REVISE
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Intersection of a curve and a line

Mark schemes for the Intersection of a curve and a line question pack

WJEC Level 2 Additional Mathematics (9550) · Coordinate geometry

Official WJEC mark schemes for the 9 questions in the matching revise.wales question pack (48 marks total), from the 2011–2024 papers.

8	$\frac{4x+1}{6} = \frac{x^2-2x+3}{2}$ or $6(\frac{1}{2}x^2 - 2x + 3) = 4x + 1$ or equivalent	M1	For M1 allow intention, with no more than 1 slip or for use of $y = 4x/6 + 1$ or $y = 4x + 1/6$
	$3x^2 - 16x + 17 = 0$ or $\frac{x^2}{2} - \frac{8x}{3} + \frac{17}{6} = 0$ or equivalent	A1	CAO. Must be equated to zero. '=0' may be implied in further work to solve, if no further work and not '=0' then A0
	$x = \frac{-(-16) \pm \sqrt{((-16)^2 - 4\times3\times17)}}{2\times3}$ or equivalent	m1	Working **must** be seen for m1 to be awarded No further FT from m0 if no working seen Allow 1 slip in substitution (not a slip with the formula) FT provided M1 awarded for equivalent level of difficulty, 3 term quadratic equation Allow $(-16 \pm\sqrt{((-16)^2 - 4\times3\times17)})/(2\times3)$ or $(-16 \pm\sqrt{(16^2 - 4\times3\times17)})/(2\times3)$ as 1 slip On FT, 'their quadratic equation' factorised correctly is awarded m1 A1 (then possible A1 A1)
	$x = \frac{16 \pm\sqrt{52}}{6}$ or equivalent	A1	Do not FT from m1 awarded when there has been 1 slip in substitution
	$x = 3.8685...$ or 3.87 **with** $x = 1.46(48...)$	A1	ISW. Allow $x = 1.465$ for $x = 1.4648\ldots$ FT from m1 awarded when there has been 1 slip in substitution
	$x = 3.87$ **and** $y = 2.75$ **with** $x = 1.46$ **and** $y = 1.14$	A1	FT provided M1, m1 previously awarded, provided not from a slip in substitution, using their unrounded values of x in $(4x + 1)/6$ or $y = x^2/2 - 2x + 3$, but not in an incorrect rearrangement of the given equations, to find y-values correct to 2 d.p. FT values must be written to 2 d.p. Accept answers given as coordinates *Note: Final A0 for premature approximation of $x = 1.4648$ leading to $y \neq 1.14$*
	Alternative using $x = (6y - 1)/4$ *$y = \frac{(6y-1)^2}{2\times4^2} - \frac{2(6y-1)}{4} + 3$ or equivalent*	*M1*	*Apply same guidance as detailed in the main method*
	$36y^2 - 140y + 113 = 0$ or equivalent	*A1*	*Must equate to zero*
	$y = \{-(-140) \pm\sqrt{((-140)^2 - 4\times36\times113)}\}/2\times36$ or equivalent	*m1*	*Working **must** be seen for m1 to be awarded Allow 1 slip in substitution (not a slip with the formula) FT provided M1 awarded for equivalent level of difficulty*
	$y = (140 \pm\sqrt{3328})/72$ or equivalent	*A1*	*Do not FT from m1 awarded when there has been a slip in substitution*
	*$y = 2.7456...$ or 2.75 **with** $y = 1.14(3...)$*	*A1*	*FT from m1 awarded when there has been 1 slip in substitution*
	*$x = 3.87$ **and** $y = 2.75$ **with** $x = 1.46$ **and** $y = 1.14$*	*A1*	*FT to final A1, provided M1, m1 previously awarded using their values of y in $(6y - 1)/4$ or $y = x^2/2 - 2x + 3$,*

		6	*but not in an incorrect rearrangement of the given equations, to find x-values correct to 2 d.p.*

8	$\frac{10x+3}{2} = 4x^2 + 2x - 3$ or $10x - 2(4x^2 + 2x - 3) + 3 = 0$ or equivalent	M1	For M1 allow intention, with no more than 1 slip or for use of $y = 5x + 3$
	$8x^2 - 6x - 9 = 0$ or equivalent	A1	CAO. Must be equated to zero. '=0' may be implied in further work to solve, if no further work and not '=0' then A0
	$x = \frac{-(-6) \pm \sqrt{((-6)^2 - 4\times8\times-9)}}{2\times8}$ or $(2x \ldots3)(4x \ldots3)$ or equivalent	m1	Working **must** be seen for m1 to be awarded If not solved by factorising, the use of correct quadratic formula must be seen, allow 1 slip in substitution (not a slip with the formula) FT provided M1awarded for equivalent level of difficulty
	$x = \frac{6 \pm\sqrt{324}}{16}$ or $(2x - 3)(4x + 3)$ (=0)	A1	Or equivalent Do not FT from m1 awarded when there has been a slip in substitution
	$x = 1.5$ **with** $x = -0.75$	A1	Or equivalent. ISW
	$x = 1.5$ **and** $y = 9$ **with** $x = -0.75$ **and** $y = -2.25$	A1	Or equivalent FT provided M1, m1 previously awarded using their values of x in $(10x + 3)/2$ or $4x^2 + 2x - 3$, but not in an incorrect rearrangement of the given equations, to find y-values Accept answers given as coordinates
	Alternative using $x = (2y - 3)/10$		
	$y = \frac{4(2y-3)^2}{10^2} + \frac{2(2y-3)}{10} - 3$ *or equivalent*	*M1*	
	$16y^2 - 108y - 324 = 0$ *or* $4y^2 - 27y - 81 = 0$ *or equivalent*	*A1*	*Must equate to zero*
	$y = \{27 \pm\sqrt{(27^2 - 4\times4\times-81)}\}/2\times4$ *or* $(4y \ldots9)(y \ldots9)$ *or equivalent*	*m1*	*Working **must** be seen for m1 to be awarded* *If not solved by factorising, the use of correct quadratic formula must be seen, allow 1 slip in substitution (not a slip with the formula)* *FT provided M1awarded for equivalent level of difficulty*
	$y = (27 \pm\sqrt{2025})/8$ *or* $(4y + 9)(y - 9)$ *or equivalent*	*A1*	*Do not FT from m1 if given when including a slip in substitution*
	$y = -2.25$ ***with*** $y = 9$	*A1*	*Or equivalent. ISW*
	$x = 1.5$ ***and*** $y = 9$ ***with*** $x = -0.75$ ***and*** $y = -2.25$	*A1*	*Or equivalent* *FT to final A1, provided M1, m1 previously awarded using their values of y in $(2y - 3)/10$* or *$y = 4x^2 + 2x - 3$, but not in an incorrect rearrangement of the given equations, to find x-values*
		6	

9	$10x + 2 = 5x^2 - 8x - 6$ $5x^2 - 18x - 8 = 0$	M1 A1	Must be equated to zero. '=0' may be implied in further work to solve, if no further work and not '=0' then A0
	$x = \frac{-(-18) \pm \sqrt{((-18)^2 - 4\times5\times-8)}}{2\times5}$ or $(x \ldots4)(5x \ldots2)$	m1	Working **must** be seen for m1 to be awarded, **must show** use of correct quadratic formula, allow 1 slip in substitution (not a slip with the formula)
	$x = \frac{18 \pm \sqrt{484}}{10}$ or $(x - 4)(5x + 2)\ (=0)$	A1	
	$x = 4$ **with** $x = -0.4$	A1	Or equivalent
	$x = 4$ **and** $y = 42$ **with** $x = -0.4$ **and** $y = -2$	A1	Or equivalent. Both solutions are required FT provided M1, m1 previously awarded using their values of x in $10x + 2$ or equivalent to find y-values Accept answers given as coordinates No marks for a single point of intersection found from trial & improvement.
	Alternative method using $x = (y - 2)/10$		
	$y = 5\frac{(y-2)^2}{10^2} - \frac{8(y-2)}{10} - 6$ or equivalent	*M1*	
	$5y^2 - 200y - 420 = 0$ or $y^2 - 40y - 84 = 0$ or equivalent (equate to zero)	*A1*	
	$y = \frac{40 \pm \sqrt{(40^2 - 4\times1\times-84)}}{2\times1}$ or $(y \ldots42)(y \ldots2)$	*m1*	*Allow 1 slip in substitution*
	$y = (40 \pm \sqrt{1936})/2$ or $(y + 2)(y - 42)$	*A1*	
	*$y = 42$ **with** $y = -2$*	*A1*	*Both solutions are required*
	*$x = 4$ **and** $y = 42$ **with** $x = -0.4$ **and** $y = -2$*	*A1*	*Or equivalent* *FT provided M1, m1 previously awarded using their values of y in $(y - 2)/10$ or equivalent to find x-values*
		6	

11	(a) Correct shaped graph with (0°,) 180° & 360° labelled on the x-axis AND 2, 7 & 12 labelled on the y-axis	B3	Ignore outside the required range *Intention for approximately (0°, 7), (90°, 2), (180°, 7), (270°, 12) and (360°, 7)* B2 awarded a for correct shape graph with conditions: • sinx reflected • with one complete period, labelled 0° to 360° • with difference in y values between maximum and minimum of 10, for their labels OR B1 for a correct shape graph with any 2 of the 3 bullet points above met, OR B1 for a graph with all 3 bullet points above met but joined by straight lines (even if turning points curved), OR B1 for a curved graph through intended points: (0°, 7), (90°, 2), (180°, 7), (270°, 12) and (360°, 7)
	(b) Maximum value 12 AND Minimum value 2	B1	Accept Maximum (270(°), 12) and Minimum (90(°), 2) Allow unsupported correct responses
		4	FT provided at least B2 previously awarded in (a)
12	(a) $(dy/dx=)\ 16x^7 + 8x$ $(d^2y/dx^2=)\ 112x^6 + 8$	B1 B1	 FT to 2nd B1 from $dy/dx = kx^n$ (+ …)
	(b) $(5/5)\ x^5 + (3/-1)\ x^{-1} + (-2/-2)x^{-2}$ $(= x^5 - 3x^{-1} + x^{-2})$	B3	B1 for each term. Accept unsimplified. ISW
	+ c (constant)	B1	Award if at least B1 given for integration
	(c) $6x^2/2 + 10x$ $[\ 6x^2/2 + 10x\ ]^3_2$ and with intention to substitute and subtract	B2 M1	B1 for $6x^2/2$ or $10x$ Intention to use 3, 2 (in either order) **and** subtract FT their integration, not the same terms as given or differentiated, this includes if there is only1 term seen.
	$=(6\times3^2/2+10\times3) - (6\times2^2/2+10\times2)$ $(=\ 57 - 32)$	A1	FT for correct use of limits provided working with 2 terms from 'their integration'
	$=\ 25$	A1 11	CAO, not FT. *Answer only, no working shown, M0 A0 A0*
13	(When x = 2) y = 27 (Gradient when x = 2, dy/dx =) $5\times2x$ 20	B1 M1 A1	 For differentiation, before substitution of x = 2
	Equation $\frac{y-27}{x-2} = 20$ or $27 = 20\times2 + c$	M1	FT values for 'their 27' and 'their 20' provided at least one of these is correct.
	$y - 27 = 20(x - 2)$ or $c = -13$	m1	Implies previous M1
	$y = 20x - 13$	A1 6	CAO. Mark final answer
14	Method to solve simultaneously, e.g. use of $y = 2x + 1$ or $x = (y - 1)/2$ into the first equation	M1	$2x + 1 = x^2 - 5x + 13$ or $y = \frac{(y-1)^2}{2^2} - \frac{5(y-1)}{2} + 13$
	$x^2 - 7x + 12 = 0$ or $y^2 - 16y + 63 = 0$	A1	Or equivalent but must '=0' or implied in further working
	$(x - 3)(x - 4)\ (=0)$ or $(y - 9)(y - 7)\ (=0)$	m1	OR $x = (7 \pm \sqrt{1})/2$ or $y = (16 \pm \sqrt{4})/2$
	(3, 7) and (4, 9)	A1	FT from their quadratic CAO Need not be in this form, accept x=3, y=7 with x=4, y=9 x & y values must be given
		4	Do not accept unsupported responses Do not accept trial & improvement

15	Working to support that (7, 10) and (2, -5) both lie on the curve	B2 2	Working, e.g. • substituting the x-values and correctly finding y-values • substituting coordinates for the points and showing "=55" (Allow sight of -5^2 in working provided -25 is not seen) B1 for either correct working for either point
16	Intention to integrate $-x^3/3 + 8x^2/2 - 12x$ Use of correct limits 6 & 2 in the correct order **and** intention to subtract 32/3 or 10.66(6…) or 10.7	M1 A2 m1 A1 5	Intention to integrate (not using given or differentiated) A1 one term correct. The limits must be used in the correct order CAO. Only allow 10.6 from correct working seen *Answer only gets no marks* *No marks for use of the trapezium rule*

	Summer 2017		
5	$3x - 1 = 4x^2 + 8x - 3$ $4x^2 + 5x - 2 = 0$	M1 A1	Must be equated to zero. '=0' may be implied in further work to solve, if no further work and not '=0' then A0 FT provided their quadratic does not factorise and equivalent level of difficulty
	$x = \{-5 \pm\sqrt{(5^2 - 4\times4\times-2)}\}/2\times4$	m1	Use of correct quadratic formula, allow 1 slip in substitution (not a slip with the formula) If completing the square used award m1 for sight of $(2x + 1.25)^2 \pm \ldots$ or $4(x + 5/8)^2 \pm \ldots$
	$x = \{-5 \pm\sqrt{57}\}/8$	A1	If m0 then A0 (leading to $x = (-1.25 \pm\sqrt{3.5625})/2$)
	$x = 0.318\ldots$ or $x = 0.32$ **and** $x = -1.5687\ldots$ or $x = -1.57$	A1	If m0 then A0
	$x = 0.32$ with $y = -0.04$ **and** $x = -1.57$ with $y = -5.71$	A1	FT provided M1, m1 previously awarded using their values of x in $3x - 1$ or equivalent to find y-values to 2 d.p. Use of 2d.p. x values in the quadratic leads to $y = -0.03$ and $y = -5.70$ *Alternative using $x = (y + 1)/3$* M1 $y = 4\frac{(y+1)^2}{3^2} + 8\frac{(y+1)}{3} - 3$ or equivalent A1 $4y^2 + 23y + 1 = 0$ or equivalent (equate to zero) m1 $y = \{-23\pm\sqrt{(23^2 - 4\times4\times1)}\}/2\times4$ or equivalent Allow 1 slip in substitution A1 $y = (-23\pm\sqrt{513})/8$ or equivalent A1 $y = -0.04(38\ldots)$ and $y = -5.7(061\ldots)$ A1 $x = 0.32$, $y = -0.04$ with $x = -1.57$, $y = -5.71$ FT to final A1, provided M1, m1 previously awarded using their values of y in $(y + 1)/3$ or equivalent to find x-values to 2 d.p.
		6	

	Additional Mathematics Summer 2017		Final Version

14	Method to solve simultaneously, e.g. use of $y = 10 - x$ or $x = 10 - y$ into the first equation	M1	$10 - x = x^2 - 6x + 14$ or $y = (10 - y)^2 - 6(10 - y) + 14$
		A1	Must '=0' or implied in further working
	$x^2 - 5x + 4 = 0$ or $y^2 - 15y + 54 = 0$	m1	OR $x = (5 \pm \sqrt{9})/2$ or $y = (15 \pm \sqrt{9})/2$
	$(x - 4)(x - 1)$ (=0) or $(y - 9)(y - 6)$ (=0)		FT from their quadratic
		A1	CAO
	(4, 6) and (1, 9)		Need not be in this form, accept x=4, y=6 with x=1, y=9
			x & y values must be given
			Do not accept unsupported responses
		4	Do not accept trial & improvement

	Summer 2015		
6	$2x+1 = x^2 + 6x - 5$ $x^2 + 4x - 6 = 0$	M1 A1	Must be equated to zero. '=0' may be implied in further work to solve, if no further work and not '=0' then A0 FT provided their quadratic does not factorise and equivalent level of difficulty
	$x = \{-4 \pm\sqrt{(4^2 - 4\times1\times-6)}\}/2$	m1	Use of correct quadratic formula, allow 1 slip in substitution (not a slip with the formula) If completing the square used award m1 for sight of $(x + 2)^2 \pm \ldots$
	$x = \{-4 \pm\sqrt{40}\}/2$	A1	
	$x = 1.16$ and $x = -5.16$	A1	
	$x = 1.16$ with $y = 3.32$ or $y = 3.31$ **and** $x = -5.16$ with $y = -9.32$ or $y = -9.33$	A1	FT provided M1,m1 previously awarded using their values of x in $2x + 1$ or equivalent to find y-values to 2 d.p. *Alternative using $x = (y - 1)/2$* M1 $y = (\underline{y-1})^2 + 6(\underline{y-1}) - 5$ or $y = y^2/4 + 5y/2 - 8 + ¼$ 2^2 2 A1 $y^2 + 6y - 31 = 0$ or equivalent (equate to zero) m1 $y = \{-6\pm\sqrt{(6^2 - 4\times1\times-31)}\}/2$ or equivalent (×¼) Allow 1 slip in substitution A1 $y = (-6\pm\sqrt{160})/2$ or equivalent A1 $y = 3.32$ and $y = -9.32$ A1 $x = 1.16$, $y = 3.32$ and $x = -5.16$, $y = -9.32$ FT to final A1, provided M1,m1 previously awarded using their values of y in $(y - 1)/2$ or equivalent to find x-values to 2 d.p.
		6	

	other (gradient) or similar	3	
14	Method to solve simultaneously, e.g. use of $y = 4 - x$ or $x = 4 - y$ into the first equation $x^2 - 6x + 8 = 0$ or $y^2 - 2y = 0$ $(x - 4)(x - 2)$ (=0) or $y(y - 2)$ (=0) (4, 0) and (2, 2)	M1 A1 m1 A1 4	$4 - x = x^2 - 7x + 12$ or $y = (4 - y)^2 - 7(4 - y) + 12$ OR $x = (6 \pm \sqrt{4})/2$. FT from their quadratic CAO Need not be in this form, accept x=4, y=0 with x=2, y=2 y values must be given Accept unsupported correct responses for all 4 marks, or from trials if coordinates of **both** points are given and no others

6	$x+1 = x^2 + 2x - 3$	M1	
	$x^2 + x - 4 = 0$	A1	Must be equate to zero
			FT provided their quadratic does not factorise and equivalent level of difficulty
	$x = \{-1 \pm\sqrt{(1^2 - 4\times1\times-4)}\}/2$	m1	Use of quadratic formula, allow 1 slip in substitution
	$x = \{-1 \pm\sqrt{17}\}/2$	A1	
	$x = 1.56$ and $x = -2.56$	A1	
	$x = 1.56$, $y = 2.56$ and $x = -2.56$, $y = -1.56$	A1	
			Alternative using $x = y - 1$:
		M1	$y = (y-1)^2 + 2(y-1) - 3$ or $y = y^2 - 4$
		A1	$y^2 - y - 4 = 0$ (equate to zero)
		m1	$y = \{1\pm\sqrt{(1^2 - 4\times1\times-4)}\}/2$
		A1	$y = (1\pm\sqrt{17})/2$
		A1	$y = 2.56$ and $y = -1.56$
		6	A1 $x = 1.56$, $y = 2.56$ and $x = -2.56$, $y = -1.56$

End of solutions