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GCE A LEVEL – APPLIED MATHEMATICS B QUESTION PACK

0981-01 (Legacy M2) · New spec Unit 4 Topic 6 · A2 unit, 15% of A-level, 80 marks, 1h 45min paper

REVISE
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MATHEMATICS – APPLIED B · KINEMATICS - R, V, F VECTOR CALCULUS

Kinematics - $r(t)$, $v(t)$, $F(t)$ Vector Calculus

Every vector kinematics question with calculus (variable r , v , F in i/j components) from the legacy WJEC M2 papers (2011-2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 12 minutes

Derived from the legacy M2 paper's pace of ~1.25 min/mark (58 marks over 6 questions).

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC M2 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 6 (2.4.5).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet and statistical tables may be referred to. Take $g = 9.8 \text{ ms}^{-2}$ for mechanics.

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Q	Source	Max	Mark
1	Jun 11 Q3	7	
2	Jun 12 Q3	8	
3	Jun 13 Q2	12	
Total		58	

Kinematics - $\mathbf{r}(t)$, $\mathbf{v}(t)$, $\mathbf{F}(t)$ Vector Calculus – what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 4: Applied Mathematics B · Topic 2.4.5.

Vector kinematics 2.4.5

- $\mathbf{r}(t), \mathbf{v}(t) = \frac{d\mathbf{r}}{dt}, \mathbf{a}(t) = \frac{d\mathbf{v}}{dt}$.
- Integrate component-wise: $\mathbf{v} = \int \mathbf{a} dt + \mathbf{C}$.
- Initial conditions fix the constants of integration.

Forces in vector form 2.4.7

- $\mathbf{F} = m\mathbf{a}$ applied component-wise.
- Momentum vector: $\mathbf{p} = m\mathbf{v}$.
- Speed at time t : $|\mathbf{v}(t)|$.

Kinematics - \mathbf{r} , \mathbf{v} , \mathbf{F} Vector Calculus in one page

Quick-reference notes – revisit before each section. Don't use during questions.

Differentiate component-wise

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}.$$

$$\mathbf{v} = x'\mathbf{i} + y'\mathbf{j} + z'\mathbf{k}, \text{ similarly for } \mathbf{a}.$$

Integrate component-wise

$$\mathbf{r} = \int \mathbf{v} dt + \mathbf{C}.$$

Use $\mathbf{r}(0)$ (or $\mathbf{v}(0)$) to pin down \mathbf{C} .

Keep track of vector \mathbf{C} , not a scalar.

$\mathbf{F} = m\mathbf{a}$ in vector form

$$\mathbf{F}(t) = m \frac{d\mathbf{v}}{dt} - \text{differentiate } \mathbf{v}, \text{ multiply by } m.$$

Magnitude of force: $|\mathbf{F}|$ at a given t .

Perpendicular tests

$\mathbf{v}(t)$ perpendicular to fixed vector \mathbf{u} :
solve $\mathbf{v} \cdot \mathbf{u} = 0$.

Often a trig or polynomial equation in t .

Crossing the axis

Particle crosses y -axis when $x(t) = 0$
(with $t > 0$).

Substitute back to find y -coordinate
and distance from origin.

Power and momentum vectors

Instantaneous power $P = \mathbf{F} \cdot \mathbf{v}$.

Momentum $\mathbf{p} = m\mathbf{v}$; $\frac{d\mathbf{p}}{dt} = \mathbf{F}$.

SECTION T6

Kinematics - $r(t)$, $v(t)$, $F(t)$ Vector Calculus

Questions 1-6 · 58 marks

3. A particle P , of mass 2 kg , is moving under the action of a force $\mathbf{F} \text{ N}$ so that its velocity $\mathbf{v} \text{ ms}^{-1}$ at time $t \text{ s}$ is given by

$$\mathbf{v} = 2\mathbf{i} + 6t\mathbf{j} + 4t^3\mathbf{k}.$$

- (a) Find an expression for \mathbf{F} at time $t \text{ s}$. [3]
- (b) Determine the value of $\mathbf{F} \cdot \mathbf{v}$ when $t = 1$ and state the units of your answer. [4]

3. A particle moves on a horizontal plane so that at time t seconds its position vector \mathbf{r} metres relative to a fixed origin O is given by

$$\mathbf{r} = (t + 2t^2)\mathbf{i} + (1.5t^2 - 2t)\mathbf{j}.$$

- (a) Determine the time when the velocity of the particle is perpendicular to the vector $(-\mathbf{i} + 2\mathbf{j})$. [5]
- (b) Show that the acceleration of the particle is constant and find its magnitude. [3]

2. A particle P , of mass 2 kg , is moving so that at time $t\text{ s}$ its velocity $\mathbf{v}\text{ ms}^{-1}$ is given by $\mathbf{v} = (13t - 3)\mathbf{i} + (2 + 3t^2)\mathbf{j}$. At time $t = 0\text{ s}$, the position vector of the particle is $(2\mathbf{i} + 7\mathbf{j})\text{ m}$.
- (a) Find the position vector \mathbf{r} of P at time $t\text{ s}$. [5]
- (b) Determine the acceleration \mathbf{a} of P at time $t\text{ s}$. [2]
- (c) Calculate the values of t when the velocity of P is perpendicular to the vector $\mathbf{i} - 2\mathbf{j}$. [5]

6. A particle of mass 3 kg moves on a horizontal plane. At time $t = 0$, the particle has position vector $-2\mathbf{i} + 3\mathbf{j}$ m, where \mathbf{i} and \mathbf{j} are unit vectors along the x -axis and y -axis respectively. At time t s, the particle moves with velocity \mathbf{v} ms⁻¹ given by

$$\mathbf{v} = 4\sin 2t\mathbf{i} + 15\cos 5t\mathbf{j}.$$

- (a) Find the magnitude of the force acting on the particle at time $t = \frac{3\pi}{2}$ s. [5]
- (b) Determine the position vector of the particle at time t s. [4]
- (c) Calculate the time and the distance of the particle from the origin when it crosses the y -axis for the first time. [4]

6. A particle moves on a horizontal plane such that its velocity vector \mathbf{v} ms^{-1} at time t s is given by

$$\mathbf{v} = 7 \sin 2t \mathbf{i} + 6 \cos 3t \mathbf{j}.$$

- (a) Find the acceleration vector of the particle at time t s. [2]
- (b) Given that when $t = 0$, the particle has position vector $(0.5\mathbf{i} + 3\mathbf{j})$ m, find the position vector of the particle when $t = \frac{\pi}{2}$. [5]

1. The position vector of a particle P at time t seconds is given by

$$\mathbf{r} = t \sin t \mathbf{i} + t \cos t \mathbf{j}.$$

- (a) (i) Find the velocity vector of P and an expression for the speed of P at time t seconds in its simplest form.
- (ii) Given that the mass of P is 3 kg, write down the momentum vector of P at time t seconds. [6]
- (b) At time $t = \frac{\pi}{6}$, the vector $b\mathbf{i} + \sqrt{3}\mathbf{j}$ is perpendicular to \mathbf{r} . Find the value of b . [5]

END OF KINEMATICS - R, V, F VECTOR CALCULUS PACK

Source: WJEC M2 (2008 modular spec) · 2011–2017

Curated for WJEC Maths 2017 spec A2 Unit 4 – Topic 6 (2.4.5)

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