revise.wales - Mark Scheme

Mock Paper B - Unit 2: Non-calculator (Higher Tier)

80 marks. R.WM-MNH-U2-002 (MS).

Notation. Mn = method mark; An = accuracy / answer mark; Bn = independent unsupported correct value; Cn = communication (OCW); ft = follow through from a prior error; oe = or equivalent; cao = correct answer only.

Question 1 **(5 marks)**

(a) **B1** LCM $= \mathbf{2^3} \times \mathbf{3^3} \times \mathbf{5^2}$ (take highest power of each prime; cao).

B1 HCF $= \mathbf{2^2} \times \mathbf{3} \times \mathbf{5}$ (take lowest power of each shared prime; cao). Do not award if a non-shared higher power appears (e.g. 3^3 or 5^2).

(b) **(i)** **B1** $\dfrac{7^5 \times 7^{-2}}{7^0} = 7^{5+(-2)-0} = \mathbf{7^3}$ (cao). Accept 7^3 unevaluated; do not penalise $= 343$.

(ii) **B1** $(a^{-2})^3 \times a^9 = a^{-6} \times a^9 = \mathbf{a^3}$ (cao). Award M1 not B1 if $(a^{-2})^3 = a^{-5}$ is seen (adding instead of multiplying indices).

(c) **B1** Rewrite $2{\cdot}5 \times 10^{-5} = 25 \times 10^{-6}$ (so the index is even and the mantissa is a perfect square); then $\sqrt{25 \times 10^{-6}} = 5 \times 10^{-3}$. Answer: $\mathbf{5 \times 10^{-3}}$ (cao). Reject $\sqrt{2{\cdot}5} \times 10^{-2{\cdot}5}$ — must be in proper standard form with integer index. Award 0 for $0{\cdot}005$ without index form.

Question 2 **(7 marks)**

(a) **M1** $\sqrt{27} = 3\sqrt{3}$, $\sqrt{12} = 2\sqrt{3}$ so $2\sqrt{12} = 4\sqrt{3}$, and $\sqrt{75} = 5\sqrt{3}$. At least two of three simplified correctly.

A1 $3\sqrt{3} + 4\sqrt{3} - 5\sqrt{3} = \mathbf{2\sqrt{3}}$ (cao; $k = 2$).

(b) **M1** Multiply numerator and denominator by the conjugate $2 + \sqrt{5}$: $\dfrac{(3+\sqrt{5})(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})}$. Expand: numerator $= 6 + 3\sqrt{5} + 2\sqrt{5} + 5 = 11 + 5\sqrt{5}$; denominator $= 4 - 5 = -1$.

A1 $\dfrac{11 + 5\sqrt{5}}{-1} = \mathbf{-11 - 5\sqrt{5}}$ (cao; $a = -11$, $b = -5$). Accept equivalent forms with both signs negated together.

(c) *"Show that" - algebra must be visible.* **M1** Let $x = 0{\cdot}14545\ldots$; state $10x = 1{\cdot}4545\ldots$ and $1000x = 145{\cdot}4545\ldots$ (multiplying by powers chosen so the recurring tails align).

M1 Subtract: $1000x - 10x = 145{\cdot}4545\ldots - 1{\cdot}4545\ldots = 144$, so $990x = 144$.

A1 $x = \dfrac{144}{990} = \dfrac{72}{495} = \mathbf{\dfrac{8}{55}}$ (divide top and bottom by 18, or stepwise by 2 then 9). Both lines of working required for full marks - final answer alone earns at most A1.

Question 3 **(8 marks)**

(a) **M1** Expand a pair first: $(x+3)(x-2) = x^2 + x - 6$ (oe), then multiply by the third bracket: $(x^2 + x - 6)(x+1) = x^3 + x^2 + x^2 + x - 6x - 6$. Award for the first expansion correct and an attempt at the second.

A1 $\mathbf{x^3 + 2x^2 - 5x - 6}$ (cao).

(b) **M1** Recognise the difference of two squares with $9x^2 = (3x)^2$ and $16y^2 = (4y)^2$; attempt $(3x)^2 - (4y)^2 = (3x - 4y)(3x + 4y)$ form.

A1 $(\mathbf{3x - 4y})(\mathbf{3x + 4y})$ (cao). Accept either order.

(c) **M1** Factorise numerator and denominator: $3x^2 + 7x + 2 = (3x + 1)(x + 2)$ (non-monic — product $3 \times 2 = 6$, sum 7: use 6 and 1, splitting middle term); $x^2 - 4 = (x - 2)(x + 2)$ (difference of two squares).

A1 Cancel $(x + 2)$: $\mathbf{\dfrac{3x + 1}{x - 2}}$ (cao; oe). Do not award A1 if cancellation is performed before factorisation (e.g. "cancelling the x^2 terms").

(d) **M1** Common denominator $(x + 1)(x - 2)$: $\dfrac{2(x - 2) + 3(x + 1)}{(x + 1)(x - 2)} = \dfrac{2x - 4 + 3x + 3}{(x + 1)(x - 2)}$.

A1 $\mathbf{\dfrac{5x - 1}{(x + 1)(x - 2)}}$ (cao; oe). Accept the denominator expanded as $x^2 - x - 2$. Award M1A0 if numerator sign error gives $\dfrac{5x + 7}{\cdots}$ or similar.

Question 4 (9 marks)

(a) **M1** Multiply throughout by 10: $2(2x - 1) + 5(x + 3) = 40$.

M1 Expand and collect: $4x - 2 + 5x + 15 = 40 \Rightarrow 9x + 13 = 40$.

A1 $\mathbf{x = 3}$ (cao; from $9x = 27$).

(b) **M1** Add the two equations to eliminate y: $(3x + 2y) + (5x - 2y) = 7 + 17 \Rightarrow 8x = 24$. (Subtraction methods that eliminate x first also score M1.)

A1 $\mathbf{x = 3}$ (cao).

A1 Substitute back: $3(3) + 2y = 7 \Rightarrow 2y = -2$, so $\mathbf{y = -1}$ (cao; ft from their x). Both values must be present for full marks; one value alone scores M1A1.

(c) **M1** Multiply both sides by 3 and divide by 2π: $3V = 2\pi r^3 \Rightarrow r^3 = \dfrac{3V}{2\pi}$. (Either order of operations is acceptable.)

M1 Take the cube root of both sides: $r = \sqrt[3]{\dfrac{3V}{2\pi}}$.

A1 $\mathbf{r} = \sqrt[3]{\dfrac{3V}{2\pi}}$ (cao; oe e.g. $\left(\dfrac{3V}{2\pi}\right)^{1/3}$). Award M1M0A0 for taking the cube root before isolating r^3, or square root in place of cube root.

Question 5 (10 (incl. 2 OCW) marks)

(a) **M1** Recognise the sequence is quadratic: first differences $5, 7, 9, 11$, second differences constant at 2, so nth term $= n^2 + bn + c$ with leading coefficient 1. (Alternatively spot pattern as $n \times (n + 2)$.)

M1 Determine b and c: substitute $n = 1 \Rightarrow 1 + b + c = 3$; $n = 2 \Rightarrow 4 + 2b + c = 8$. Solve to give $b = 2,\ c = 0$. (Or by inspection: $n^2 + 2n = n(n + 2)$ matches all four terms.)

A1 nth term $= \mathbf{n^2 + 2n}$ (oe $n(n + 2)$; cao).

A1 Substitute $n = 20$: $20^2 + 2(20) = 400 + 40 =$ **440** tiles (ft from their quadratic nth term).

(b) *OCW - working must be in connected sentences.* **M1** Find the term value at $n = 14$:

$14^2 + 2(14) = 196 + 28 = 224$.

M1 Find the term value at $n = 15$: $15^2 + 2(15) = 225 + 30 = 255$.

M1 State the comparison: $224 < 250 < 255$, so 250 lies strictly between two consecutive terms of the sequence. (Equivalent algebraic approach: solve $n^2 + 2n = 250 \Rightarrow n^2 + 2n - 250 = 0$; discriminant $4 + 1000 = 1004$ is not a perfect square (since $31^2 = 961$ and $32^2 = 1024$), so no integer solution.)

A1 Conclusion: **250** is not a term of the sequence; the student is **not correct**. (cao - must explicitly reject the claim.)

C1 (OCW) Working reads as connected English sentences: identifies the consecutive terms straddling 250, draws the comparison, and states the conclusion.

C1 (OCW) Correct mathematical notation throughout (clear use of n as the pattern number; explicit reference to integer or consecutive-term reasoning).

Question 6 **(11 marks)**

(a) **M1** Factorise: $(2x-1)(x+3) = 0$ (product $2 \times -3 = -6$, sum $+5$: split as $2x^2 + 6x - x - 3$, or trial).

A1 $\mathbf{x} = \mathbf{\frac{1}{2}}$ or $\mathbf{x} = -\mathbf{3}$ (both required; cao). Accept $0{\cdot}5$ for $\frac{1}{2}$.

(b) **(i)** **M1** Take half the coefficient of x: $\frac{1}{2} \times (-6) = -3$; write $x^2 - 6x = (x-3)^2 - 9$.

A1 $x^2 - 6x + 2 = (x-3)^2 - 9 + 2 = (\mathbf{x} - \mathbf{3})^{\mathbf{2}} - \mathbf{7}$, so $\mathbf{p} = \mathbf{3}$ and $\mathbf{q} = -\mathbf{7}$ (cao).

(ii) **B1** Turning point at $(\mathbf{3},\ -\mathbf{7})$ (ft from their p, q).

B1 Minimum (since the coefficient of x^2 is positive, the parabola opens upwards; cao). "Minimum point" suffices.

(c) **B1** Table completed: $x = 1 \Rightarrow y = -2$; $x = 2 \Rightarrow y = -3$; $x = 5 \Rightarrow y = 6$ (all three required; one error costs the B1).

M1 Plot all seven points correctly to within half a small square (at least five of seven correct for the M1).

A1 Smooth parabolic curve through all seven points (no straight-line segments; symmetric about $x = 2$).

B1 Recognise $x^2 - 4x + 3 = 0 \Leftrightarrow x^2 - 4x + 1 = -2$, so draw the horizontal line $\mathbf{y} = -\mathbf{2}$ on the same grid. Award only if the line is drawn (not merely stated).

B1 Read off intersections of the parabola with $y = -2$: $\mathbf{x} = \mathbf{1}$ and $\mathbf{x} = \mathbf{3}$ (ft from their curve, accept $\pm 0{\cdot}1$). Both roots required.

Question 7 **(11 marks)**

(a) **M1** Sum of interior angles of an n-gon $= (n-2) \times 180$; form the equation $(n-2) \times 180 = 1620$, so $n - 2 = 9$.

A1 $\mathbf{n} = \mathbf{11}$ sides (cao).

(b) **M1** State that a tangent meets the radius at the point of contact at $90°$: so $\angle OTP = \angle OSP = 90°$. Recognise $OTPS$ is a (kite-shaped) quadrilateral whose interior angles sum to $360°$.

A1 $\angle TOS = 360 - 90 - 90 - 50 = \mathbf{130}°$ (cao).

B1 Both reasons explicitly stated: "the angle between a tangent and the radius at the point of contact is $90°$" *and* "angles in a quadrilateral sum to $360°$". Missing either reason caps at A1.

(c) **A1** $\angle BDC = \mathbf{38}°$ (cao).

B1 Reason: "the alternate segment theorem" - the angle between the tangent BT and chord BC equals the angle in the alternate segment subtended by BC at D on the major arc (oe). Both the named theorem *and* a clear statement of which two angles are being equated are required for full marks; a bare "alternate segment" without the angle identification scores B0 but keeps the A1.

(d) **M1** Recognise $ABCD$ is a cyclic quadrilateral, so opposite angles sum to $180°$: $\angle BAD + \angle BCD = 180$.

A1 $\angle BCD = 180 - 78 =$ **102**$°$ (cao).

B1 Reason: "opposite angles of a cyclic quadrilateral sum to $180°$" explicitly stated.

Question 8 **(8 marks)**

(a) **M1** Reflection in $y = x + 1$ maps $(x, y) \to (y - 1, x + 1)$ (equivalently: the line passes through $(0, 1)$ and $(-1, 0)$; points on the line are fixed). Apply to at least two vertices: $(1, 1) \to (0, 2)$, $(3, 1) \to (0, 4)$, $(1, 2) \to (1, 2)$ (this last vertex lies on the mirror line and so is its own image).

A1 Triangle A drawn correctly with all three vertices at $(0, 2)$, $(0, 4)$, $(1, 2)$ and labelled A (cao).

(b) **M1** Rotation $180°$ about $(2, 0)$ maps $(x, y) \to (4 - x,\ -y)$. Apply to at least two vertices: $(1, 1) \to (3, -1)$, $(3, 1) \to (1, -1)$, $(1, 2) \to (3, -2)$.

A1 Three image vertices at $(3, -1)$, $(1, -1)$, $(3, -2)$ correctly plotted.

B1 Triangle B drawn and labelled B (cao).

(c) **M1** Test mapping: $(1, 1) \to (-3, -3)$, $(3, 1) \to (-9, -3)$, $(1, 2) \to (-3, -6)$ - each image vertex is -3 times the corresponding original vertex.

A1 Identify as an **enlargement** with **scale factor** -3 and **centre** $(0, 0)$ (the origin).

B1 All three descriptors stated together - type, scale factor (with sign), and centre. Missing any one descriptor caps at A1.

Question 9 **(11 marks)**

(a) **B1** Venn diagram completed correctly: "both" region $(W \cap F) = 35 + 28 - (60 - 8) = 11$; "$W$ only" $= 35 - 11 = 24$; "F only" $= 28 - 11 = 17$; "neither" $= 8$. Total check: $24 + 11 + 17 + 8 = 60$.

M1 Number studying at least one language $= 60 - 8 = 52$ (equivalently $|W \cup F| = 52$). Set up $P(\text{both} \mid \text{at least one}) = \dfrac{|W \cap F|}{|W \cup F|} = \dfrac{11}{52}$.

A1 $\mathbf{\dfrac{11}{52}}$ (cao; already in simplest form - gcd$(11, 52) = 1$ as 11 is prime and does not divide 52). Award M1A0 for the unconditional probability $\frac{11}{60}$ (a common error: dividing by the whole school instead of by $W \cup F$).

(b) **B1** Tree-diagram second-draw probabilities all correct: from R: $\frac{3}{8}$ (R), $\frac{5}{8}$ (B); from B: $\frac{4}{8}$ (R), $\frac{4}{8}$ (B).

M1 Identify "same colour" as RR or BB: $P(RR) = \frac{4}{9} \times \frac{3}{8} = \frac{12}{72}$, $P(BB) = \frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$.

M1 Sum: $\frac{12}{72} + \frac{20}{72} = \frac{32}{72}$.

A1 $\mathbf{\dfrac{4}{9}}$ (cao; must be in simplest form - $\frac{32}{72}$ alone scores M1A0).

(c) *"Show that" - algebra must be visible.* **M1** Use the complement: $P(\text{at least one yellow}) = 1 - P(\text{no yellow}) = \dfrac{9}{14}$, so $P(\text{both blue}) = 1 - \frac{9}{14} = \frac{5}{14}$. State $P(\text{both blue}) = \dfrac{n}{n+3} \times \dfrac{n-1}{n+2} = \dfrac{5}{14}$.

M1 Cross-multiply: $14n(n - 1) = 5(n + 3)(n + 2)$; expand to $14n^2 - 14n = 5(n^2 + 5n + 6) = 5n^2 + 25n + 30$.

A1 Rearrange to $9n^2 - 39n - 30 = 0$; divide by 3 to obtain $\mathbf{3n^2 - 13n - 10 = 0}$ (intermediate line required).

A1 Factorise $(3n+2)(n-5) = 0$; reject $n = -\frac{2}{3}$ (a count of counters must be a positive integer); so $\mathbf{n = 5}$. Justification required for full marks.

Total: $5 + 7 + 8 + 9 + 10 + 11 + 11 + 8 + 11 =$ **80 marks.**

OCW marks (Q5): the 2 OCW marks are included in the question total of 10. To award full OCW the candidate's working must read as connected English sentences with correct mathematical notation throughout, including an explicit conclusion that addresses the student's claim.