

**REVISE**

*.wales*

## **Completing the square & minimum values**

*Mark schemes for the Completing the square & minimum values question pack*

*WJEC Level 2 Additional Mathematics (9550) · Algebra*

Official WJEC mark schemes for the 7 questions in the matching revise.wales question pack (31 marks total), from the 2011–2024 papers. Pack layout © revise.wales.

16	$(x+14)^2 (\pm \dots)$ or $(x+28/2)^2 (\pm \dots)$ or $(x+a)^2 + b$ with $a = 14$ or equivalent  (Minimum value when $x =$ ) $-14$ (Minimum value is) $-96$	M1	Ignore 'their $(\pm \dots)$ ' or ' $=0$ ' Do not accept method $dy/dx = 2x+28$
		A1	CAO. Must be from $(x+14)^2$
		A1	CAO. Must be from completing the square, $(x+14)^2 - 96$ or $(x+14)^2 - 196 + 100$
			Use of quadratic formula is awarded M0 A0 A0
		3	

2	<p><math>x^2 = 20x - 28</math> written correctly showing correct completing of the square, e.g., for sight of any one of the following:</p> <ul style="list-style-type: none"> <li>• <math>(x - 10)^2 - 100 = -28</math></li> <li>• <math>(x - 10)^2 - 100 + 28 = 0</math></li> <li>• <math>100 - (x - 10)^2 = 28</math></li> <li>• <math>100 - (x - 10)^2 - 28 = 0</math></li> <li>• <math>(x - 10)^2 = 100 - 28</math></li> <li>• <math>-(x - 10)^2 + 100 = 28</math></li> <li>• <math>-(x - 10)^2 + 100 - 28 = 0</math></li> <li>• <math>-(x - 10)^2 = -100 + 28</math></li> <li>• <math>(x - 10)^2 = 72</math></li> <li>• <math>(x - 10)^2 - 72 = 0</math></li> <li>• <math>-(x - 10)^2 = -72</math></li> <li>• <math>-(x - 10)^2 + 72 = 0</math></li> <li>• <math>-(-x + 10)^2 + 100 - 28 = 0</math></li> <li>• <math>-(-x + 10)^2 + 72 = 0</math> or equivalent</li> </ul> <p style="text-align: center;"><math>(x =) 10 \pm 6\sqrt{2}</math> or <math>(x =) 10 + 6\sqrt{2}</math> with <math>10 - 6\sqrt{2}</math></p>	<p>M2 <i>No working, no marks</i> '=0' may be implied in previous working, e.g., from sight of <math>x^2 - 20x + 28 (= 0)</math>, or further working</p> <p>M1 for sight of any one of the following, including if embedded in incorrect working:</p> <ul style="list-style-type: none"> <li>• <math>(x - 10)^2</math> (-100)</li> <li>• <math>-(x - 10)^2</math> (+100)</li> <li>• <math>-(-x + 10)^2</math> (+100)</li> </ul> <p>OR</p> <p>Allow M1 for any one of the following, if not corrected in further correct working (which may then be M2):</p> <ul style="list-style-type: none"> <li>• <math>(x - 10x)^2 - 100 + 28</math></li> <li>• <math>-(x - 10x)^2 + 100 - 28</math></li> <li>• <math>(x + 10)^2 - 100 + 28</math></li> </ul> <p>A2 ISW for award of A2 or A1 Allow A2 for strict FT from M1 and:</p> <ul style="list-style-type: none"> <li>• <math>(x - 10)^2 = 100 + 28</math> for an answer of <math>10 \pm 8\sqrt{2}</math></li> <li>• <math>(x + 10)^2 = 100 - 28</math> for an answer of <math>-10 \pm 6\sqrt{2}</math></li> </ul> <p>A1 for any of the following, <math>(x =)</math></p> <ul style="list-style-type: none"> <li>• <math>10 \pm 2\sqrt{18}</math></li> <li>• <math>10 \pm 3\sqrt{8}</math></li> <li>• <math>10 + 6\sqrt{2}</math></li> <li>• <math>10 - 6\sqrt{2}</math></li> </ul> <p>OR A1 for strict FT from M1 and <math>(x - 10)^2 = 100 + 28</math>, for:</p> <ul style="list-style-type: none"> <li>• <math>10 \pm 2\sqrt{32}</math></li> <li>• <math>10 \pm 4\sqrt{8}</math></li> <li>• <math>10 + 8\sqrt{2}</math></li> <li>• <math>10 - 8\sqrt{2}</math></li> </ul> <p>OR A1 for strict FT from M1 and <math>(x + 10)^2 = 100 - 28</math>, for:</p> <ul style="list-style-type: none"> <li>• <math>-10 \pm 2\sqrt{18}</math></li> <li>• <math>-10 \pm 3\sqrt{8}</math></li> <li>• <math>-10 + 6\sqrt{2}</math></li> <li>• <math>-10 - 6\sqrt{2}</math></li> </ul> <p>If no marks, Award SC1 for either:</p> <ul style="list-style-type: none"> <li>• <math>-10 \pm 8\sqrt{2}</math> from <math>(x + 10)^2 = 128</math></li> <li>• 'their simplified <math>a \pm b\sqrt{c}</math>' from their <math>(x \dots)^2 = \dots</math> provided equivalent level of difficulty with <math>a &gt; 0</math>, <math>c &gt; 0</math> and <math>b</math> is a product of at least 2 integers</li> </ul> <p>No FT for A mark when attempting to write the square root of a negative number, e.g. <math>\sqrt{-72}</math> or <math>\sqrt{-128}</math></p> <p>4 If no marks, award SC1 for <math>\sqrt{72} = \pm 6\sqrt{2}</math> or <math>\sqrt{128} = \pm 8\sqrt{2}</math></p>
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8	$(x+20)^2$ ( $\pm \dots$ )	M1	Ignore 'their ( $\pm \dots$ )' or ' $=0$ '
		A1	Do not accept method $dy/dx = 2x+40$
		A1	CAO
		3	CAO, from $(x+20)^2 - 300$



**GCSE ADDITIONAL MATHEMATICS  
SUMMER 2019 MARK SCHEME**

		Mark	Comment
1	(a)(i) $(9x + 5)(3x - 1)$	B2	Mark final answer. <i>Ignore inclusion of '=0'</i>  B1 $(9x \pm 5)(3x \pm 1)$ or $9x(3x - 1) + 5(3x - 1)$ or or $(x - 1/3)(27x + 15)$ or $\frac{(27x - 9)(27x + 15)}{27}$ or sight of $(9x + 5)$ AND $(3x - 1)$
	(ii) $-5/9$ with $1/3$ or equivalent or $-0.55(5\dots)$ or $-0.556$ with $0.33(3\dots)$	B2	If a restart in (ii) to factorise, do not alter marking in (i), unless the candidate is clearly replacing their answer (i) Ignore sight of '=0' Must be from factorising. STRICT FT for their factors. B1 for each answer Do not accept from the use of the quadratic formula
	(b)(i) $(x+5)^2 \pm \dots$ ..... +10	B1 B1	Sight of $(x+5)^2$ . Ignore sight of '=0' Do not accept '= -10' or '=10' $(x + 5)^2 + 10$ , B1, B1 ISW Allow +35 -25 for 10 provided 10 seen in later working
	(ii) Least value (+)10	B1	Must follow completing the square FT their value but not 35 or - 10
	(iii) $(x = ) -5$	B1	FT from 'their $(x + 5)^2$ ' Do not accept $(-5, 10)$
	(c) $(x - 22/2)^2 - 121 = -5$ or $(x - 22/2)^2 - 121 + 5 = 0$ $(x - 11)^2 = 121 - 5$ $x = 11 \pm 2\sqrt{29}$	M1 M1 A2	Allow for sight of $(x - 22/2)^2 - 121 + 5$ or $(x - 22/2)^2 - 116$  FT from 1 slip, e.g. for sight of $(x - 11)^2 = 126$ or $(x - 11)^2 - 126 = 0$ Allow A2 for $x = 11 \pm \sqrt{116}$ ISW A1 for $11 + \sqrt{116}$ or $11 + 2\sqrt{29}$ or $11 - \sqrt{116}$ or $11 - 2\sqrt{29}$ or FT $11 \pm \sqrt{126}$ or $11 \pm 3\sqrt{14}$
		12	FT $11 \pm \sqrt{126}$ or $11 \pm 3\sqrt{14}$ <i>No working in (c), no marks</i>
2	(a) $40x^3 + 6x (+0)$	B3	<i>Penalise '+c' shown -1 only throughout</i> B1 for $40x^3$ (not $10 \times 4x^3$ ), B1 for $+6x$ (not $3 \times 2x$ ), and B1 for $+0$ (or blank) provided at least 1 other mark awarded. Mark final answer
	(b) $-22x^{12}$ or $-\frac{22}{x^{12}}$	B1	Mark final answer
	(c) $\frac{7}{8}x^{1/8}$ or $\frac{7}{8x^{1/8}}$	B1	Index needs to be simplified. Mark final answer
		5	

		□	
7	$(x+9)^2$ (± ...)	M1	Ignore 'their (± ...)' or '=0'
	(Minimum value at $x =$ ) -9	A1	Do not accept method $dy/dx = 2x+18$
	(Minimum value is) (+) 11	A1	CAO
		3	CAO



3	$(x+11)^2$ ( $\pm \dots$ ) or $(x+22/2)^2$ ( $\pm \dots$ ) (Minimum value at $x =$ ) -11 (Minimum value is) (+) 2	$\cup$ M1 A1 A1 3	Ignore 'their ( $\pm \dots$ )' or '=0' Do not accept method $dy/dx = 2x+22$ CAO. Must be from sight of completing the square CAO. Must be from sight of completing the square
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2	<p>(a) <math>(x+7)^2</math> (<math>\pm \dots</math>)</p> <p>(Minimum value at <math>x =</math>) -7</p> <p>(b) -40</p>	<p>4</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>3</p>	<p>Ignore 'their (<math>\pm \dots</math>)' or '=0'</p> <p>Do not accept method <math>dy/dx = 2x+14</math></p> <p>Unsupported -7 is M0, A0</p> <p>CAO</p> <p>Must be given in (b), do not accept shown in (a).</p> <p>Do not accept reversed answers of '-40' in (a) and '-7' in (b), maximum mark would be possible M1 in (a)</p> <p>However, if no marks in (a), but a full statement, e.g. 'minimum value -40 when <math>x = -7</math>' is given in (b), then award B1 in (b) and M1, A0 in (a)</p>
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End of solutions