>  <math>\delta y = 2x\delta x + (\delta x)^2 + 2\delta x</math>            Dividing by <math>\delta x</math> and <math>\lim_{\delta x \rightarrow 0}</math>  <math>\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \delta y / \delta x = 2x + 2</math></p> <p>(b) <math>2x + 2 = 12</math>  <math>x = 5</math></p>	<p>B1            M1            A1            M1            A1            M1            A1            7</p>	<p>Or alternative notation. Allow if final bracket omitted</p> <p>Accept <math>\delta x^2</math> as meaning <math>(\delta x)^2</math></p> <p>FT equivalent level of difficulty</p> <p>CAO. Must follow from correct working and notation</p> <p>Use of <math>dy/dx</math> throughout max 4 marks only, final AO</p> <p>FT from their response in (a) into (b)</p>

End of solutions