Name	Date started	Target end date

GCE A LEVEL — SHM THEORY QUESTION PACK

Legacy PH4 · New spec Unit 3 Topic 2a · A2 unit, 25% of A-level

REVISE
.wales

PHYSICS — UNIT 3 · SHM THEORY

3.2 Vibrations — SHM equation, period and energy

Defining equation $a = -\omega^2 x$, sinusoidal solutions for displacement / velocity / acceleration, the period of mass-spring and pendulum systems, and the constant-total-energy condition for SHM.

NEW 2015 SPEC · UNIT 3 TOPIC 2A

Estimated time for entire question pack: ~1h 56m

Derived from the legacy PH4 paper's pace of 120 marks in 1h 45m.

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every question from the legacy WJEC PH4 papers (2008 modular spec) that maps onto new-spec Unit 3 Topic 2a (3.2).

Questions are ordered chronologically within each section.

INSTRUCTIONS

Use black ink or black ball-point pen. Answer all questions in the spaces provided.

The number of marks is given in brackets at the end of each question or part-question. A calculator is required. The Data Booklet is allowed.

For Examiner's use only

Q	Source	Max	Mark
1	PH4 Jan 10 Q1	14	
2	PH4 Jun 10 Q1	9	
3	PH4 Jun 15 Q2	15	
4	PH4 Jun 14 Q4	12	
5	PH4 Jan 13 Q1	17	
6	PH4 Jan 14 Q3	16	
Total		83	

SHM Theory — what the new spec asks

WJEC GCE A Level Physics (from 2015) · Unit 3: Oscillations & Nuclei · Topic 3.2.

Defining SHM A

- Acceleration proportional to displacement and directed towards equilibrium: $a = -\omega^2 x$.
- $\omega = 2\pi f = 2\pi/T$ — the angular frequency.

Sinusoidal solutions A

- $x = A\cos(\omega t)$ (released from $x = A$); $x = A\sin(\omega t)$ (released from $x = 0$).
- $v = \pm\omega\sqrt{(A^2 - x^2)}$; $v_{max} = A\omega$ at $x = 0$.
- $a_{max} = A\omega^2$ at $x = \pm A$.

Period of standard systems B

- Mass on a spring: $T = 2\pi\sqrt{(m/k)}$.
- Simple pendulum (small angles): $T = 2\pi\sqrt{(L/g)}$.
- Period independent of amplitude — the hallmark of SHM.

Energy in SHM B

- Total energy constant: $E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} kA^2$.
- $KE = \frac{1}{2} m\omega^2(A^2 - x^2)$; $PE = \frac{1}{2} m\omega^2 x^2$.
- KE max at $x = 0$; PE max at $x = \pm A$; sum constant throughout.

SHM Theory in one page

Quick-reference notes — revisit before each section.

Defining equation

Acceleration ∝ displacement, directed back to equilibrium.

$\omega = 2\pi f = 2\pi/T$.

Solutions

$x = A\cos(\omega t)$ if released from $x = A$.

$x = A\sin(\omega t)$ if released from $x = 0$ moving outward.

$v(t)$ and $a(t)$ follow by differentiation.

Velocity formula

$v = \pm\omega\sqrt{(A^2 - x^2)}$.

$v_{max} = A\omega$ at $x = 0$.

$v = 0$ at $x = \pm A$.

Acceleration

$a = -\omega^2 x$.

$a_{max} = A\omega^2$ at $x = \pm A$.

Mass-spring

k is the spring stiffness in N m^{-1}. Independent of amplitude (linear restoring force).

Pendulum

$T = 2\pi\sqrt{(L/g)}$ for $\theta < \sim 10°$.

Length L = pivot to centre of mass of bob.

Energy

$E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$.

$KE = \frac{1}{2}m\omega^2(A^2 - x^2)$.

$PE = \frac{1}{2}m\omega^2 x^2$.

Energy graphs

KE vs x is an inverted parabola; PE vs x is an upright parabola.

KE + PE = E (horizontal line on graph).

Strategy

1. Identify A, ω from data.
2. Pick the right sinusoid for the initial condition.
3. Use $v = \pm\omega\sqrt{(A^2-x^2)}$ when t isn't given.
4. Use energy when t isn't given and v isn't asked directly.

Section index

Use this index to jump straight to the section you need.

1. *(a)* Define simple harmonic motion. [2]

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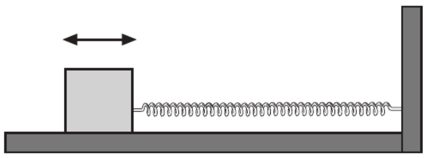
(b) (i) A mass on the end of a light spring performs simple harmonic motion. The stiffness k of the spring is $28{\cdot}5\,\mathrm{N m^{-1}}$ and the period of oscillation is $0{\cdot}42\,\mathrm{s}$. Calculate the mass. [3]



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(ii) Show that ω, the angular frequency of the oscillation is approximately $15\ \mathrm{rad\ s^{-1}}$. [1]

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(c) The amplitude of oscillation is 1·30 cm. Calculate

(i) The maximum speed of oscillation. [2]

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(ii) The maximum acceleration. [2]

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(d) The time of oscillation starts when the mass is passing the equilibrium position moving to the right. The position x of the mass at time t is then given by

$$x = A\sin(\omega t)$$

(i) At what time will the acceleration of the mass **first** be of maximum magnitude? [1]

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(ii) Calculate a time when the acceleration of the mass is $2{\cdot}10\,\mathrm{m\,s^{-2}}$ to the left. [3]

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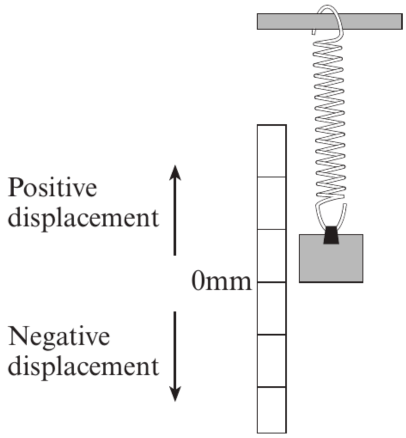
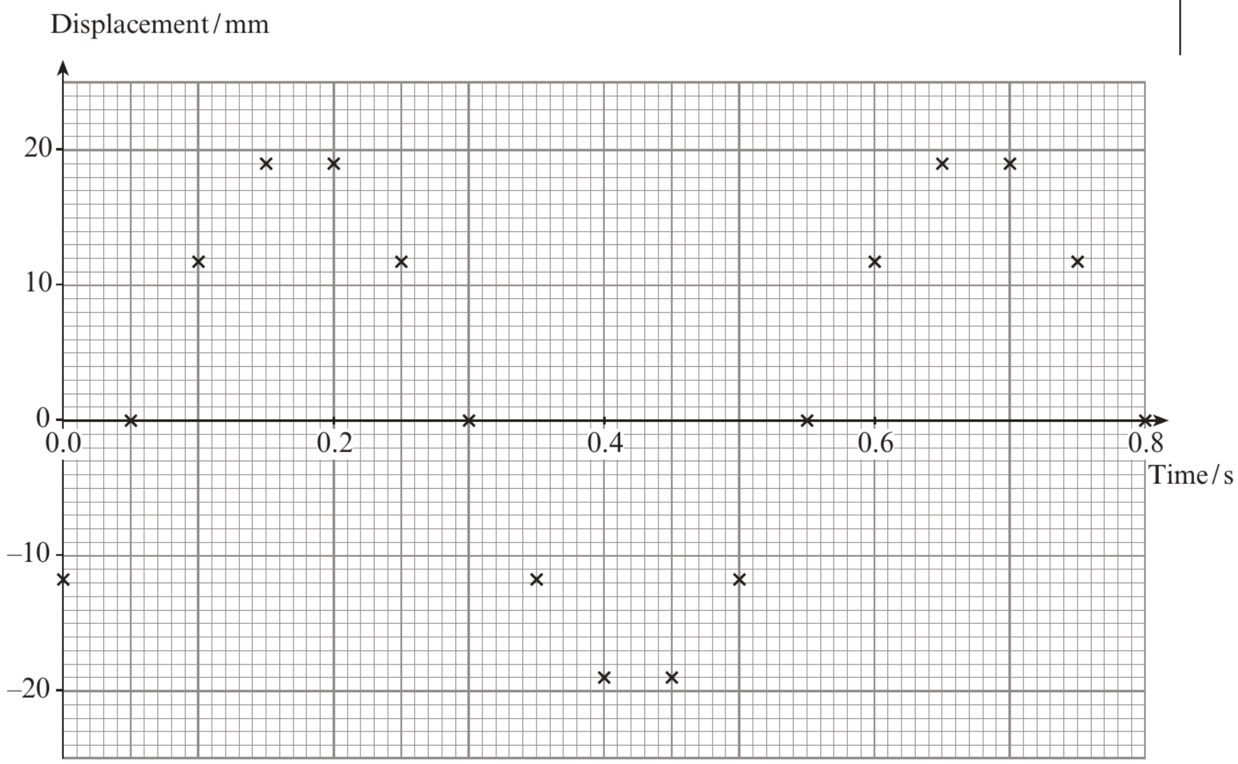
1. In a laboratory experiment a block with a mass of 0.04 kg is suspended from a vertical spring. The diagram shows the block in its equilibrium position. When it is pulled down and released it oscillates with simple harmonic motion (SHM).

The motion is recorded by a high speed video camera, and the displacement of the bottom of the block at regular times is shown on the graph.





(a) Define simple harmonic motion (SHM).

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(b) On the graph, plot carefully a curve through the points that shows how the displacement varies with time. [1]

(c) Show that the angular frequency, ω, of the oscillation is 12.57 rad s^{-1}.

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(d) Calculate the maximum speed of the block.

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(e) Find the stiffness, k, of the spring (force per unit extension).

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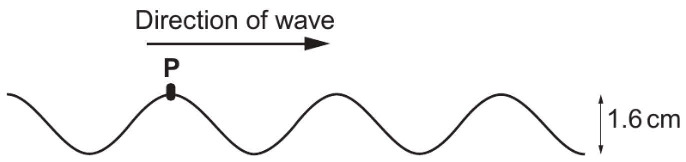
2. A cork, **P**, floats on the surface of a pond. When a wave travels over the surface the cork oscillates vertically with Simple Harmonic Motion (SHM). The cork completes 20 oscillations in 24 s and has a total vertical range of 1.6 cm.



(a) Define *Simple Harmonic Motion.* [2]

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(b) Calculate the period of oscillation. [1]

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(c) Show that the angular velocity, ω, of oscillation is approximately 5 rad s^{-1}. [2]

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(d) If the cork is at its highest point when t = 0, complete the expression for the upward displacement of the cork, x, by inserting numerical values into the boxes. [3]

$$x = \boxed{}\ \sin\left(\boxed{}\ t + \boxed{}\right)\ \text{cm}$$

Examiner only

(e) Determine the time it takes for the cork to move directly downward from 0.4 cm above the centre of oscillation to 0.3 cm below the centre. [3]

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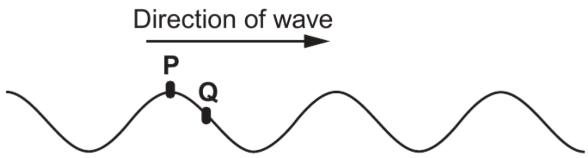
(f) A second cork, **Q**, also oscillates on the surface at a **quarter of a wavelength** from **P** as shown in the diagram.



Sketch the vertical displacements of the two corks during the time interval $t = 0$ to $t = 2.4$ s. Use the same axes for both curves and label both curves clearly for cork **P** and cork **Q**. [3]

(g) Hence write an expression for the upward displacement of cork **Q** in terms of t. [1]

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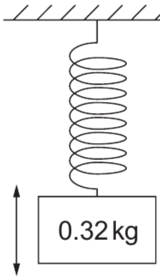
4. A mass of 0.32 kg oscillates with simple harmonic motion vertically on a spring with a frequency of 3.47 Hz.



(a) Calculate the spring constant of the spring. [3]

(b) Show that the angular velocity, ω, of the oscillations is 21.8 rad s^{-1}. [1]

(c) The amplitude of oscillation of the spring is 8.5 cm. Calculate:

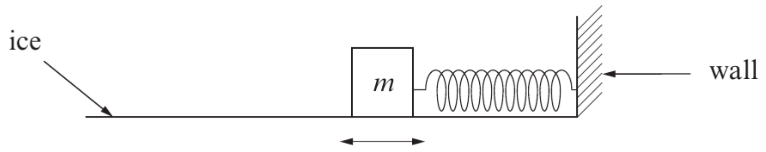
(i) the maximum kinetic energy of the mass; [3]

(ii) the maximum resultant force acting on the mass. [2]

(d) The displacement of the mass is given by the equation $x = A\sin(\omega t + \varepsilon)$. Calculate a valid value for ε given that the displacement of the mass is –1.4 cm at time t = 0.100 s. [3]

Examiner only

1. *(a)* A mass, m, is attached to a spring and oscillates horizontally with simple harmonic motion on the floor of an ice rink. Its frequency of oscillation is 0.625 Hz and the spring constant of the spring is $2\,640\,\text{N m}^{-1}$.



(i) Show that the mass, m, is approximately 170 kg. [3]

(ii) The maximum kinetic energy of the mass is 2.15 kJ. Calculate its maximum speed. [2]

(iii) State the maximum potential energy stored in the spring and explain your reasoning. [2]

(iv) Calculate the amplitude of oscillation. [2]

Examiner only

(v) At time $t = 0$, the displacement of the mass is zero. Calculate the **acceleration** of the mass at time $t = 1.40$ s. [3]

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(b) Explain briefly, why pushing the mass every 1.60 s would result in large amplitude oscillations. [2]

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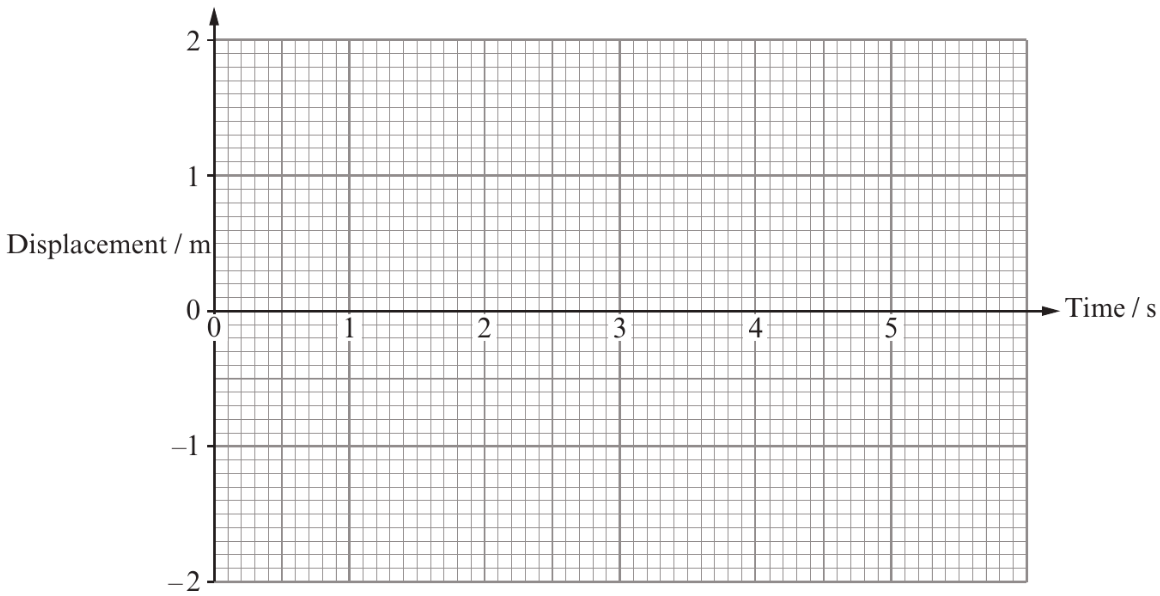
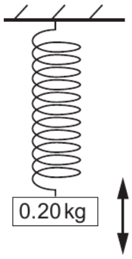
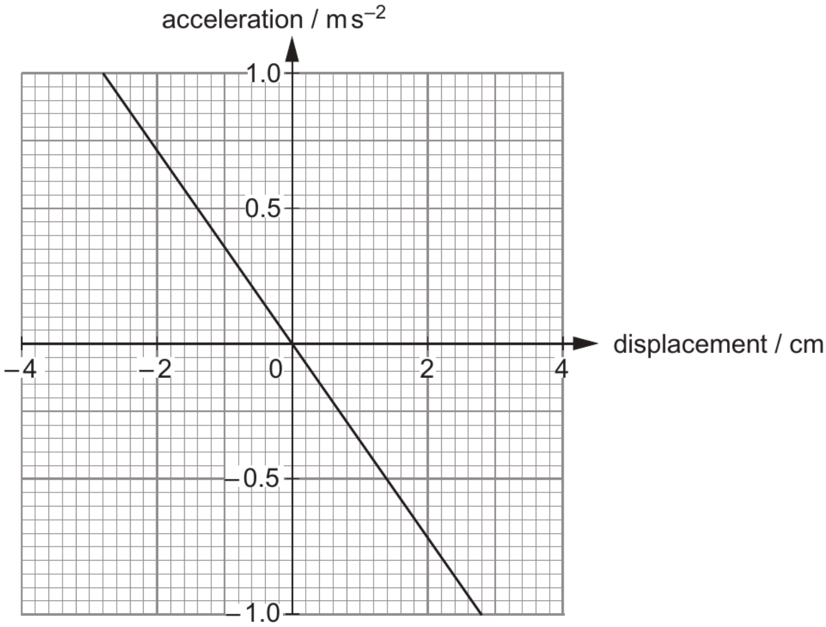
(c) Later, when the mass is released from its maximum displacement of 2.00 m an observer starts a stopwatch. After 5.0 s, the amplitude of oscillation has decreased to 1.40 m. Sketch a displacement-time graph of the damped oscillations on the grid below. [3]



3. A mass oscillates vertically on a spring as shown.



(a) The graph below shows the variation of acceleration with displacement of the mass on the spring.



(i) Explain how the graph verifies that the mass will perform simple harmonic motion. [2]

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(ii) Use the graph to show that the frequency of oscillation of the mass on the spring is approximately 1 Hz. [3]

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(iii) The amplitude of oscillation of the mass on the spring is 2.8 cm. Write down (or calculate) the maximum acceleration of the mass. [1]

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(iv) Calculate the maximum kinetic energy of the 0.20 kg mass. [3]

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(v) If the mass is moving upwards at its maximum speed when t = 0 s, calculate the first time that the mass moves upwards with a speed of 0.100 m s^{-1}. [3]

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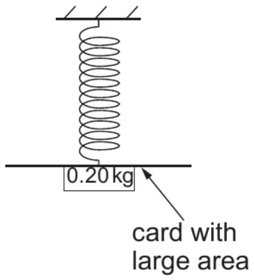
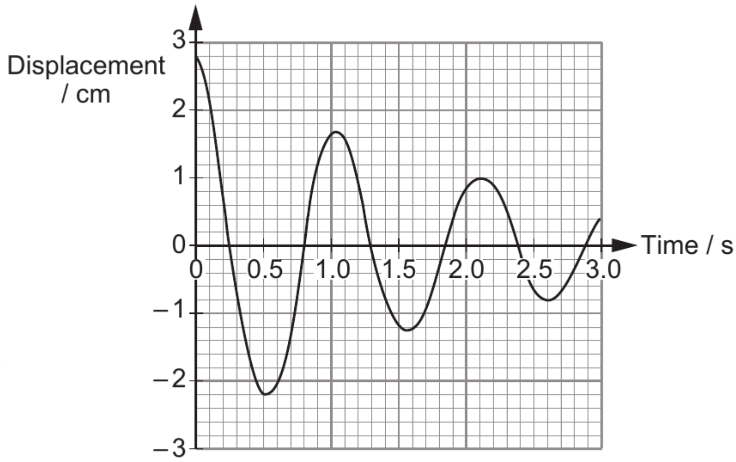
Examiner only

(b) When damping is introduced the following graph of displacement against time is obtained. Explain how the principle of conservation of energy applies during the cycles shown. [4]





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END OF QUESTION PACK

6 questions · 83 marks · ~1h 56m

Source: WJEC PH4 (2008 modular spec)

Curated for WJEC Physics 2015 spec A2 Unit 3 — Topic 2a (3.2)