

# REVISE

*.wales*

## 2.17 – Circle theorems

*Mark schemes for the 2.17 question pack*

*Spec 3.4.4, 3.4.5 – Unit 2*

**SOLUTIONS · 2025 SPECIFICATION**

*Mark schemes for the 18 questions in the corresponding revise.wales question pack (68 marks total). Sources: legacy WJEC GCSE papers, WJEC SAM, and custom-authored mark schemes. Pack layout © revise.wales.*

Autumn 2016			
8.	$BDC = 28(^{\circ})$ $BCD = 90(^{\circ})$  $BD = \frac{4.7}{\sin 28}$  $BD = 10(\dots)(\text{cm})$	✓ ✓  ✓✓ ✓	B1 B1  M2 A1  <u>Alternative method</u> $COB = 56(^{\circ})$ $OB = \frac{2.35}{\sin 28}$ (M1 for $\frac{2.35}{\sin 28} = \sin 28$ ) $OB = 5(\dots)(\text{cm})$ $BD = 10(\dots)(\text{cm})$
Organisation and Communication		✓	OC1 For OC1, candidates will be expected to: <ul style="list-style-type: none"> <li>• present their response in a structured way</li> <li>• explain to the reader what they are doing at each step of their response</li> <li>• lay out their explanation and working in a way that is clear and logical</li> </ul>
Accuracy of writing		✓	W1 For W1, candidates will be expected to: <ul style="list-style-type: none"> <li>• show all their working</li> <li>• make few, if any, errors in spelling, punctuation and grammar</li> <li>• use correct mathematical form in their working</li> </ul> use appropriate terminology, units, etc.

<p>13. <math>RPQ = x</math> (Reason =) Alternate segment (theorem)</p> <p><math>PQR = 90 - x</math> (Reason =) Angle in a semicircle is <math>90^\circ</math> (and angles in a triangle add up to <math>180^\circ</math>)</p>	<p>✓ ✓ ✓ ✓</p>	<p>B1 E1 B1 E1</p>	<p>Check diagram. Dependent on B1.</p> <p>F.T. <math>90 -</math> 'their RPQ'.</p> <p><u>Accept correct alternative methods e.g.</u></p> <p>(i) <math>PRA = 90 - x</math> <span style="float:right">B1</span>              (Reason =) Angle in a semicircle is <math>90^\circ</math>              (and angles on a straight line add up to <math>180^\circ</math>) <span style="float:right">E1</span>  <math>PQR = 90 - x</math> FT <math>PQR =</math> 'their PRA' <span style="float:right">B1</span>              (Reason =) Alternate segment (theorem) <span style="float:right">E1</span></p> <p>(ii) <math>ORQ = 90 - x</math> <span style="float:right">B1</span>              'radius and tangent meet at <math>90^\circ</math>' <span style="float:right">E1</span>  <math>PQR = 90 - x</math> <span style="float:right">B1</span>              Isosceles triangle <span style="float:right">E1</span></p>
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		= 28 C.A.O. B1
12.		<i>All 'E1' marks are dependent on associated 'B1' marks.</i>
ACB = 74(°)	✓	B1 By applying the alternate segment theorem. Check diagram.
Alternate segment (theorem)	✓	E1
CAB (= 180 – 53 – 74) = 53(°)	✓	B1 FT from B1 E0, but not from B0 E0.
(Sum of) angles in a triangle (equals 180°)	✓	E1
Concluding statement	✓	E1
e.g. 'triangle ABC is isosceles as it has two equal angles.'		Must justify <u>why</u> the triangle is isosceles i.e. stating (only) 'triangle is isosceles' at this stage is insufficient.
		<u>Alternative method</u>
		CAX = 53(°) (by applying the alternate segment theorem) B1
		Alternate segment (theorem) E1
		CAB (= 180 – 53 – 74) = 53(°) B1
		(Sum of) angles on a straight line (equals 180°) E1
		Concluding statement. E1
		<i>Be aware of equivalent methods (e.g. drawing a radius to the centre O etc.). These methods must</i>

<p>4.</p> <p style="text-align: center;"> <math>(RQP \text{ or } QRP =) \frac{180 - 30}{2}</math>  <math>= 75^\circ</math> </p> <p>Tangents (from external point) are equal (in length)  OR a geometric consequence based on this fact  e.g. 'QPR is isosceles' or 'PQOR is a kite'.</p> <p style="text-align: center;"> <math>(OQR = 90 - 75 =) 15^\circ</math> </p> <p>Tangent and radius (at any point) are perpendicular</p>	<p>M1</p> <p>A1</p> <p>E1</p> <p>B1</p> <p>E1</p> <p>OC1</p> <p>W1</p>	<p><i>Note: Both E1 marks are awarded for a suitable/valid attempt at statement (not an implied reason from a calculation). Both E marks are dependent on attempt at related work. Look for angles seen on the diagram. For this question allow angles shown in diagram to take precedence over answer space.</i></p> <p>Accept any suitable attempt at a valid statement. Allow PQ = PR. Also allow unambiguous indication on the diagram. 'Angles in a triangle' not sufficient.</p> <p>F.T. 'their derived 75' provided acute.</p> <p>Accept any suitable attempt at a valid statement. Also allow unambiguous indication on the diagram.</p> <p><u>Alternative method 1</u>  <math>(ROQ = 360 - 90 - 90 - 30 =) 150^\circ</math>      B1  Tangent and radius (at any point) are perpendicular.      E1  <math>OQR = \frac{180 - 150}{2}</math>      M1  <math>= 15^\circ</math>      F.T. 'their derived 150'  Radii form an isosceles triangle.      E1</p> <p><u>Alternative method 2 (with line OP drawn)</u>  <math>(POQ \text{ or } RQP =) 180 - 90 - 15</math>      M1  <math>= 75^\circ</math>      A1  Tangents (from external point) are equal (in length)  OR a geometric consequence based on this fact  e.g. 'QPR is isosceles' or 'PQOR is a kite'.      E1  <math>(OQR = 90 - 75 =) 15^\circ</math>      B1  F.T. 'their derived 75' provided acute  Tangent and radius (at any point) are perpendicular.      E1</p> <p>[Note: Do not 'mix and match' marks from alternative methods.]</p> <p>Organisation and Communication.  For OC1, candidates will be expected to:</p> <ul style="list-style-type: none"> <li>• present their response in a structured way</li> <li>• explain to the reader what they are doing at each step of their response</li> <li>• lay out their explanation and working in a way that is clear and logical</li> </ul> <p>Accuracy of writing.  For W1, candidates will be expected to:</p> <ul style="list-style-type: none"> <li>• show all their working</li> <li>• make few, if any, errors in spelling, punctuation and grammar</li> <li>• use correct mathematical form in their working</li> <li>• use appropriate terminology, units, etc</li> </ul>
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<p>10. For a correct proof: i.e. each angle <u>within the triangle</u> is correctly evaluated as <math>60^\circ</math> AND with correct reasons. No assumptions can be made at any stage of the proof for the evaluation of any angles.</p>	<p>E1 E1 E1</p>	<p>If any other angle is used within the proof e.g. angle FYH, then a correct reason must again be stated (there is no E1 for this angle as it is working towards one of the angles within the triangle).</p> <p>Award E2 for two angles <u>within the triangle</u> correctly evaluated as <math>60^\circ</math> AND with correct reasons.</p> <p>Award E1 for one angle <u>within the triangle</u> correctly evaluated as <math>60^\circ</math> AND with correct reason(s).</p> <p>*Do not accept 'Z' angles for alternate angles.</p> <p><u>Examples</u>  <math>\angle EYF = 60^\circ</math> AND Alternate Segment Theorem; E1  <math>\angle FEY = 60^\circ</math> AND Alternate angles; E1  <math>(\angle EYF = 60^\circ</math> AND) angles in a triangle (therefore equilateral) OR  <math>(\angle EYF = 60^\circ</math> AND) therefore equilateral E1</p> <p><math>\angle FEY = 60^\circ</math> AND Alternate angles E1  <math>\angle FYH = 60^\circ</math> AND Alternate Segment Theorem E1  <math>\angle FYE = (180^\circ - 60^\circ - 60^\circ) = 60^\circ</math> AND straight line E1  <math>(\angle EYF = 60^\circ</math> AND) angles in a triangle (therefore equilateral) OR  <math>(\angle EYF = 60^\circ</math> AND) therefore equilateral E1</p> <p><math>\angle EYF = 60^\circ</math> AND Alternate Segment Theorem E1  <math>\angle FYH = 60^\circ</math> AND Alternate angles E1  <math>\angle FYE = (180^\circ - 60^\circ - 60^\circ) = 60^\circ</math> AND straight line E1  <math>(\angle FEY = 60^\circ</math> AND) angles in a triangle (therefore equilateral) OR  <math>(\angle FEY = 60^\circ</math> AND) therefore equilateral E1</p> <p><math>\angle EYF = 60^\circ</math> AND Alternate Segment Theorem E1  <math>\angle FYE = 60^\circ</math> AND Interior angles E1  <math>(\angle EYF = (180^\circ - 60^\circ - 60^\circ) = 60^\circ</math> AND) angles in a triangle (therefore equilateral) OR  <math>(\angle EYF = 60^\circ</math> AND) therefore equilateral E1</p>
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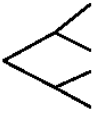
<p>19.(a) <math>(AE^2 =) 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos 31^\circ</math>  <math>(AE =) 5.8(\dots \text{cm})</math></p>	<p>M1 A2</p>	<p>Award A2 for an answer of 6(cm) from correct working.  A1 for <math>(AE^2 =) 34.1(\dots)</math></p>
<p>19.(b) <math>\sin CAE = \frac{8 \times \sin 31^\circ}{5.8(\dots)}</math>   <math>(CAE =) 44.8(\dots^\circ)</math>   <math>(CED =) 44.8(\dots^\circ)</math></p>	<p>M2  A1  B1</p>	<p>FT 'their AE' from 19(a). Check the diagram.  M1 for <math>\sin CAE = \frac{\sin 31^\circ}{5.8(\dots)}</math> or equivalent  Accept answers in the range <math>44.7^\circ</math> to <math>45.3^\circ</math>.   Strict FT of 'their CAE', provided not <math>31^\circ</math>. Must be convincing (check the diagram).  Accept answers in the range <math>44.7^\circ</math> to <math>45.3^\circ</math>.</p>
<p><u>Alternative method 1</u>   <math>\cos(CAE) = \frac{11^2 + 5.8(\dots)^2 - 8^2}{2 \times 11 \times 5.8(\dots)}</math>   <math>(CAE =) 44.8(\dots^\circ)</math>   <math>(CED =) 44.8(\dots^\circ)</math></p>	<p>M2  A1  B1</p>	<p>FT 'their AE' from 19(a). Check the diagram.  M1 for  <math>8^2 = 11^2 + 5.8(\dots)^2 - 2 \times 11 \times 5.8(\dots) \times \cos(CAE)</math>  Accept answers in the range <math>44.7^\circ</math> to <math>45.3^\circ</math>.   Strict FT of 'their CAE', provided not <math>31^\circ</math>. Must be convincing (check the diagram).  Accept answers in the range <math>44.7^\circ</math> to <math>45.3^\circ</math>.</p>
<p><u>Alternative method 2 (Initially evaluating CEA)</u>   <math>\sin CEA = \frac{11 \times \sin 31^\circ}{5.8(\dots)}</math> OR   <math>\cos CEA = \frac{5.8(\dots)^2 + 8^2 - 11^2}{2 \times 5.8(\dots) \times 8}</math>   <math>(CEA =) 104.1(\dots)</math> [or <math>75.8(\dots)</math> from sine rule]   <math>(CAE =) 180 - 31 - 75.8(\dots) = 73.2(\dots)</math> or  <math>180 - 31 - 104.1(\dots) = 44.8(\dots)</math>   <math>(CED =) 44.8(\dots)</math> or <math>73.2(\dots)</math></p>	<p>M2  A1  B1</p>	<p>FT 'their AE' from 19(a). Check the diagram.  M1 for <math>\sin CEA = \frac{\sin 31^\circ}{11}</math> or equivalent OR  <math>\frac{11}{5.8(\dots)}</math>   M1 for  <math>11^2 = 5.8(\dots)^2 + 8^2 - 2 \times 5.8(\dots) \times 8 \times \cos CEA</math>  Accept answers in the range <math>103.7</math> to <math>104.3</math> or <math>75.7</math> to <math>77.7</math> OR <math>78</math>.   Strict FT of 'their CAE', provided not <math>31^\circ</math>. Must be convincing (check the diagram).  Accept answers in the range <math>44.7^\circ</math> to <math>45.3^\circ</math>.</p>

14. (Total area =) (2 ×) $\pi \times 30^2 \times 20 / 360$ or equivalent (= $100\pi$ ) $100\pi = \pi r^2$ $r = 10$ (cm)	M1	Accept use of 3·14 for $\pi$ .
	m1	Equating 'their derived $100\pi$ ' or equivalent
	A1	CAO

<p>8.</p> <p>One correct evaluation <math>1 \leq x \leq 2</math>                  2 correct evaluations <math>1.55 \leq x \leq 1.75</math>,                  one <math>&lt; 0</math>, one <math>&gt; 0</math>.                  2 correct evaluations <math>1.55 \leq x \leq 1.65</math>,                  one <math>&lt; 0</math>, one <math>&gt; 0</math>.</p> <p style="text-align: center;"><math>x = 1.6</math></p>	<p>B1 B1 M1 A1</p>	<p><i>Correct evaluation regarded as enough to identify if 'too high' or 'too low'. If evaluations not seen accept 'too high' or 'too low'.</i></p> <p style="text-align: center;"><math>x</math>                      <math>2x^3 + x - 10</math> (or check <math>2x^3 + x = 10</math>)</p> <p>1                      -7</p> <p>1.1                    -6.238</p> <p>1.2                    -5.344</p> <p>1.3                    -4.306</p> <p>1.4                    -3.112                    1.45   - 2.452...</p> <p>1.5                    -1.75                    1.55   - 1.002...</p> <p><b>1.6                    -0.208                    1.65   0.634...</b></p> <p><b>1.7                    1.526                    1.75   2.468...</b></p> <p>1.8                    3.464                    (1.62 0.123..)</p> <p>1.9                    5.618                    (1.63 0.291..)</p> <p>2                      8                          (1.64 0.461..)</p>
<p>9.                    <math>85\% \equiv \frac{6154}{85}</math>  <math>\frac{6154 \times 100}{85}</math> OR <math>\frac{6154}{0.85}</math>  <span style="margin-left: 150px;"><math>= 7240</math></span></p>	<p>B1 M1 A1</p>	<p>Accept any indication.                  Implies the B1.</p>
<p>10.                    <math>x = 54^\circ</math>  <u>Opposite angles</u> (of a) <u>cyclic quad.</u> (add up to <math>180^\circ</math>).</p> <p style="text-align: center;"><math>y = 108^\circ</math>  <u>Angle at the centre</u> (is twice the angle at the circumference).</p>	<p>B1 E1  B1 E1</p>	<p>Dependent on an attempt at <math>180 - 126</math>.</p> <p>FT <math>2 \times</math> 'their <math>54^\circ</math>' only if less than <math>360^\circ</math>                  Dependent on an attempt at <math>2 \times</math> 'their <math>54^\circ</math>'.</p>
<p>11.                    Correct enlargement</p>	<p>B2</p>	<p>Otherwise B1 for 2 correct vertices within a triangle.                  OR for 3 correct vertices in the correct location not joined to form the triangle                  OR triangle of correct shape, size and orientation in incorrect position                  OR consistent correct use of an incorrect negative scale factor.</p>
<p>12(a).                <math>(9p + 1)(9p - 1)</math></p>	<p>B2</p>	<p>B1 for <math>(9p \dots 1)(9p \dots 1)</math></p>
<p>12(b).                <math>(7t - 2)(t + 3)</math></p>	<p>B2</p>	<p>B1 for <math>(7t \dots 2)(t \dots 3)</math></p>
<p>13.                    Sight of 297.5 AND 6.5  <span style="margin-left: 100px;"><math>297.5 \div 6.5</math></span>  <span style="margin-left: 150px;"><math>= 45.77(\text{km/h})</math></span></p>	<p>B1 M1  A1</p>	<p>Accept 6 hours 30 minutes, but not 6.3 hours.                  If other calculations shown, then the relevant calculation must be identified.                  Award M1 for their values provided <math>295 \leq d &lt; 300</math> AND <math>6 &lt; t \leq 7</math> (but not 6 hours 30 minutes).                  CAO. Correct answer must be clearly identified.</p>
<p>14.                    <math>\sin \text{BAD} = (2 \times 70) / (8 \times 19)</math> or equivalent</p> <p style="text-align: center;"><math>(\text{BAD} =) 67(.08 \dots)^\circ</math></p> <p>(Area of sector ABD =) <math>67(.08 \dots) / 360 \times \pi \times 8^2</math></p> <p>Accept answers in the range <math>37.4(\text{cm}^2)</math> to <math>37.5(\text{cm}^2)</math>                  OR <math>37(\text{cm}^2)</math></p>	<p>M2  A1  M1  A1</p>	<p>Allow any unambiguous indication of angle BAD.                  M1 for the <u>correct use</u> of the formula when <math>\sin \text{BAD}</math> is <u>not</u> the subject, for example: <math>70 = 1/2 \times 8 \times 19 \times \sin \text{BAD}</math>.</p> <p>Allow any answer that rounds to <math>67^\circ</math>.</p> <p>Accept <math>292.9(\dots) / 360 \times \pi \times 8^2</math> OR <math>293 / 360 \times \pi \times 8^2</math> for the area of the major sector ABD.                  FT their derived or stated value of angle BAD.</p> <p>Accept an answer in the range <math>163.5(\text{cm}^2)</math> to <math>163.7(\text{cm}^2)</math> OR <math>164(\text{cm}^2)</math> for the area of the major sector ABD.</p>

<p>14.</p> $EBC \text{ or } ECB = (180 - 58) / 2$ $= 61(^{\circ})$ $BAC = 61(^{\circ})$ $ABC (= 180 - 35 - 61) = 84(^{\circ})$	<p>M1 A1</p> <p>B1</p> <p>B1</p>	<p>Check diagram. Angles in an isosceles triangle.</p> <p>Alternate segment theorem. FT 'their <i>EBC</i> or <i>ECB</i>'.</p> <p>FT <math>180 - 35 -</math> 'their <i>BAC</i>'.</p>
<p><u>Alternative method 1</u></p> $EBC \text{ or } ECB = (180 - 58) / 2$ $= 61(^{\circ})$ $DBA = 35(^{\circ})$ $ABC (= 180 - 35 - 61) = 84(^{\circ})$	<p>M1 A1</p> <p>B1</p> <p>B1</p>	<p>Check diagram. Angles in an isosceles triangle.</p> <p>Alternate segment theorem.</p> <p>Angles on a straight line FT <math>180 -</math> 'their <i>EBC</i>' - 'their <i>DBA</i>'.</p>
<p><u>Alternative method 2</u></p> $EBC \text{ or } ECB = (180 - 58) / 2$ $= 61(^{\circ})$ $ACF (= 180 - 35 - 61) = 84(^{\circ})$ $ABC = 84(^{\circ})$	<p>M1 A1</p> <p>B1</p> <p>B1</p>	<p>Check diagram. Angles in an isosceles triangle.</p> <p>Angles on a straight line. FT <math>180 - 35 -</math> 'their <i>ECB</i>'.</p> <p>Alternate segment theorem. FT 'their <i>ACF</i>'.</p>
<p><u>Alternative method 3</u> (using isosceles triangle <i>BOC</i>, where <i>O</i> is the centre of the circle)</p> $BOC = 360 - 90 - 90 - 58$ $= 122$ $BAC = 61$ $ABC (= 180 - 35 - 61) = 84(^{\circ})$	<p>M1 A1</p> <p>B1</p> <p>B1</p>	<p>Check diagram.</p> <p>Angles in kite <i>BOCE</i></p> <p>Use of angle in the centre FT 'their <i>BOC</i>' <math>\div 2</math> FT <math>180 - 35 -</math> 'their <i>BAC</i>'</p>

<p>11. <math>I \propto 1/d^2</math> OR <math>I = k/d^2</math> or equivalent</p> <p><math>5 = k/2^2</math> OR <math>k = 20</math></p> <p><math>I = 20/d^2</math> OR <math>I = 20/0.5^2</math> or equivalent</p> <p><math>I = 80</math> (lux)</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Allow <math>I \propto k/d^2</math></p> <p>M1 implies B1.</p> <p>F.T. (for possible B0 M2 A0) for use of <math>I \propto d^2</math> or <math>I \propto 1/d^n</math> with <math>n &gt; 0</math> and <math>n \neq 2</math>.</p> <p>CAO.</p> <p>Use of <math>I \propto 1/d</math>, leading to <math>I = 10/d</math> (or an answer of <math>I = 20</math> (lux)) is awarded B0 FT M2 A0.</p> <p>Use of <math>I \propto d^2</math>, leading to <math>I = 1.25 d^2</math> (or an answer of <math>I = 0.3125</math> (lux)) is awarded B0 FT M2 A0.</p> <p>Use of <math>I \propto 1/\sqrt{d}</math>, leading to <math>I = 5\sqrt{2}/\sqrt{d}</math>, (or an answer of <math>I = 10</math> (lux)) is awarded B0 FT M2 A0.</p>
<p>12. <math>CAD = 2x</math></p> <p>(Reason =) Alternate segment (theorem)</p> <p><math>BCD = 180 - 3x</math> OR <math>BCD = 3(60 - x)</math></p> <p>(Reason =) Opposite angles in a cyclic quadrilateral (add up to <math>180^\circ</math>)</p>	<p>B1</p> <p>E1</p> <p>B1</p> <p>E1</p>	<p>Check diagram. (If this is the only B mark awarded, then <math>2x</math> marked on diagram must be unambiguous. Otherwise, ignore spurious angles on diagram.)</p> <p>Dependent on B1.</p> <p>Allow 'opposite segments'. Do not accept 'alternate angles' or 'opposite angles'.</p> <p>F.T. <math>180 - (x + \text{'their CAD'})</math>. Must be in simplest form.</p> <p>Mark final answer</p> <p>e.g. do not accept <math>60 - x</math> or <math>x = 60</math></p> <p>If B0, E mark may be awarded provided there is a clear attempt to apply the circle theorem.</p>
<p>13.(a) <math>48x^2 + 6x - 48x^2 + 12x - 12x + 3</math></p> <p>OR <math>48x^2 + 6x - 48x^2 + 3</math>.</p> <p><math>6x + 3</math></p>	<p>B2</p> <p>B1</p>	<p>Accept <math>48x^2 + 6x - (48x^2 - 12x + 12x - 3)</math> or <math>48x^2 + 6x - (48x^2 - 3)</math></p> <p>B1 for <math>16x^2 [-4x + 4x] - 1</math> or <math>48x^2 [-12x + 12x] - 3</math> or <math>-48x^2 [+12x - 12x] + 3</math>.</p> <p>OR</p> <p>B1 if one error or incorrect (or extra or missing) term within entire expression.</p> <p>(An incorrect term may be implied e.g. <math>-24x</math> implies <math>-12x - 12x</math>).</p> <p>Must be convincing.</p> <p>For last B1, do not accept <math>48x^2 + 6x - (48x^2 - 12x + 12x - 3)</math> or <math>48x^2 + 6x - (48x^2 - 3)</math> without further correct work seen before final <math>6x + 3</math>.</p> <p>If <u>no work</u> seen in (a), allow marks in (a) for work shown in (b)</p>
<p>13.(b) <math>-\frac{1}{2}</math> or <math>-\frac{3}{6}</math> or <math>-0.5</math> or equivalent</p>	<p>B1</p>	<p>Mark final answer.</p>

<p>7.(a) Correct framework</p>  <p>Suitable labelling on both 1<sup>st</sup> pair of branches AND on both of at least one pair of 2<sup>nd</sup> set of branches. e.g. 'Car', 'No car', 'Before 8', 'After 8'. OR Titles of 'Car' and 'Before 8' with branch endings of 'Yes' and 'No'.</p> <p>Correct probabilities on first pair of branches 0·7 AND 0·3 (for 'Car', 'No car') OR 0·4 AND 0·6 (for 'Before 8', 'After 8')</p> <p>Correct probabilities on second two sets of branches 0·4 AND 0·6 correctly placed (following 0·7 and 0·3) OR 0·7 AND 0·3 correctly placed (following 0·4 and 0·6)</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p>	<p>Accept any unambiguous wording.</p> <p>Must be consistent with their labelling. Allow this B1 if no headings given, <u>unless</u> contradicted by, or inconsistent with, further labelling.</p> <p>Allow this B1 if no headings given, <u>unless</u> contradicted by, or inconsistent with, further labelling.</p> <p>Allow this B1 if only shown on one set of branches. Provided not contradicted on the other set of branches.</p>
<p>7.(b) <math>0·7 \times 0·4</math> or equivalent. <math>= 0·28</math> or equivalent.</p>	<p>M1</p> <p>A1</p>	<p>No FT. M1A0 for a final answer of 0·28%. Mark final answer.</p>
<p>8.(a) <math>PA = 12(\text{cm})</math> AND correct theorem given, e.g. 'tangents from an external point are equal in length'.</p>	<p>E1</p>	<p>Must use the words '<u>tangents</u>' AND '<u>equal (identical/same)</u>'.  Do not accept e.g. 'PA = PB'. (E0) Accept alternative correct answers.</p>
<p>8.(b) <math>\hat{PAO} = 90(^{\circ})</math> AND correct theorem given, e.g. 'the tangent at any point on a circle is perpendicular to the radius at that point'.</p>	<p>E1</p>	<p>Must use the words '<u>tangent</u>' AND '<u>radius (diameter)</u>'. Allow e.g. 'radius and tangent meet at 90'. (E1) Do not accept e.g. 'PA and OA meet at 90'. (E0)</p>
<p>8.(c) (Area PAOB =) <math>2 \times \frac{12 \times 4}{2}</math> or equivalent.  <math>= 48 (\text{cm}^2)</math></p>	<p>M1</p> <p>A1</p>	<p>OR FT '<u>their PA</u>' <math>\times 4 + \frac{12 \times 4}{2}</math> M0 for <math>48 \times 2</math> or <math>12 \times 4 \times 2 (= 96)</math>  An unsupported final answer of 48 gains both marks. If no marks gained allow SC1 for sight of <math>24(\text{cm}^2)</math> OR a correct evaluation of ('their PA' <math>\times 4) / 2</math>.</p>
<p>9.(a) <math>y = 2·5x + 3</math></p>	<p>B1</p>	
<p>9.(b) <math>y = 3x - 5</math></p>	<p>B1</p>	
<p>9.(c) Line D</p>	<p>B1</p>	
<p>10.(a) <math>t \propto 1/g</math> OR <math>t = k/g</math> <math>36 = k/25</math> OR <math>k = 900</math>  <math>t = 900/g</math></p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>Allow <math>t \propto k/g</math> FT from <math>y \propto 1/x^n</math> with <math>n \neq 1, n &gt; 0</math> No FT from direct proportion M1 implies B1. May be seen explicitly in part (b). Do not allow <math>t \propto 900/g</math> for the A mark</p>
<p>10.(b) <math>(900/20 =)</math> 45 (days)</p>	<p>B1</p>	<p>FT 'their formula' only if non-linear.</p>
<p>10.(c) Sight of 900/40  22 (goats)</p>	<p>M1</p> <p>A1</p>	<p>FT 'their formula' only if non-linear and of equivalent difficulty  M1 A0 for an answer of 22·5 or 23 For A1, FT for equivalent difficulty i.e. need to round down an answer with a decimal part of 0.5 or over. Allow use of trial and improvement for M1, provided 22 or 23 seen. A0 for incorrect working e.g. <math>90/4</math> given as 22.2, leading to 22.</p>
<p>11. (a) <math>(\sqrt[3]{m})^2</math></p>	<p>B1</p>	
<p>11. (b) <math>p^{\frac{1}{3}}</math></p>	<p>B1</p>	

<p>10.(a)(i) Correct reason given. e.g. 'An angle at the circumference subtended by a diameter is a right angle'. 'line AC is a diameter'</p>	<p>E1</p>	<p>Accept any correct unambiguous wording. The key word is 'diameter'.  Allow eg 'angle in a semicircle is 90°', 'line AC goes through the centre', 'opposite a diameter' Do not accept 'because it's a right angle'.</p>
<p>10.(a)(ii) <math>\tan x = \frac{7.5}{4.7}</math> <math>x = \tan^{-1}(7.5 / 4.7)</math> or <math>\tan^{-1} 1.6</math> or <math>\tan^{-1} 1.59(\dots)</math> <math>= 57.9(\dots)^\circ</math> or <math>57.8(\dots)^\circ</math> or <math>58^\circ</math></p>	<p>M1 m1 A1</p>	<p>Implies M1.  C.A.O. <u>Alternative method to find x</u> A correct and complete method (using Pythagoras's theorem and a trigonometric relationship). M2 <math>x = 57.9(\dots)^\circ</math> or <math>57.8(\dots)^\circ</math> or <math>58^\circ</math> CAO A1</p>
<p>10.(b) <math>(y =) 58^\circ</math>  Correct circle theorem given. e.g. 'angles (at the circumference) subtended by the same chord (or arc) are equal', 'angles in the same segment (are equal)'.</p>	<p>B1 E1</p>	<p><u>Strict</u> FT of 'their x'.  Accept any correct unambiguous wording. Allow eg 'angles on the same chord (are equal)' Do not accept e.g. 'they are equal' on its own.</p>
<p>11. <math>2^{400}</math></p>	<p>B2</p>	<p>B1 for <math>(2^{100})^4</math> OR sight of <math>2^4</math></p>
<p>12. (Height =) <math>\frac{3 \times 5533}{825}</math> OR <math>\frac{5533}{\frac{1}{3} \times 825}</math>  <math>= 20.1(2 \text{ cm})</math> ----- <i>Alternative method (finding the radius first):</i>  Use <math>A = \pi r^2</math> to evaluate <math>r</math> or <math>r^2</math>.  (Height =) <math>\frac{3 \times 5533}{\pi \times 16.2(05\dots)^2}</math> OR <math>\frac{5533}{\frac{1}{3} \times \pi \times 16.2(05\dots)^2}</math> OR  <math>\frac{3 \times 5533}{\pi \times 262.6(\dots)}</math> OR <math>\frac{5533}{\frac{1}{3} \times \pi \times 262.6(\dots)}</math>  <math>= 20.1(2\dots \text{ cm})</math></p>	<p>M2 A1 M2 A1</p>	<p>M1 for <math>5533 = 1/3 \times \text{height} \times 825</math> or equivalent.  Allow an answer of 20(cm) from correct working.  <i>Allow use of <math>\pi = 3.14, 3.142</math> or <math>3.14(59\dots)</math>. When using the <math>\pi</math> button on the calculator, <math>r = 16.2(05\dots)</math> OR <math>r^2 = 262.6(\dots)</math>.</i>  <i>There will be no FT for any radius other than <math>r = 16\text{cm}</math>, from working seen.</i>  M1 for <math>5533 = 1/3 \times \text{height} \times \pi \times 16.2(05\dots)^2</math> or equivalent. Allow M1 for use of <math>r = 16</math> (cm)  A1 Allow an answer of 20(cm) from correct working. Accept an answer in the range 20.10 to 20.143(cm) <u>FT base radius = 16 cm</u>: Allow an answer in the range 20.6(cm) to 20.65(cm) OR 21(cm) from correct working.</p>
<p>13.(a) <math>(2x + 9)(2x - 9)</math></p>	<p>B2</p>	<p>B1 for <math>(2x \dots 9)(2x \dots 9)</math></p>
<p>13.(b) <math>(7x - 4)(x + 2)</math></p>	<p>B2</p>	<p>B1 for <math>(7x \dots 4)(x \dots 2)</math></p>
<p>13.(c) <math>(x + 2)^2(x + 7)</math> OR <math>(x + 2)(x + 2)(x + 7)</math></p>	<p>B2</p>	<p>B1 for <math>(x + 2)^2(x + 2 + 5)</math> OR <math>(x + 2)[(x + 2)^2 + 5(x + 2)]</math> OR <math>(x + 7)(x^2 + 4x + 4)</math> OR <math>(x + 2)(x^2 + 9x + 14)</math>. Allow B1 for <math>(x + 2)^2(x + k)</math> where <math>k \neq 0, 2</math> or <math>7</math>.</p>
<p>14. <math>-\frac{1}{2}</math> or equivalent</p>	<p>B2</p>	<p>B1 for <math>-2</math> or <math>\frac{1}{2}</math>.</p>
<p>15. <math>2n^2 + 1</math> or equivalent  <math>= 20001</math></p>	<p>B2 B1</p>	<p>B1 for sight of <math>2n^2</math> OR for sight of consistent 2<sup>nd</sup> difference 4. B1 FT from their <math>2n^2 \pm k</math>, where <math>k \neq 0</math> OR from their <math>2n^2 \pm an</math>, where <math>a \neq 0</math> OR from their <math>2n^2 \pm an \pm k</math>, where <math>a \neq 0, k \neq 0</math>. An unsupported answer of 20001 gains all 3 marks. If no marks, award SC1 for an unsupported answer of 20000.</p>

<p><math>= 1.8(\dots m^2)</math></p> <p>14.(a) <math>68^\circ</math> AND alternate segment theorem.</p>	<p>A1</p> <p>B2</p>	<p>Allow <math>1.9(\dots m^2)</math> from correct working.</p> <p>Do not accept 'alternate (angle) theorem' or 'alternate angles' only as the given angle property. <math>68^\circ</math> may be seen on the diagram at ACB. B1 for <math>68^\circ</math> Award B0 for any angle other than ACB clearly identified as <math>68^\circ</math></p>
<p><u>Alternative method</u> Allow a <b>correct and complete method</b> that results in an angle of <math>68(\cdot 219\dots^\circ)</math> AND relevant angle property (e.g. angles on a straight line OR angles in a triangle OR using the sine rule).</p>	<p>B2</p>	<p>Allow B1 for a <b>correct and complete method</b> that results in an angle of <math>68(\cdot 219\dots^\circ)</math>.</p>
<p>14.(b) <math>\frac{1}{2} \times 7 \times 13 \times \sin 68^\circ</math></p> <p><math>= 42.1(\dots cm^2)</math> OR <math>42.2(\dots cm^2)</math></p>	<p>M1</p> <p>A1</p>	<p>FT 'their 68' identified as their ACB from part (a). Award M1 for a complete alternative method leading to a correct answer of <math>42.1(\dots cm^2)</math> OR <math>42.2(\dots cm^2)</math>.</p>

<p>10.(a) (AOY =) 36(°)</p> <p>(% shaded =) <math>\frac{36}{360} (\times 100)</math> or equivalent</p> <p>= 10(%)</p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>Check diagram.</p> <p>FT 'their derived or stated angle AOY' provided not 54°.</p> <p>Award M0A0 for <math>\frac{360(^\circ)}{36(^\circ)} = 10</math>, but award M1A1 if a final answer of 10% is seen.</p> <p>If no marks awarded, award:</p> <ul style="list-style-type: none"> <li>• SC2 for unsupported 10% (AOY not shown or stated to be 36(°))</li> <li>• SC1 for a final answer of 15% (from using 54(°)).</li> </ul>
<p>10.(b) Statement explaining that, 'The <u>tangent</u> at any point on a circle is <u>perpendicular</u> (or equivalent) to the <u>radius</u> at that point'.</p>	<p>E1</p>	<p>Accept unambiguous similar wording. e.g. 'Radius and tangent 90(°)'. Diameter could be used in place of radius. Must refer to <u>tangent</u> and <u>radius</u> by name (not simply AY and OA or description).</p>

<p>10.</p> <p>(AOB =) <math>180 - 90 - 42</math> or <math>90 - 42</math></p> <p style="text-align: right;"><math>48(^{\circ})</math></p> <p style="text-align: right;"><math>x = 24(^{\circ})</math></p>	<p>M1</p> <p>A1</p> <p>B1</p>	<p>Check diagram for answers.</p> <p>Note: <math>180 - 132</math></p> <p>May be implied by sight of a final answer of 24.</p> <p>FT 'their 48' + 2, provided 'their 48' <math>\neq</math> 42.</p>
<p>10. <u>Alternative method</u></p> <p style="text-align: center;"><math>(x = ) \frac{180 - 90 - 42}{2}</math> or <math>\frac{90 - 42}{2}</math></p> <p style="text-align: right;"><math>x = 24(^{\circ})</math></p>	<p>M2</p> <p>A1</p>	<p>Check diagram for answers.</p> <p>Award M2 for complete method.</p>

Unit 1: Higher Tier	Mark	Comments
<p>13. <math>\widehat{DAF} = x</math></p> <p>(Reason:) <u>Alternate segment</u> (theorem)</p> <p><math>\widehat{BAD} (= 180 - 114) = 66(^{\circ})</math> [OR <math>\widehat{BAD} = 180 - 3x = 180 - 114</math>]</p> <p>(Reason:) (Opposite angles in a) <u>cyclic quadrilateral</u> (add up to <math>180^{\circ}</math>)</p> <p><math>x = 38(^{\circ})</math></p>	<p>B1</p> <p>E1</p> <p>B1</p> <p>E1</p> <p>B1</p>	<p>Check diagram. Must be unambiguous.</p> <p>Dependent on B1. Allow 'opposite segments (theorem)'.</p> <p>Check diagram. Must be unambiguously identified.</p> <p>Dependent on clearly attempting [or stating] <math>180 - 114</math>. (Sight of equation <math>66 + 2x + x = 180</math> may imply previous B marks.)</p> <p>FT 'their <math>180 - 114</math>'. An unsupported answer of 38 is awarded B3.</p>
<p>13. <u>Alternative method 1:</u> <math>\widehat{ADB} = 2x</math></p> <p>(Reason:) <u>Alternate segment</u> (theorem)</p> <p><math>\widehat{BAD} (= 180 - 114) = 66(^{\circ})</math> [OR <math>\widehat{BAD} = 180 - 3x = 180 - 114</math>]</p> <p>(Reason:) (Opposite angles in a) <u>cyclic quadrilateral</u> (add up to <math>180^{\circ}</math>)</p> <p><math>x = 38(^{\circ})</math></p>	<p>B1</p> <p>E1</p> <p>B1</p> <p>E1</p> <p>B1</p>	<p>Check diagram. Must be unambiguous.</p> <p>Dependent on B1. Allow 'opposite segments (theorem)'.</p> <p>Check diagram. Must be unambiguously identified.</p> <p>Dependent on clearly attempting [or stating] <math>180 - 114</math>. (Sight of equation <math>66 + 2x + x = 180</math> may imply previous B marks.)</p> <p>FT 'their <math>180 - 114</math>'.</p>
<p>13. <u>Alternative method 2:</u> (using additional line AC) <math>\widehat{ADB} = 2x</math></p> <p>(Reason:) <u>Alternate segment</u> (theorem)</p> <p><math>\widehat{ACD} = x</math> and <math>\widehat{ACB} = 2x</math></p> <p>(Reason:) <u>Angles in the same segment (are equal)</u> or <u>Angles on the same arc (are equal)</u></p> <p><math>x = 38(^{\circ})</math></p>	<p>B1</p> <p>E1</p> <p>B1</p> <p>E1</p> <p>B1</p>	<p>Check diagram. Must be unambiguous.</p> <p>Dependent on B1. Allow 'opposite segments (theorem)'.</p> <p>Check diagram. Must be unambiguous.</p> <p>Dependent on B1</p> <p>(<math>\widehat{ACD} + \widehat{ACB} = \widehat{BCD}</math>) (Sight of equation <math>2x + x = 114</math> may imply previous B marks.)</p>

13.  $(\hat{CDE}) = 20^\circ$	B2	All angles should be clearly stated or seen on the diagram. Answer line takes precedence. An unsupported answer of $20^\circ$ gains full marks, but must be clearly stated or seen on the diagram in the correct place. Award B1 for $\hat{CED} = 50^\circ$ OR $\hat{BEF} = 70^\circ$
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15.	Angle CAB = $x$	B1	May be indicated on the diagram.
	(Reason) Alternate segment theorem.	E1	E1 dependent on previous B1.
	Angle ABC = $\frac{180 - x}{2}$ ( $= 90 - \frac{x}{2}$ )	B1	
	(Reason) isosceles triangle.	E1 4	E1 dependent on previous B1.

*End of solutions*