

Name	Date started	Target end date
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## GCE A LEVEL – APPLIED MATHEMATICS B QUESTION PACK

0981-01 (Legacy M2) · New spec Unit 4 Topic 5 · A2 unit, 15% of A-level, 80 marks, 1h 45min paper

# REVISE

.wales

## MATHEMATICS – APPLIED B · KINEMATICS - VECTORS (POSITION & CLOSEST APPROACH)

### *Kinematics - Position Vectors and Closest Approach*

*Every position-vector and closest-approach question (constant-velocity 2D motion) from the legacy WJEC M2 papers (2011-2016)*

LEGACY 2008 SPECIFICATION

### Estimated time for entire question pack: ~0 hours 56 minutes

*Derived from the legacy M2 paper's pace of ~1.25 min/mark (45 marks over 5 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

### ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC M2 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 5 (2.4.5).

Questions are ordered chronologically.

### INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

*A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet and statistical tables may be referred to. Take  $g = 9.8 \text{ ms}^{-2}$  for mechanics.*

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Q	Source	Max	Mark
1	Jun 11 Q7	10	
2	Jun 12 Q8	10	
3	Jun 14 Q4	10	
4	Jun 15 Q1	6	
5	Jun 16 Q3	9	
<b>Total</b>		<b>45</b>	

# Kinematics - Position Vectors and Closest Approach – what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 4: Applied Mathematics B · Topic 2.4.5.

## Position vectors 2.4.5

- Particle at  $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$  for constant-velocity motion.
- Displacement between two particles:  $\overrightarrow{AB} = \mathbf{r}_B - \mathbf{r}_A$ .
- Distance is  $|\overrightarrow{AB}|$ ; closest when  $\frac{d}{dt}|\overrightarrow{AB}|^2 = 0$ .

## Scalar (dot) product 2.4.5

- $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ .
- Perpendicular vectors:  $\mathbf{a} \cdot \mathbf{b} = 0$ .
- Angle between:  $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$ .

# Kinematics - Vectors (Position & Closest Approach) in one page

Quick-reference notes – revisit before each section. Don't use during questions.

## Constant velocity model

$$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}.$$

Linear in  $t$  – straight-line motion in 2D / 3D.

$$\text{Speed} = |\mathbf{v}|.$$

## Distance between particles

$$\overrightarrow{AB}(t) = \mathbf{r}_B - \mathbf{r}_A.$$

$AB^2 = |\overrightarrow{AB}|^2$  is quadratic in  $t$  – minimise by differentiating or completing the square.

## Closest approach

$\frac{d}{dt}(AB^2) = 0 \Rightarrow$  time of closest approach.

Substitute back for minimum distance.

Quicker:  $\overrightarrow{AB} \cdot \frac{d\overrightarrow{AB}}{dt} = 0$  at closest point.

## Perpendicular vectors

$$\mathbf{a} \cdot \mathbf{b} = 0.$$

For 3D:  $a_1b_1 + a_2b_2 + a_3b_3 = 0$ .

Solve the trig / algebraic equation that pops out.

## Speed and momentum

$$\text{Speed } |\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}.$$

Momentum vector:  $\mathbf{p} = m\mathbf{v}$ ;

magnitude  $|\mathbf{p}| = m|\mathbf{v}|$ .

## Intercept (boat meeting ship)

Set  $\mathbf{r}_S(t) = \mathbf{r}_B(t)$  at intercept; solve for time + unknown velocity.

Sometimes given target time; sometimes the unknown is direction.

# SECTION T5

*Kinematics - Position Vectors and Closest Approach*

Questions 1-5 · 45 marks

7. At time  $t$ , the position vectors relative to a fixed origin  $O$ , of two particles  $A$  and  $B$  are given by  $\mathbf{OA} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k} + t(2\mathbf{i} - 6\mathbf{j} + 9\mathbf{k})$  and  $\mathbf{OB} = 5\mathbf{i} - 8\mathbf{j} + 10\mathbf{k} + t(3\mathbf{i} - 6\mathbf{j} + 7\mathbf{k})$ .
- (a) Find the speed of particle  $A$ . [3]
- (b) Show that the distance  $AB$  at time  $t$  is given by  $AB^2 = 5t^2 - 30t + 211$ . Determine the time at which the particles  $A$  and  $B$  are closest together. [7]

8. A ship  $S$  is moving in a straight line with constant velocity. At time  $t = 0$ , its position vector relative to a fixed origin  $O$  is  $(8\mathbf{i} + 7\mathbf{j})$ . At time  $t = 3$ , its position vector is  $(14\mathbf{i} - 5\mathbf{j})$ .

(a) Show that the velocity of  $S$  is  $(2\mathbf{i} - 4\mathbf{j})$ . [2]

(b) Find an expression, in terms of  $t$ , for the position vector of  $S$  at time  $t$ . [2]

At time  $t = 10$ , a boat  $B$  leaves  $O$  and travels with constant velocity  $x\mathbf{i} + y\mathbf{j}$ , intercepting  $S$  at time  $t = 50$ .

(c) Calculate the value of  $x$  and the value of  $y$ . [6]

4. At time  $t = 0$ , an aeroplane  $A$  has position vector  $(3\mathbf{i} + 5\mathbf{j} + 20\mathbf{k})$  m and is flying with constant velocity  $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$   $\text{ms}^{-1}$ .  
At time  $t = 0$ , another aeroplane  $B$  has position vector  $(-2\mathbf{i} + x\mathbf{j} + 15\mathbf{k})$  m, and is flying with constant velocity  $(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$   $\text{ms}^{-1}$ .
- (a) Find expressions for the position vector of  $A$  and the position vector of  $B$  at time  $t$  s. [3]
- (b) Determine an expression for  $AB^2$ , where  $AB$  is the distance between  $A$  and  $B$  at time  $t$  s. [4]
- (c) Given that the shortest distance between  $A$  and  $B$  occurs at  $t = 5$ , calculate the value of  $x$ . [3]

1. The vectors  $\mathbf{x}$  and  $\mathbf{y}$  are given by

$$\begin{aligned}\mathbf{x} &= \sin\theta\mathbf{i} + 2\cos2\theta\mathbf{j}, \\ \mathbf{y} &= 2\mathbf{i} - \mathbf{j}.\end{aligned}$$

Find the values of  $\theta$  between 0 and  $2\pi$  such that  $\mathbf{x}$  is perpendicular to  $\mathbf{y}$ .

[6]

3. At time  $t = 0$  s, the position vector of an object  $A$  is  $\mathbf{i}$  m and the position vector of another object  $B$  is  $3\mathbf{i}$  m. The constant velocity vector of  $A$  is  $2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$   $\text{ms}^{-1}$  and the constant velocity vector of  $B$  is  $\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$   $\text{ms}^{-1}$ . Determine the value of  $t$  when  $A$  and  $B$  are closest together and find the least distance between  $A$  and  $B$ . [9]

**END OF KINEMATICS - VECTORS (POSITION & CLOSEST APPROACH)  
PACK**

Source: WJEC M2 (2008 modular spec) · 2011–2017  
Curated for WJEC Maths 2017 spec A2 Unit 4 – Topic 5 (2.4.5)

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