



Remainder & factor theorem

Mark schemes for the Remainder & factor theorem question pack

WJEC Level 2 Additional Mathematics (9550) · Algebra

Official WJEC mark schemes for the 12 questions in the matching revise.wales question pack (95 marks total), from the 2011–2024 papers. Pack layout © revise.wales.

2	(a) $1024x^5$ (b) $7x^{3/5} - 6x^{-1}$ or $7x^{3/5} - 6/x$	B2 B2 4	<u>indices must be simplified throughout</u> Mark final answer B1 for nx^5 where $n \neq 0$, including $2^{10}x^5$ For award of B2 mark final answer B1 for sight of $x^{3/5}$, $(-) 6x^{-1}$ or $(-) 6/x$
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6	(a) Substitution of $x = -5$, $4(-5)^3 - 2(-5)^2 - (-5) (= -500 - 50 + 5)$ -545	M1	Or division method giving $4x^2 - 22x \dots$ Allow for intention, e.g. $4(-5)^3 - 2(-5)^2 - 5 (= -555)$
		A1	Mark final answer
	(b)(i) Substitute $x = 2$, $(2)^3 - 6(2)^2 - 13(2) + 42$ $(= 8 - 24 - 26 + 42)$ Showing $f(2) = 0$	M1	Or division method giving $x^2 - 4x \dots$
		A1	Accept sight of substitution with ‘=0’ shown
	(ii) $(x-2)(x^2 + bx + c)$ or intention to divide by $(x - 2)$ with x^2 shown	M1	If any values are inserted at least 1 needs to be correct. Appropriate sight of $-4x$ or -21 implies M1 (and possible A1 to follow)
	$((x-2)) \quad (x^2 - 4x - 21)$	A2	A1 for $-4x$ or -21 Or use of factor theorem A1 $(x + 3)$, A1 $(x - 7)$
	$(x - 2)(x + 3)(x - 7)$	A1	CAO, with all 3 factors shown, ignore sight of “=0”, ISW Must be from sight of $x^2 - 4x - 21$ previously No working to show factorising is M0 A0 A0
		8	

		4	
6	(a) $3(2)^2 + 4(2)^2 + 3(2) + 1$ $(= 24 + 16 + 6 + 1)$ 47	M1	Or division method giving $3x^2 + 10x \dots$
		A1	
	(b)(i) Substitute $x = -6$ Showing $f(-6) = 0$	M1	Or division method giving $x^2 - 5x \dots$
		A1	Accept sight of substitution with ‘=0’ shown
	(ii) $(x+6)(x^2 + bx + c)$ or intention to divide by $(x+6)$ with x^2 shown $(x+6) \) \ (x^2 - 5x + 6)$ $(x+6)(x - 2)(x - 3)$	M1	If any values are inserted at least 1 needs to be correct, appropriate sight of $-5x$ or $+6$ implies M1 (and A1 to follow)
	A2	A1 for $-5x$ or $+6$ Or use of factor theorem A1 $(x - 2)$, A1 $(x - 3)$	
	A1	CAO, with all 3 factors shown, ignore sight of ‘=0’, ISW Must not be from previous incorrect work	
	8		

7	<p>(a) $(3)^3 + 8(3)^2 - 2(3) + 6 (= 27 + 72 - 6 + 6)$ $= 99$</p> <p>(b)(i) Substitute $x = -3$ Showing $f(-3) = 0$</p> <p>(ii) $(x + 3)(x^2 + bx + c)$ or intention to divide by $(x + 3)$ with x^2 shown $(x + 3) (x^2 - 2x - 35)$ $(x + 3)(x + 5)(x - 7)$</p>	<p>M1 A1 M1 A1 M1 A2 A1 8</p>	<p>Or division method giving $x^2 + 11x \dots$</p> <p>Or division method giving $x^2 - 2x \dots$ Convincing from working shown (not if incorrect working seen), allow $(-3)^3 + (-3)^2 - 41(-3) - 105 = 0$, also allow for sight of $-3^3 + -3^2 - 41 \times -3 - 105 = 0$ provided no incorrect calculation is given such as -3^2 as -9</p> <p>A1 for $-2x$ or -35. Or use of factor theorem A1 $(x+5)$, A1 $(x-7)$ CAO. Mark final answer, but ignore attempts to 'solve'</p>
8	<p>$(dy/dx) = 12x^2 - 6x$ $dy/dx = 0$ or $12x^2 - 6x = 0$ or $12x^2 = 6x$ $x = 0$ and $y = 20$ $x = 1/2$ and $y = 19\frac{3}{4}$</p> <p>$d^2y/dx^2 = 24x - 6$</p> <p>(0, (20)): $d^2y/dx^2 < 0$, point is a maximum $(1/2, (19\frac{3}{4}))$: $d^2y/dx^2 > 0$, point is a minimum</p>	<p>B1 M1 A1 A1 M1 A1 A1</p>	<p>FT their dy/dx form $ax^2 \pm bx$</p> <p>If A0, A0 here, award A1 for $x = 0$ with $x = 1/2$ Answer only, no working shown MOAOAO</p> <p>Or first derivative test, interpretation of first derivative test. Or alternative (e.g. full graphical method with explanation)</p> <p>FT for their x value FT for their other x value provided this does not have the same interpretation as the first x value</p> <p>Answer only, no working shown MOAOAO If $d^2y/dx^2 = cx + d$ where $c \neq 0$ and test applied correctly then SC2 instead of final A1, A1 (as M1 has not been awarded) provided one minimum and one maximum</p>
9	<p>$\frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4}$</p>	<p>B1 1</p>	<p>Working must be shown</p>
10	<p>(a) $FG^2 = (-4 - 8)^2 + (10 - 28)^2$ $(= 12^2 + 18^2 = 468)$ $FG = 6\sqrt{13}$</p> <p>(b) Gradient $FG (28-10)/(8 - -4)$ $= 18/12 (= 9/6 = 3/2)$</p> <p>(c) $(-4 + 8)/2$ or $(10 + 28)/2$ Mid point (2, 19) Perpendicular gradient $-2/3$ (or $-6/9$ or $-12/18$)</p> <p>$\frac{y - 19}{x - 2} = \frac{-2}{3}$ or $19 = -2/3 \times 2 + c$</p> <p>$y - 19 = -2/3(x - 2)$ or $3(y - 19) = -2(x - 2)$ or $3y = -2x + 61$ or $c = 20\frac{1}{3}$ or $c = 61/3$</p> <p>$2x + 3y - 61 = 0$ or $-2x - 3y + 61 = 0$</p>	<p>M1 A1 M1 A1 M1 A1 B1 M1 m1 A1 10</p>	<p>Or equivalent. Allow 1 slip or error M1, A0 for answers $\sqrt{468}$ or $21.6(3\dots)$ CAO</p> <p>Do not ignore incorrect cancelling, mark final answer</p> <p>Sight of (2, ...) or (... , 19) implies M1 provided no incorrect working is seen</p> <p>FT $-1/$ 'their answer in (b)'</p> <p>OR for an alternative correct method of finding the equation of a straight line, for the idea of how an equation of a straight line can be found. FT 'their perpendicular gradient' or 'their answer in (b)' AND 'their mid point' or for 'points F or G' used</p> <p>Do not allow use gradient from their answer in (b), and/or points F or G as the mid-point of FG. Only FT for 'their perpendicular gradient' (not 'their answer' from (b)) AND 'their mid point'</p> <p>CAO. Must be in this form with '=' with terms in any order</p>

5		4	
	(a) $2(-3)^3 - (-3)^2 + 2(-3) + 1$ (= -54-9-6+1) -68	M1 A1	Or division method giving $2x^2 - 7x \dots$
	(b)(i) Substitute $x = -2$ Showing $f(-2) = 0$	M1 A1	Or division method giving $x^2 - 8x \dots$ Accept sight of substitution with '=0' shown
	(ii) $(x+2)(x^2 + bx + c)$ or intention to divide by $(x+2)$ with x^2 shown	M1	If any values are inserted at least 1 needs to be correct, appropriate sight of $-8x$ or -33 implies M1 (and A1 to follow)
	$(x+2)(x^2 - 8x - 33)$	A2	A1 for $-8x$ or -33 Or use of factor theorem A1 $(x+3)$, A1 $(x-11)$
	$(x+2)(x+3)(x-11)$	A1 8	CAO, but ignore sight of '=0', ISW

4	(a) $2(-4)^3 - 5(-4)^2 + 8(-4) - 6$ (= -128-80-32-6) -246	M1 A1	Or division method giving $2x^2 - 13x \dots$
	(b)(i) Substitute $x = 2$ Showing $f(2) = 0$	M1 A1	Or division method giving $x^2 + 11x \dots$ Accept sight of substitution with ‘=0’ shown Or equivalent ie. $X^2 + 11x - 60$ with no remainder
	(ii) $(x-2)(x^2 + bx + c)$ or intention to divide by $(x-2)$ with x^2 shown	M1	If any values are inserted at least 1 needs to be correct, appropriate sight of $+11x$ or $+30$ implies M1 (and A1 to follow)
	$((x-2) \quad (x^2 + 11x + 30))$ $((x-2) \quad (x + 5)(x + 6))$	A2 A1 8	A1 for $(+)11x$ or $(+)30$ Or use of factor theorem A1 $(x+5)$, A1 $(x+6)$ CAO, but ignore sight of ‘=0’, ISW

	Additional Mathematics Summer 2017		Final Version
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Summer 2016			
10	(a) $(4)^3 + 6(4)^2 - (4) - 30 (= 64 + 96 - 4 - 30)$ $= 126$	M1 A1	Or division method giving $x^2 + 10x \dots$
	(b)(i) Substitute $x = 2$ Showing $f(2) = 0$	M1 A1	Or division method giving $x^2 + 8x \dots$ Convincing, working shown Allow $2^3 + 6(2)^2 - (2) - 30 = 0$
	(ii) $(x-2)(x^2 + bx + c)$ or intention to divide by $(x-2)$ with x^2 shown	M1	
	$((x - 2)) (x^2 + 8x + 15)$	A2	A1 for $+8x$ or $+15$.
	$((x - 2))(x + 3)(x + 5)$	A1 8	Or use of factor theorem A1 $(x+3)$, A1 $(x+5)$ CAO. Mark final answer, but ignore attempts to 'solve'

7	<p>(a) $3(-2)^3 - 2(-2)^2 + 5(-2) - 1$ (= -24-8-10-1) -43</p> <p>(b)(i) Substitute $x = 2$ Showing $f(2) = 0$</p> <p>(ii) $(x-2)(x^2 + bx + c)$ or intention to divide by $(x-2)$ with x^2 shown</p> <p>$(x-2) (x^2 + 10x + 21)$</p> <p>$(x-2) (x+3)(x+7)$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A2</p> <p>A1 8</p>	<p>Or division method giving $3x^2 - 8x \dots$</p> <p>Or division method giving $x^2 + 10x \dots$ Accept sight of substitution with '=0' shown</p> <p>If any values are inserted at least 1 needs to be correct, appropriate sight of +10x or +21 implies M1 (and A1 to follow)</p> <p>A1 for (+)10x or (+)21 Or use of factor theorem A1 (x+3), A1 (x+7)</p> <p>CAO, but ignore sight of "=0", ISW</p>
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		mark	final answer
10	(a) $(3)^3 + 5(3)^2 + 2(3) - 8$ ($= 27 + 45 + 6 - 8$) $= 70$	M1 A1 M1	Or division method giving $x^2 + 8x \dots$
	(b)(i) Substitute $x = 1$ Showing $f(1) = 0$	A1 M1	Or division method giving $x^2 + 6x \dots$ Convincing, working shown ($1 + 5 + 2 - 8$) Allow $1^3 + 5(1)^2 + 2(1) - 8 = 0$
	(ii) $(x-1)(x^2 + bx + c)$ or intention to divide by $(x-1)$ with x^2 shown $(x-1)(x^2 + 6x + 8)$ $(x-1)(x+4)(x+2)$	A2 A1 8	A1 for $+6x$ or $+8$. Or use of factor theorem A1 $(x+4)$, A1 $(x+2)$ CAO. Mark final answer, <i>but ignore attempts to 'solve'</i>

		11	ANSWER ONLY, NO WORKING SHOWN (NO AO AO)
8	(a) $7(2)^3 - 4(2)^2 + (2) - 2$ ($= 56 - 16 + 2 - 2$) $= 40$	M1 A1	Or division method giving $7x^2 + 10x \dots$
	(b)(i) Substitute $x = -3$ Showing $f(-3) = 0$	M1 A1	Or division method giving $x^2 + x \dots$ Accept sight of substitution with equate to zero
	(ii) $(x+3)(x^2 + bx + c)$ or intention to divide by $(x+3)$ with x^2 shown $(x+3) \mid (x^2 + x - 20)$	M1 A2	A1 for $+1x$ or -20 . Or use of factor theorem A1 $(x-4)$, A1 $(x+5)$
	$(x+3) \mid (x-4)(x+5)$	A1 8	CAO. Final answer, but ignore sight of “=0”

Q	Additional Mathematics Summer 2013	Marks	Final
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4	<p>(a) $(-3)^3 - 2(-3)^2 - 9(-3) + 18 = 0$ $(x+3)$ is a factor OR divisible by $x+3$</p> <p>(b) $(x+3)(ax^2 + bx + c)$ or intention to $\div (x+3)$</p> <p>$(x+3)(x^2 - 5x + 6)$</p> <p>$((x+3)(x-3)(x-2))$</p>	<p>3</p> <p>M1 A1 E1</p> <p>M1</p> <p>A2</p> <p>A1</p> <p>7</p>	<p>Depends on M1, A1. Do not accept contradictions</p> <p>Division method needs to show x^2 and attempt to find the next term</p> <p>May be division by $x-3$ or $x-2$, mark in the same way as described for division by $x+3$</p> <p>A1 for $-5x$ or $+6$. Or use of factor theorem A1 for each factor</p> <p>CAO. Mark final answer. Do not ignore continuing to solve.</p> <p>An answer of $(x-2)(x^2 - 9)$ is awarded M1 A2</p>
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		4	
5	(a) $6(-3)^3 - 13(-3)^2 + (-3) + 2$ (= $-162 - 117 - 3 + 2$) = -280	M1 A1	Or division method giving $6x^2 - 31x \dots$
	(b)(i) Substitute $x = 2$ Showing $f(2) = 0$	M1 A1	Or division method giving $6x^2 - x \dots$ Convincing, working shown ($48 - 52 + 2 + 2$)
	(ii) $(x-2)(6x^2 + bx + c)$ or intention to divide by $(x-2)$ with $6x^2$ shown ($(x-2)$) ($6x^2 - x - 1$)	M1 A2	A1 for $-x$ or -1 . Or use of factor theorem A1 ($3x+1$), A1 ($2x-1$)
	($(x-2)$) ($3x+1$)($2x-1$) ISW	A1 o	CAO

End of solutions