Name	Date started	Target end date

WJEC GCSE Mathematics and Numeracy (Double Award) — Question Pack

Solving cubic equations numerically by trial and improvement — substituting trial values of x, observing where the sign of $f(x)$ changes, and

REVISE
.wales

3.04 — Cubic equations by trial & improvement

Spec 2.2.9 — Unit 3 (calculator allowed)

Solving cubic equations numerically by trial and improvement — substituting trial values of x, observing where the sign of $f(x)$ changes, and narrowing the interval until the required degree of accuracy is reached. Sourced from legacy WJEC GCSE Mathematics Higher calculator-allowed Unit 2 papers, organised for revision under the 2025 spec.

2025 SPECIFICATION

Estimated time for entire question pack: ~1 hours 28 minutes

*Derived from the GCSE Higher pace of **~1.5 min/mark** (59 marks across 14 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **focused single-topic practice pack**, not a single mock paper. Questions are organised against the 2025 specification. Questions are ordered chronologically by sitting, with custom-written and SAM questions at the end.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed on every question in this pack (Unit 3 is the calculator-allowed paper).

Cubic equations by trial & improvement — what the new spec asks

WJEC GCSE Mathematics (first teaching 2025) · Unit 3: calculator-allowed.

Setting up the search 2.2.9

- Identify $f(x)$ from the cubic in the form $f(x) = 0$.
- Use the given integer bounds as your starting bracket.
- Compute $f(\text{lo})$ and $f(\text{hi})$ to confirm the sign change.

Iterative narrowing 2.2.9

- Try the midpoint (or a well-chosen value) and evaluate $f(x)$.
- Replace the endpoint with the same sign as your trial.
- Repeat until the interval is narrower than the required precision.

Stating the answer 2.2.9

- Once trapped in a 0.01-wide interval (for 1 d.p.), use the midpoint as tie-breaker.
- Pick the side that rounds to the correct d.p.
- Write the final answer at the requested precision.

Communication marks 2.2.9

- ‘QWC’ / written-communication questions reward a clearly tabulated method.
- State conclusions in words: ‘therefore the solution is … correct to … d.p.’
- Show enough trials to demonstrate convergence, not just the final value.

Crib sheet »

Cubic equations by trial & improvement in one page

Quick-reference notes — revisit before each question. Don't use during the questions.

The trial & improvement idea

You're given a cubic $f(x) = 0$ and told a solution lies between two integers, e.g. 2 and 3.

Pick a trial x, work out $f(x)$, and decide whether the true solution is bigger or smaller than your trial.

Repeat with a narrower interval each time.

Sign-change rule

If $f(2) < 0$ and $f(3) > 0$, the root lies strictly between 2 and 3.

Always look for a sign change — that's your bracket.

If $f(x) = 0$ exactly, you've hit the root — done.

Tabulate your trials

Use a 3-column table: trial x, $f(x)$, 'too high / too low'.

Examiners want to see the working — a list of trial values with their function values is the cleanest way to show it.

Lose the table, lose the method marks.

Worked example

$x^3 - 3x = 37$, root between 3 and 4.

$f(3) = 27 - 9 - 37 = -19$ (low); $f(4) = 64 - 12 - 37 = 15$ (high).

$f(3.5) = 42.875 - 10.5 - 37 = -4.625$ (low) $\Rightarrow$ root is in $(3.5, 4)$.

Continue narrowing.

Choosing the next trial

After a sign change in (a, b), try the midpoint $\frac{a+b}{2}$ first.

If that's too high, replace b; if too low, replace a.

Speed up by leaning towards whichever endpoint had the smaller $|f|$.

Achieving the required d.p.

To find the root correct to 1 d.p., test the midpoint of the final 0.1-wide interval (e.g. 3.55).

That midpoint is the 'tie-breaker': its sign tells you which 0.1-step rounds correctly.

Always state the final answer at the requested d.p.

Calculator workflow

Unit 3 is calculator-allowed — use x^3 and ANS buttons.

Store the trial in memory: substitute and read off $f(x)$ each time.

Keep your table on paper — don't rely on screen scrollback.

Common traps

- Not showing enough trials — aim for at least 3 inside the narrowed range plus the tie-breaker.
- Stopping too early — verify with the half-step before quoting the d.p.
- Mis-reading negatives, especially with $-3x$ terms.
- Forgetting to give the final answer to the d.p. asked for.

Examiner only

4. A solution to the equation

$$2x^3 - 3x - 17 = 0$$

lies between 2 and 3.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

Examiner only

3. A solution to the equation

$$x^3 - 2x - 45 = 0$$

lies between 3 and 4.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

Examiner only

2. A solution of the equation

$$x^3 + 2x = 91$$

lies between 4 and 5.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

Examiner only

3. *In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.*

A solution of the equation

$$x^3 - 7x - 51 = 0$$

lies between 4 and 5.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4 + 2 OCW]

Examiner only

5. A solution of the equation

$$x^3 - 13x = 107$$

lies between 5 and 6.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

Examiner only

8. A solution of the equation

$$2x^3 + x - 10 = 0$$

lies between 1 and 2.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

Examiner only

1. A solution of the equation

$$x^3 - 3x = 37$$

lies between 3 and 4.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

Examiner only

3. A solution of the equation

$$x^3 - 5x - 350 = 0$$

lies between 7·2 and 7·3.

Use the method of trial and improvement to find this solution correct to 2 decimal places. You must show all your working. [4]

Examiner only

3. A solution of the equation

$$x^3 + 3x = 20$$

lies between 2 and 3.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

Examiner only

8. This cuboid has:

- length = 5 cm
- width = x cm
- height = $(x^2 + 3)$ cm
- volume = 132 cm^3.

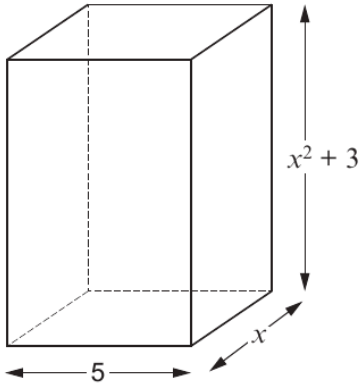

Diagram not drawn to scale

(a) Show that $5x^3 + 15x = 132$. [1]

(b) (i) A solution of the equation

$$5x^3 + 15x = 132$$

lies between 2 and 3.
Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

Examiner only

(ii) Hence, find the height of the cuboid. [1]

.....

.....

Height of the cuboid = ... cm

Examiner only

2. A solution to the equation

$$x^3 + 5x - 8 = 0$$

lies between 1 and 2.
Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

Examiner only

3. A solution of the equation

$$x^3 - 8x + 3 = 0$$

lies between 2 and 3.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

Examiner only

5. A solution of the equation

$$x^3 + 6x = 80$$

lies between 3 and 4.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

Examiner only

2. A solution of the equation

$$x^3 + 5x - 4 = 0$$

lies between 0 and 1.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]