Name	Date started	Target end date

REVISE
.wales

3.09 — Reciprocal, exponential & cubic graphs

Spec 2.4.8, 2.4.9, 2.4.11 — Unit 3 (calculator allowed)

Sketching and recognising non-linear graphs: cubics $y = ax^3 + bx^2 + cx + d$, reciprocals $y = a/x$ (two branches with asymptotes), and implicit/exponential curves including circles $x^2 + y^2 = r^2$. Sourced from legacy WJEC GCSE Mathematics Higher calculator-allowed papers plus custom-authored questions to fill historically thin coverage, organised for revision under the 2025 spec.

2025 SPECIFICATION

Estimated time for entire question pack: ~1 hours 12 minutes

*Derived from the GCSE Higher pace of **~1.5 min/mark** (48 marks across 12 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **focused single-topic practice pack**, not a single mock paper. Questions are organised against the 2025 specification. Questions are ordered chronologically by sitting, with custom-written and SAM questions at the end.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed on every question in this pack (Unit 3 is the calculator-allowed paper).

Reciprocal, exponential & cubic graphs — what the new spec asks

WJEC GCSE Mathematics (first teaching 2025) · Unit 3: calculator-allowed.

Cubic graphs 2.4.8

- Recognise the S-shape of $y = ax^3 + \ldots$.
- Find x-intercepts by factorising (or reading the graph).
- Plot from a table of values and join smoothly.

Reciprocal graphs 2.4.9

- $y = a/x$ has two branches and two asymptotes.
- Sign of a controls which pair of quadrants the branches lie in.
- Curve approaches but never touches the x- and y-axes.

Exponential & circular graphs 2.4.11

- $y = ab^x$: passes through $(0, a)$, asymptote at $y = 0$.
- $x^2 + y^2 = r^2$: circle, radius r, centred at origin.
- Recognise these shapes from a sketch without scales.

Solving graphically 2.4.10

- To solve $f(x) = g(x)$, draw both curves and read intersections.
- Horizontal line $y = k$ gives solutions of $f(x) = k$.
- Quote answers to the precision the grid allows (usually 1 d.p.).

Reciprocal, exponential & cubic graphs in one page

Quick-reference notes — revisit before each question. Don't use during the questions.

Cubic graphs

$y = x^3$ has an S-shape: passes through the origin, increases throughout.

$y = -x^3$ is the mirror image: decreases throughout.

$y = (x-a)(x-b)(x-c)$ crosses the x-axis at $x = a, b, c$ and looks like a flattened S/W.

Reciprocal graphs

$y = \frac{a}{x}$ has **two branches**: one in each of opposite quadrants.

$a > 0$: branches in 1st and 3rd quadrants.

$a < 0$: branches in 2nd and 4th.

Asymptotes are the x-axis ($y = 0$) and y-axis ($x = 0$) — the curve never touches them.

Exponential graphs

$y = ab^x$ with $b > 1$: grows rapidly to the right, asymptotic to x-axis on the left.

$y = ab^{-x}$ or $0 < b < 1$: decays.

Always passes through $(0, a)$ — the y-intercept equals a.

Circles centred at origin

$x^2 + y^2 = r^2$ is a circle of radius r centred at $(0, 0)$.

Crosses both axes at $\pm r$.

Symmetric in both axes and through the origin.

Recognising shapes

Two arms going to infinity in opposite quadrants ⇒ reciprocal.

S-shape rising/falling through the origin ⇒ cubic.

Closed loop ⇒ circle (or ellipse).

One arm growing rapidly ⇒ exponential.

Tables for cubics

For $y = x^3 - 6x + 4$: compute each value of y from the given x.

$x = -3$: $-27 + 18 + 4 = -5$. $x = 0$: 4. $x = 2$: $8 - 12 + 4 = 0$ — a root.

Plot all points then join with a smooth curve.

Solving graphically

To solve $x^3 - 6x + 4 = 2$: draw $y = 2$ across your cubic and read crossings.

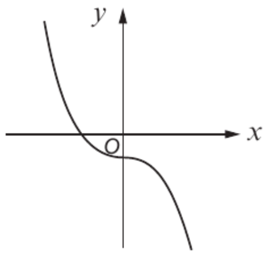
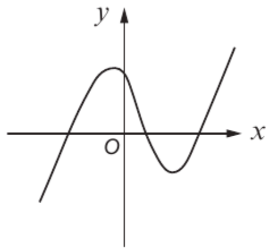
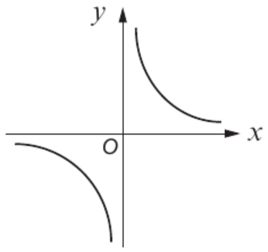
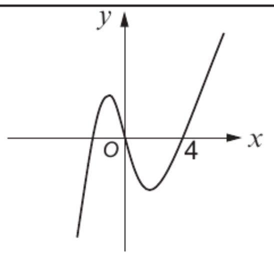
To solve $x^3 = 3x + 1$: draw $y = 3x + 1$ on $y = x^3$ and read crossings.

Common traps

- Joining the two branches of a reciprocal across the y-axis — they don't connect.
- Forgetting to label asymptotes when asked.
- Drawing a cubic with the wrong overall direction (negative leading coefficient flips it).
- Sign errors in tables for x^3 with negative x.

Examiner only

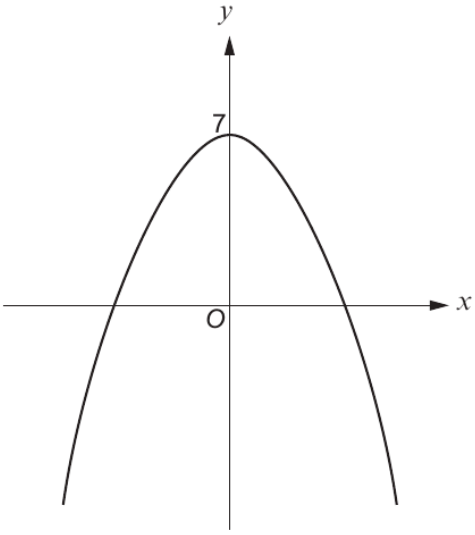
15. Circle either TRUE or FALSE for each statement given below. [2]

GRAPH	STATEMENT		
	The equation of this graph could be $y = -x^3 - 2$.	TRUE	FALSE
	The equation of this graph could be $y = x^3 - 9x$.	TRUE	FALSE
	The equation of this graph could be $y = x^{-1}$.	TRUE	FALSE
	The equation of this graph could be $y = x^3 + 4$.	TRUE	FALSE

Examiner only

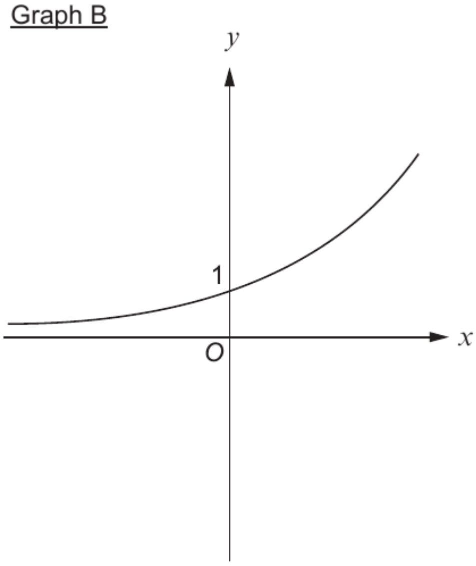
16. Each of the two graphs below is described by **one** of the equations on the right.
Put a **tick** in the box next to the equation which correctly describes each graph. [2]

Graph A



	Equation describing graph A
$y = 7x^2$	
$y = -(x + 7)^2$	
$y = (x - 7)^2$	
$y = 7 - x^2$	
$y = x^2 + 7$	

Graph B



	Equation describing graph B
$y = x^2 + 1$	
$y = 2^x$	
$y + 1 = x^2$	
$y = \frac{1}{x}$	
$y = x^0$	

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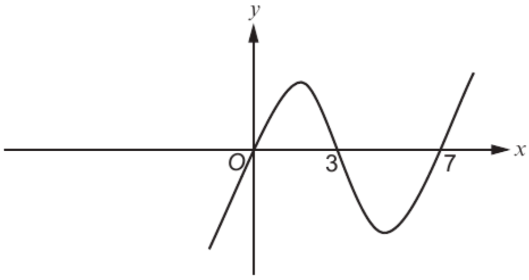
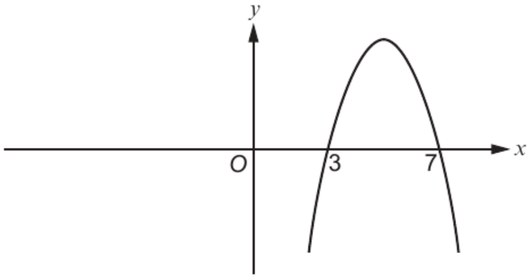
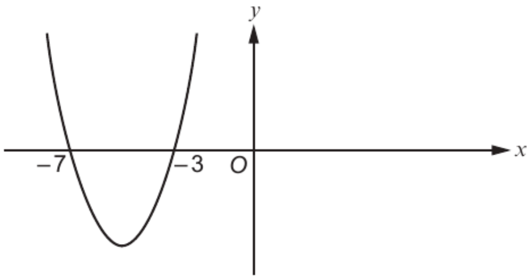
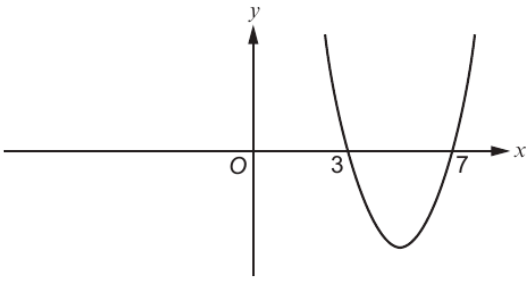
Examiner only

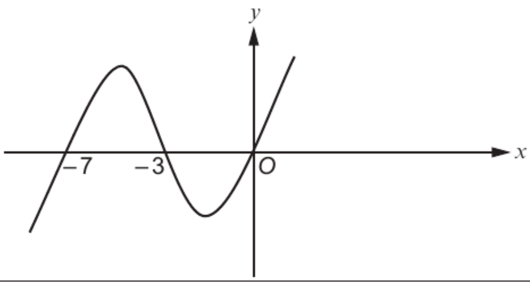
17. The equation $y = (x-3)(x-7)$ describes only **one** of the graphs below.
Put a tick (✓) in the box next to the graph that correctly shows this equation. [1]





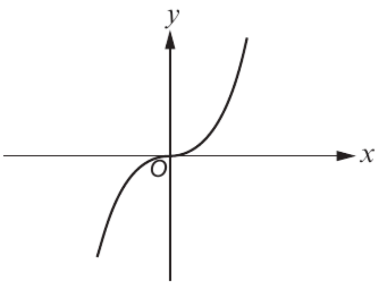


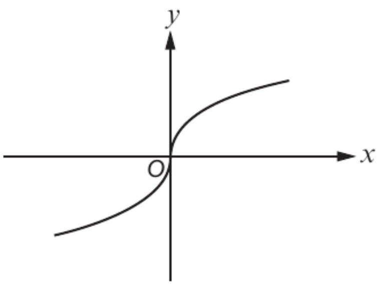


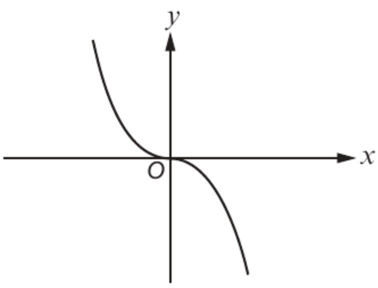


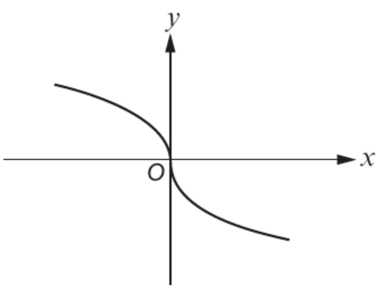
Examiner only

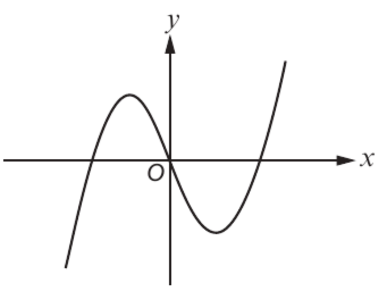
16. (a) Put a tick next to the graph that represents the equation $y = -x^3$. [1]





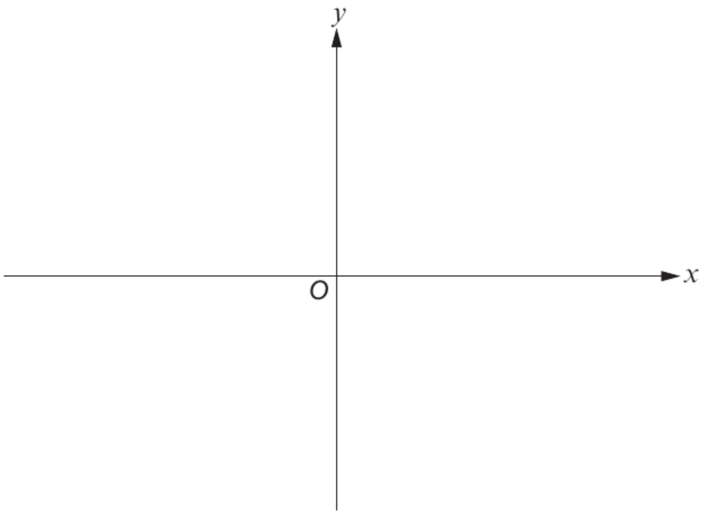






Examiner only

(b) Use the axes below to sketch the graph of $y = -\frac{1}{x}$. [2]



Spec wording (2.4.8): "draw, interpret, recognise and sketch the graphs of $y = ax^2 + b$, $y = (ax + b)(cx + d)$, $y = a/x$, $y = ax^3$"

Spec wording (2.4.9): "draw and interpret graphs of the form $y = ax^3 + b$ and $y = ax^3 + bx^2 + cx + d$"

Spec wording (2.4.11): "draw and interpret graphs when y is given implicitly in terms of x"

Assessed in Unit 3 (calculator). Each question references axes provided in the answer booklet; descriptions of axis ranges are given in the question setup.

On the axes provided below (x from −4 to 4 and y from −60 to 60, with 1 cm = 1 unit on x and 1 cm = 10 units on y):

(a) Sketch the graph of $y = x^3$. Label clearly any point where the graph crosses an axis. [2]

(b) On the same axes, sketch the graph of $y = x^3 + 20$. Label the y-intercept. [2]

On the axes provided below (x from -8 to 8 and y from -8 to 8):

(a) Sketch the graph of $y = 6/x$. [3]

(b) Write down the equations of both asymptotes. [1]

(a) Find the x-intercepts of the curve

$y = (x + 2)(x - 1)(x - 3)$. [1]

(b) On the axes provided below (x from −4 to 5 and y from −10 to 10), sketch the curve $y = (x + 2)(x - 1)(x - 3)$. Label clearly the x-intercepts and the y-intercept. [4]

The table shows some values of $y = x^3 - 6x + 4$, correct to 1 decimal place where appropriate.

x	-3	-2	-1	0	1	2	3
y	-5	8	9	4	-1	0	13

(a) On the axes provided (x from -3 to 3 and y from -6 to 14), plot the points from the table and join them with a smooth curve. [3]

(b) Use your graph to estimate the values of x for which $x^3 - 6x + 4 = 0$. Give your answers correct to 1 decimal place. [3]

Five graphs are shown below, labelled A to E. Five equations are listed underneath, labelled (i) to (v).

[Sketch placeholders for the rendered pack:

A: cubic S-curve through origin ($y = x^3$ style);

B: rectangular hyperbola in 1st and 3rd quadrants ($y = a/x$, $a > 0$);

C: positive-quadratic parabola translated downwards ($y = x^2 - 4$);

D: cubic with 3 real roots and positive leading coefficient — shape like a cubic with two turning points crossing x-axis 3 times;

E: rectangular hyperbola in 2nd and 4th quadrants ($y = a/x$, $a < 0$).]

Equations:

(i) $y = -4/x$

(ii) $y = (x + 1)(x - 2)(x - 3)$

(iii) $y = x^3$

(iv) $y = x^2 - 4$

(v) $y = 2/x$

Match each graph (A–E) to the correct equation (i–v).

The diagram below shows the graph of a cubic function passing through the origin. The curve has the following properties:

- it passes through the origin (0, 0)
- it has rotational symmetry about the origin
- it crosses the x-axis at $x = -3$ and $x = 3$

State whether each of the following statements is **TRUE** or **FALSE**. Give a reason for each answer.

(a) The equation of the graph could be $y = x^3 - 9x$. [2]

(b) The equation of the graph could be $y = x^3 + 9x$. [2]

(c) The equation of the graph could be $y = x(x^2 - 9) + 1$. [1]

The diagram below shows the graph of the equation $x^2 + y^2 = 25$.

(a) Write down the coordinates of the points where the graph crosses the x-axis. [1]

(b) The point P lies on the curve and has x-coordinate 3. Find the two possible y-coordinates of P. [2]

(c) Explain why y is not a function of x for this equation. [2]

The diagrams below (provided on the answer booklet) show, on separate axes (x from −4 to 4 and y from −10 to 20):

- the curve $y = x^3$
- the line $y = 3x + 2$

(a) On the same axes, by sketching, show the points of intersection of the two graphs. [2]

(b) Use your graph to estimate the solutions of the equation $x^3 - 3x - 2 = 0$. Give your answers correct to 1 decimal place. [3]

(c) State the number of real roots of the equation $x^3 - 3x - 2 = 0$. [1]

End of question pack