

# REVISE

*.wales*

## 3.17 – Trigonometric function graphs

*Mark schemes for the 3.17 question pack*

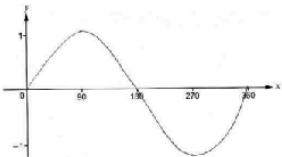
*Spec 3.7.8 – Unit 3*

SOLUTIONS · 2025 SPECIFICATION

*Mark schemes for the 12 questions in the corresponding revise.wales question pack (46 marks total). Sources: legacy WJEC GCSE papers, WJEC SAM, and custom-authored mark schemes. Pack layout © revise.wales.*

15.(a) Concave down curve with y-coordinate of maximum = 7 x-coordinate of maximum = 5 Points (2,0) AND (8, 0) shown.		B1 B1 B1	<i>Allow appropriate marking of axes if coordinates not given.</i>
15.(b) Cosine curve Correct cosine curve with 2 shown on the y-axis and 180° and 360° shown or implied.		M1 A1	Intention to sketch a portion of a cosine curve with minimum period of 360°.

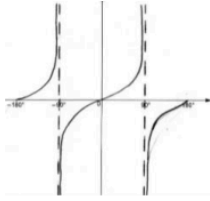
Summer 2017			
15.(a)	Correct sine curve with 1 and -1 shown on the y-axis and 180° and 360° shown on the x-axis.		<p>B2</p> <p>Multiple cycles gain both marks only if both axes fully correctly labelled for x between 0 and 360.</p> <p>If B2 not awarded, B1 for a sine curve (single cycle) with missing values on axes OR B1 for multiple cycles with (only) x axis correctly labelled.</p>
15.(b)	$\sin 340^\circ$		R1

<p>14.(a)</p> 	C1	<p>calculating with replacement.</p> <p>Clear Intention to draw a curve.  Curve must pass through (0,0), (180,0) and (360,0)  AND intention to have maximum at (90,1) and minimum at (270,-1).  Ignore curve shown for values <math>x &lt; 0^\circ</math> or <math>x &gt; 360^\circ</math>.</p>
<p>14.(b)(i)</p> <p>17 AND 163  OR  17.5 AND 162.5  OR  17.4(576...) AND 162.5(423...)</p>	B2	<p>If more than two answers offered award B1 for sight of one correct angle.  Allow embedded answers.</p> <p>Rounded angles must add up to <math>180^\circ</math>.</p> <p>B1 for sight of one correct angle OR,  B1 for two angles which total <math>180^\circ</math>. Allow different degrees of accuracy in rounding.</p>
<p>14.(b)(ii)      270(°)</p>	B1	<p>Allow an embedded answer.</p>

18. (a) $159^\circ$ and $201^\circ$ with no other values	B2	B1 for either angle. Check diagram. Penalise -1 for each extra value (beyond 2 attempts). Ignore extra (correct) values outside the required range.
18. (b) (i) Vertical enlargement upwards <u>and</u> downwards  Scale factor of 2	B1  B1	Mark clear intention. Must be the correct shape, i.e. a single cycle of a cosine <u>curve</u> , with x-intercepts at $x = 90^\circ$ and $x = 270^\circ$ , minimum at $x = 180^\circ$ , maxima at $x = 0$ and $x = 360^\circ$ .  Accept any clear indication. Must have correct x and y-intercepts, correct minimum and correct point for $x = 360^\circ$ .
18. (b) (ii) Vertical translation  Vertical $-1$	B1  B1	Mark clear intention. Must be the correct shape, i.e. a single cycle of a cosine <u>curve</u> , with x-intercepts at $x = 0^\circ$ and $x = 360^\circ$ , minimum at $x = 180^\circ$ , maxima at $x = 0$ and $x = 360^\circ$ .  Accept any clear indication. Must have correct x and y-intercepts, correct minimum and correct point for $x = 360^\circ$ .  Award SC1 for a fully labelled sketch of $y = \cos x + 1$ .

11.

Correct sketch, with inflection points at  $(-180,0)$ ,  $(0,0)$  and  $(180,0)$  AND graph tending towards the vertical asymptotes at  $x = -90$  and  $x = 90$ .

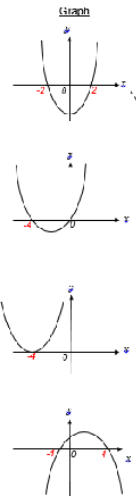


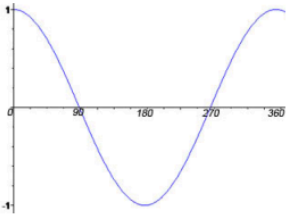
B2

If vertical asymptotes are not seen, they may be implied by a break in the curve of 'their sketch' at  $x = -90$  and  $x = 90$  provided there is asymptotic behaviour.

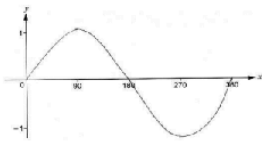
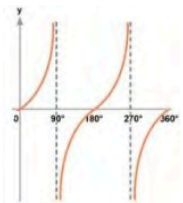
Graph must be attempted from  $x = -180$  to  $x = 180$ .  
*Ignore continuation of sketch for  $x < -180$  and  $x > 180$ .*

B1 for a sketch with inflection points at  $(-180,0)$ ,  $(0,0)$  and  $(180,0)$  only OR vertical asymptotes seen at  $x = -90$  and  $x = 90$  only.

<p>15.</p>  <p>Graph</p> <p>Equation</p> <p><math>y = (x + 1)(x - 4)</math></p> <p><math>y = (x - 4)^2</math></p> <p><math>y = x(x + 4)</math></p> <p><math>y = (x - 1)(x + 4)</math></p> <p><math>y = (x - 2)(x + 2)</math></p> <p><math>y = x(x - 4)</math></p> <p><math>y = (x + 1)(4 - x)</math></p> <p><math>y = (1 - x)(x + 4)</math></p> <p><math>y = (x + 4)^2</math></p>	<p>B2</p>	<p>B1 for any 1 or 2 correct.</p>
<p>16.(a) General sine curve with appropriate orientation and position.</p> <p>-1 and 1 indicated on the y-axis, curve passes through <math>(-180^\circ, 0)</math>, <math>(0^\circ, 0)</math> and <math>(180^\circ, 0)</math> and approximately <math>(-90^\circ, -1)</math> and <math>(90^\circ, 1)</math>.</p>	<p>M1</p> <p>A1</p>	<p>Ignore curve shown for values <math>x &lt; -180^\circ</math> or <math>x &gt; 180^\circ</math>.</p>
<p>16(b). <math>-30^\circ</math> AND <math>-150^\circ</math></p>	<p>B2</p>	<p>Accept embedded answers. Penalise further incorrect answer(s) -1. Ignore further answer(s) outside of the range.</p> <p>Award B1 for sight of an answer <math>-30^\circ</math> or <math>-150^\circ</math> (but not for sight of <math>-30</math> as part of working).</p>
<p>17.(a)</p> $\frac{3}{100} \times \frac{1}{99}$ $= \frac{3}{9900} \left( = \frac{1}{3300} \right) \text{ ISW}$	<p>M1</p> <p>A1</p>	<p>Allow <math>3(.03\dots) \times 10^{-4}</math> OR <math>0.0003(03\dots)</math> or equivalent. A0 for <math>0.0003(03\dots)\%</math>. An unsupported <math>0.000303(\dots)</math> gains M1A1. An unsupported <math>3/10000</math> OR <math>0.0003</math> gains no marks.</p>
<p>17(b)</p> $2 \times \frac{3}{100} \times \frac{1}{99} \left( = \frac{6}{9900} = \frac{1}{1650} \right)$ $+ \frac{3}{100} \times \frac{2}{99} \left( = \frac{6}{9900} = \frac{1}{1650} \right)$ <p>OR</p> $\frac{4}{100} \times \frac{3}{99}$ $= \frac{12}{9900} \left( = \frac{1}{825} \right) \text{ ISW}$	<p>M2</p> <p>A1</p>	<p>M1 for sight of <math>\left( \frac{3}{100} \times \frac{1}{99} \right) + \left( \frac{3}{100} \times \frac{1}{99} \right)</math> OR <math>\left( \frac{3}{100} \times \frac{1}{99} \right) + \left( \frac{1}{100} \times \frac{3}{99} \right)</math> OR <math>2 \times \frac{3}{100} \times \frac{1}{99}</math> OR <math>\left( \frac{3}{100} \times \frac{1}{99} \right) + \left( \frac{3}{100} \times \frac{2}{99} \right)</math></p> <p>A1 Allow <math>1(.21\dots) \times 10^{-3}</math> OR <math>0.001(21\dots)</math> or equivalent. An unsupported answer of <math>0.00121(2\dots)</math> gains M2A1. A0 for <math>0.001(21\dots)\%</math>. SC1 for working with replacement leading to an answer of <math>12/10000</math> (<math>3/2500</math>) OR <math>0.001(2)</math> [may be unsupported].</p>

<p>16. Use of 7175 AND (1)·2345 or (1)23·45(÷100) 7175 × 1·2345</p> <p style="text-align: right;">= (£)8858</p>	<p>B1 M1  A1</p>	<p>Or equivalent complete method. FT for 'their 7175' provided <math>7170 \leq x &lt; 7180</math> and 'their 1·2345' provided <math>1·234 \leq y &lt; 1·235</math> Sight of (£)8857·53(75) or (£)8857·54 implies B1M1. CAO.</p>
<p>17.(a) General cosine <u>curve</u> with appropriate orientation and position.</p> <p>Correct sketch with curve passing through (0°,1), (90°,0) and (270°,0) and approximately (180°,-1) and (360°,1) AND 90(°), 180(°), 270(°), 360(°) indicated on the x-axis AND -1 and 1 indicated on the y-axis.</p> 	<p>M1  A1</p>	<p>Ignore curve shown for values <math>x &lt; 0^\circ</math> or <math>x &gt; 360^\circ</math>.</p> <p>Accept 180° as mid-way between 0° and 360° if unlabelled. Accept 360° as unlabelled provided the sketch does not exceed 360°.</p>
<p>17.(b) 46(°) AND 314(°) OR 45·6(°) AND 314·4(°) OR 45·57(29...°) AND 314·4(27...°).</p>	<p>B2</p>	<p>B1 for sight of one correct angle. Allow embedded answers. If more than two answers offered award B1 for sight of one correct angle.</p> <p>If no marks, awarded SC1 for truncated answers 45(°) AND 315(°) OR 45·5(°) AND 314·5(°).</p>
<p>18. <math>0·7 \times 0·2 \times 0·1 \times 6</math></p> <p style="text-align: right;">= 0·084 or equivalent</p>	<p>M2  A1</p>	<p>M1 for sight of <math>0·7 \times 0·2 \times 0·1</math> OR <math>0·014</math> OR <math>7/500</math> or equivalent. A1 Fractional answer: <math>21/250</math> or equivalent. (ISW)</p>
<p>19. Sight of <math>25x^2 + 15x - 15x - 9</math> <math>25x^2 - 19x - 9 = 0</math></p> $x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4 \times 25 \times (-9)}}{2 \times 25}$ $x = \frac{19 \pm \sqrt{1261}}{50}$ <p><math>x = 1·09</math> with <math>x = -0·33</math> (answers to 2dp)</p>	<p>B1 B1  M1  A1  A1</p>	<p>Or equivalent. '= 0' required, but may be implied by an attempt to use the quadratic formula or if <math>a = 25, b = -19, c = -9</math> used in the quadratic formula.</p> <p><b>This substitution into the formula must be seen for M1, otherwise award M0A0A0.</b> FT 'their derived quadratic equation' of equivalent difficulty (<math>a, b</math> and <math>c</math> must be non-zero). Allow one slip in substitution <b>for M1 only</b>, but must be correct formula.</p> <p>Can be implied from at least one correct value of <math>x</math> evaluated, provided M1 awarded.</p> <p>CAO for their quadratic equation.</p>

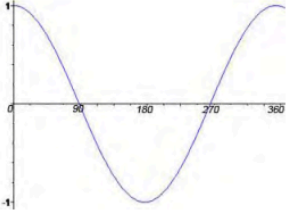
17. $218^\circ$ and $322^\circ$ with no other values	B2	<p>B1 for either angle. Check diagram. Ignore extra (correct or incorrect) values outside the required range. Penalise -1 for each extra value within range (beyond 2 attempts).</p> <p>If no marks, SC1 for accurate evaluations from consistent use of <math>180+n</math> AND <math>360-n</math> (with <math>n</math> acute). Method must be seen for this mark.</p>
18. (a) $\frac{1}{7} \times \frac{3}{6} \times \frac{2}{5}$ or equivalent $8/210 (= 4/105)$	M1 A1	<p>Penalise once only throughout for a repeated error in calculating the denominator (of 210)</p> <p>ISW</p>
18. (b) $1 - P(3, 3, 3)$ $1 - \frac{1}{7} \times \frac{3}{6} \times \frac{2}{5}$ $\frac{186}{210} (= \frac{93}{105} = \frac{62}{70} = \frac{31}{35})$	M1 M1 A1	<p>ISW</p> <p>If no other marks, award SC1 for an answer of <math>\frac{279}{343}</math> (from working 'with replacement')</p> <p>OR</p> <p>SC1 for sight of <math>\frac{1}{7} \times \frac{3}{6} \times \frac{2}{5}</math></p>
<p><u>Alternative method</u> (<math>P(\text{total}10)+P(\text{total}11)+P(\text{total}12)+P(\text{total}13)+P(\text{total}14)=</math>)</p> <p><math>P(3,3,4) \times 3 + P(3,3,5) \times 3 + P(3,4,5) \times 6</math> <math>+ P(3,5,5) \times 3 + P(4,5,5) \times 3</math></p> <p><math>= \frac{1}{7} \times \frac{3}{6} \times \frac{1}{5} \times 3 + \frac{1}{7} \times \frac{3}{6} \times \frac{2}{5} \times 3 + \frac{1}{7} \times \frac{1}{6} \times \frac{2}{5} \times 6 + \frac{1}{7} \times \frac{2}{6} \times \frac{1}{5} \times 3 + \frac{1}{7} \times \frac{2}{6} \times \frac{1}{5} \times 3</math></p> <p><math>\frac{186}{210} (= \frac{93}{105} = \frac{62}{70} = \frac{31}{35})</math></p>	M1 M1 A1	<p><i>M0 if orderings are not considered</i></p> <p>ISW</p> <p>If no marks awarded, award SC1 for the correct method for calculating any individual total, e.g. <math>P(\text{total } 10) = \frac{1}{7} \times \frac{3}{6} \times \frac{1}{5} \times 3</math> or equivalent</p> <p>For information only: <math>P(10) = \frac{36}{210} (= \frac{6}{35})</math> <math>P(11) = \frac{72}{210} (= \frac{12}{35})</math> <math>P(12) = \frac{48}{210} (= \frac{8}{35})</math> <math>P(13) = \frac{24}{210} (= \frac{4}{35})</math> <math>P(14) = \frac{6}{210} (= \frac{1}{35})</math></p> <p>OR</p> <p>award SC1 for a calculation leading to an answer of <math>\frac{54}{210}</math> (from adding probabilities without accounting for different ordering)</p> <p>OR</p> <p>award SC1 for an answer of <math>\frac{279}{343}</math> (from working 'with replacement').</p>

<p>9.(a)</p> 	C1	<p>Clear intention to draw a curve.            Curve must pass through <math>(0,0)</math>, <math>(180,0)</math> and <math>(360,0)</math>.            AND intention to have maximum at <math>(90,1)</math> and minimum at <math>(270,-1)</math>.            Ignore curve shown for values <math>x &lt; 0^\circ</math> or <math>x &gt; 360^\circ</math>.</p>
<p>9.(b)</p> 	C1	<p>Clear intention to draw a curve with positive gradient.            Curve must pass through <math>(0,0)</math>, <math>(180,0)</math> and <math>(360,0)</math>.            AND have inflection point at <math>(180,0)</math>.            There must be an intention not to cross the asymptotes at <math>x = 90^\circ</math>, <math>x = 270^\circ</math>.            Ignore curve shown for values <math>x &lt; 0^\circ</math> or <math>x &gt; 360^\circ</math>.</p>

20. $155^\circ$ and $205^\circ$ with no other values	B2	B1 for either angle. Check diagram. Ignore extra (correct or incorrect) values outside the required range Penalise -1 for each extra value within range (beyond 2 attempts).
21. $x(4x - 5) = 2(x + 1)$ or equivalent  $4x^2 - 7x - 2 [= 0]$  $(4x + 1)(x - 2) [= 0]$  $x = -\frac{1}{4}$ AND $x = 2$	M1  A1  B2  B1	M1 for sight of $x(4x - 5)$ AND $2(x + 1)$ or equivalent  A1 Ignore presence of denominator (provided correct).  B2 B1 for $(4x \dots 1)(x \dots 2)$ OR for $(2x \pm 1)(2x \mp 2)$ FT their quadratic equation, provided of equivalent difficulty.  B1 Both answers required. Strict FT 'their <u>derived</u> brackets'.  <u>Allow use of quadratic formula</u> FT their quadratic equation, provided of equivalent difficulty.  $(x =) \frac{7 \pm \sqrt{[-7]^2 - 4(4)(-2)}}{2(4)} \quad M1$  For M1, allow one error, in sign or substitution, but not in formula. $x = \frac{7 \pm \sqrt{81}}{8} \quad A1$ $x = -\frac{1}{4}$ AND $x = 2$ (both answers required) A1  No marks for a trial and improvement method.

16. (a) $242^\circ$ and $298^\circ$ with no other values	B2	B1 for either angle. Check diagram. Penalise –1 for each extra value within range (beyond 2 attempts). Ignore extra (correct or incorrect) values outside the required range. If only two angles offered and no marks gained, award SC1 for sight of both $180^\circ + 62^\circ$ and $360^\circ - 62^\circ$ .
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with maximum at $(270^\circ, +1)$ , minimum at $(90^\circ, -1)$		<p>Must be the correct shape, i.e. a single cycle of a <u>negative sine curve</u>, with <math>x</math>-intercepts at <math>x = 0</math>, <math>x = 180^\circ</math> and <math>x = 360^\circ</math>, minimum at <math>x = 90^\circ</math>, maximum at <math>x = 270^\circ</math>.</p> <p>Accept any clear indication of <math>y</math>-coordinates.</p> <p>If not B2, award B1 for one of the following:</p> <ul style="list-style-type: none"> <li>Fully correct shape and position (both for <math>0 \leq x \leq 180^\circ</math> and for <math>180^\circ \leq x \leq 360^\circ</math>) without correct coordinates indicated</li> </ul> <p>OR</p> <ul style="list-style-type: none"> <li>Correct shape and position either for <math>0 \leq x \leq 180^\circ</math> or for <math>180^\circ \leq x \leq 360^\circ</math> (a sine curve entirely between <math>y = -1</math> and <math>y = 1</math>) AND indication of <math>y = -1</math> and <math>y = 1</math>.</li> </ul> <p>SC1 for a graph which is fully correct (including labelling) other than having pointed minimum and maximum (formed from straight lines).</p>
16.(b)(ii) Vertical translation +1 with maximum at $(90^\circ, +2)$ , minimum at $(270^\circ, 0)$ , with $y$ -intercept at +1.	B2	<p>Mark clear intention.</p> <p>Must be the correct shape, i.e. a single cycle of a <u>sine curve</u>, with consistent <math>y</math> values at <math>x = 0</math>, <math>x = 180^\circ</math> and <math>x = 360^\circ</math>, maximum at <math>x = 90^\circ</math>, minimum at <math>x = 270^\circ</math>.</p> <p>Accept any clear indication of <math>y</math>-coordinates.</p> <p>Must have correct points for <math>x = 180^\circ</math> and <math>x = 360^\circ</math>. 1 and 2 indicated on the <math>y</math>-axis.</p> <p>If not B2, award B1 for one of the following:</p> <ul style="list-style-type: none"> <li>Fully correct shape and position (both for <math>0 \leq x \leq 180^\circ</math> and for <math>180^\circ \leq x \leq 360^\circ</math>) without correct coordinates indicated</li> </ul> <p>OR</p> <ul style="list-style-type: none"> <li>Correct shape and position either for <math>0 \leq x \leq 180^\circ</math> or for <math>180^\circ \leq x \leq 360^\circ</math> (a sine curve entirely between <math>y = 0</math> and <math>y = 2</math>) AND indication of <math>y = 1</math> and <math>y = 2</math>.</li> </ul> <p>SC1 for a graph which is fully correct (including labelling) other than having pointed minimum and maximum (formed from straight lines).</p>

<p>16.(a) Cosine <u>curve</u> with appropriate orientation and position AND passing through <math>(0^\circ, 1)</math>, <math>(90^\circ, 0)</math> and <math>(270^\circ, 0)</math> and approximately <math>(180^\circ, -1)</math> and <math>(360^\circ, 1)</math> AND <math>90^\circ</math>, <math>180^\circ</math>, <math>270^\circ</math>, <math>360^\circ</math> indicated on the x-axis AND -1 and 1 indicated on the y-axis.</p> 	<p>B2</p>	<p>answer of <math>0.882/(16\dots)</math>.</p> <p>Ignore curve shown for values <math>x &lt; 0^\circ</math> or <math>x &gt; 360^\circ</math>. Accept <math>180^\circ</math> as mid-way between <math>0^\circ</math> and <math>360^\circ</math> if unlabelled. Accept <math>360^\circ</math> as unlabelled provided the sketch does not exceed <math>360^\circ</math>.</p> <p>B1 for:</p> <ul style="list-style-type: none"> <li>• General cosine <u>curve</u> with appropriate orientation and appropriate position (ignore missing or incorrect labelling) <b>OR</b></li> <li>• A continuous graph passing through <math>(0^\circ, 1)</math>, <math>(90^\circ, 0)</math> and <math>(270^\circ, 0)</math> and approximately <math>(180^\circ, -1)</math> and <math>(360^\circ, 1)</math> AND <math>90^\circ, -270^\circ</math>, indicated on the x-axis AND -1 and 1 indicated on the y-axis. Accept <math>180^\circ</math> as mid-way between <math>0^\circ</math> and <math>360^\circ</math> if unlabelled. Accept <math>360^\circ</math> as unlabelled provided the sketch does not exceed <math>360^\circ</math>.</li> </ul>									
<p>16.(b)</p> <p style="text-align: center;"><math>60^\circ</math> AND <math>300^\circ</math></p>	<p>B2</p>	<p><u>Ignore any angle outside of the range <math>0^\circ &lt; x &lt; 360^\circ</math>.</u> Note B2 for <math>60^\circ</math> AND <math>300^\circ</math> and no other angle within the range <math>0^\circ &lt; x &lt; 360^\circ</math>. Allow embedded answers.</p> <p>If not B2, award B1 for either of the following:</p> <ul style="list-style-type: none"> <li>• <math>60^\circ</math> AND <math>300^\circ</math> and one incorrect angle within the range <math>0^\circ &lt; x &lt; 360^\circ</math></li> <li>• <math>60^\circ</math> OR <math>300^\circ</math> and up to one incorrect angle within the range <math>0^\circ &lt; x &lt; 360^\circ</math></li> </ul> <p>If B2 or B1 awarded, penalise -1 for <u>each</u> further incorrect answer.</p> <table border="1" data-bbox="852 1111 1369 1254"> <thead> <tr> <th></th> <th>Radians</th> <th>Gradians</th> </tr> </thead> <tbody> <tr> <td><math>60^\circ</math></td> <td><math>\pi/3</math> or 1.047...</td> <td>66.666...</td> </tr> <tr> <td><math>300^\circ</math></td> <td>358.952...</td> <td>293.333...</td> </tr> </tbody> </table>		Radians	Gradians	$60^\circ$	$\pi/3$ or 1.047...	66.666...	$300^\circ$	358.952...	293.333...
	Radians	Gradians									
$60^\circ$	$\pi/3$ or 1.047...	66.666...									
$300^\circ$	358.952...	293.333...									