Name	Date started	Target end date

GCE A LEVEL — APPLIED MATHEMATICS B QUESTION PACK

0984-01 (Legacy S2) · New spec Unit 4 Topic 3 · A2 unit, 15% of A-level, 80 marks, 1h 45min paper

REVISE
.wales

MATHEMATICS — APPLIED B · HYPOTHESIS TESTING - NORMAL (SAMPLE MEAN)

Hypothesis Testing - Normal Distribution Sample Mean

Every hypothesis test on a population mean / difference of two means (Normal model) from the legacy WJEC S2 papers (2011-2016)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 21 minutes

*Derived from the legacy S2 paper's pace of **~1.25 min/mark** (65 marks over 6 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC S2 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 3 (2.4.4).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet and statistical tables may be referred to. Take $g = 9.8\,\text{ms}^{-2}$ for mechanics.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q4	10	
2	Jun 12 Q5	11	
3	Jun 13 Q3	11	

Q	Source	Max	Mark
4	Jun 14 Q3	10	
5	Jun 15 Q1	9	
6	Jun 16 Q4	14	
Total		**65**	

Hypothesis Testing - Normal Distribution Sample Mean — what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 4: Applied Mathematics B · Topic 2.4.4.

Setup (one and two-sample tests) 2.4.4

- H_0: $\mu = \mu_0$ (or $\mu_X = \mu_Y$). H_1: one or two-sided.
- Test statistic: $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$ or $Z = \dfrac{\bar{X} - \bar{Y}}{\sqrt{\sigma_X^2/n_X + \sigma_Y^2/n_Y}}$.
- p-value: $P(Z \leq z)$ for one-sided $<$; $2P(Z \geq |z|)$ for two-sided.

Conclusion 2.4.4

- Compare p-value with significance level α (e.g. 0.05).
- If p $< \alpha$: reject H_0 in favour of H_1.
- State conclusion in context, not just "reject" / "don't reject".

Crib sheet »

Hypothesis Testing - Normal (Sample Mean) in one page

Quick-reference notes — revisit before each section. Don't use during questions.

State hypotheses

H_0: $\mu = \mu_0$ (or $\mu_X = \mu_Y$).

H_1: one-sided ($<$ or $>$) or two-sided ($\neq$), driven by context.

Test statistic (one sample)

$Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$, assuming $X \sim N(\mu, \sigma^2)$ with known σ.

Under H_0: $Z \sim N(0, 1)$.

Test statistic (two sample)

$Z = \dfrac{\bar{X} - \bar{Y}}{\sqrt{\sigma_X^2/n_X + \sigma_Y^2/n_Y}}$.

Common variance σ^2: $Z = \dfrac{\bar{X} - \bar{Y}}{\sigma\sqrt{1/n_X + 1/n_Y}}$.

p-value

One-sided ('greater than'): $p = P(Z \geq z_{\text{obs}})$.

Two-sided: $p = 2P(Z \geq |z_{\text{obs}}|)$.

Critical region method

Reject H_0 if $|Z| > z_{\alpha/2}$ (two-sided) or $Z > z_\alpha$ (one-sided).

'$|\bar{X} - \bar{Y}| > k$' form: $k = z_{\alpha/2}\sqrt{\sigma_X^2/n_X + \sigma_Y^2/n_Y}$.

Conclusion in context

'Since $p < 0.05$, we reject H_0 and conclude there is significant evidence at the 5% level that . . .'

Always restate in the context of the question, not just statistical jargon.

SECTION T3

Hypothesis Testing - Normal Distribution Sample Mean

Questions 1–6 · 65 marks

4. A zoologist believes that the mean weights of the adult males and females of a certain species of animal are equal. In order to test this belief, she weighs random samples of males and females with the following results.

Weights of males (kg)	14·3	15·8	13·9	13·4	14·5	15·1	13·6	14·2
Weights of females (kg)	13·2	14·8	13·7	14·7	15·0	13·1	13·5	

You may assume that these are random samples from normal populations with a common standard deviation of 0·5.

(a) State suitable hypotheses for carrying out a two-sided test. [1]

(b) Determine the *p*-value of these results and state whether or not the zoologist's belief is supported at the 5% level of significance. [9]

(984-01)

5. David and Frank are golfers and they wish to determine whether or not there is a difference between the mean distances that they can hit a golf ball. They decide that they should each hit six balls and measure the distances travelled in yards by these balls. The results are shown below.

Distances hit by David	152·1	148·3	150·6	145·4	144·7	149·3
Distances hit by Frank	143·4	147·9	150·8	144·1	145·6	147·2

You may assume that these are random samples from normal populations with a common standard deviation of 1·5.

(a) State suitable hypotheses for testing whether or not there is a difference between the mean distances. [1]

(b) Determine the *p*-value of these results and state your conclusion in context. [10]

3. A teacher in a large college wishes to investigate whether or not boys and girls perform equally well in examinations in practical mathematics. She therefore selects a random sample of 8 boys and 8 girls and gives them an examination.
The marks obtained were as follows.

Boys	52, 47, 62, 75, 51, 69, 56, 70
Girls	48, 39, 56, 69, 71, 45, 43, 59

You may assume that these are random samples from normal populations with a common standard deviation of 7·5.

(a) State suitable hypotheses for this investigation. [1]

(b) Determine the *p*-value of these results and state your conclusion in context. [10]

3. A new species of animal has been found on an uninhabited island. A zoologist wishes to investigate whether or not there is a difference in the mean weights of males and females of the species. She traps some of the animals and weighs them with the following results.

Males (kg)	5·3, 4·6, 5·2, 4·5, 4·3, 5·5, 5·0, 4·8
Females (kg)	4·9, 5·0, 4·1, 4·6, 4·3, 5·3, 4·2, 4·5, 4·8, 4·9

You may assume that these are random samples from normal populations with a common standard deviation of 0·5 kg.

(a) State suitable hypotheses for this investigation. [1]

(b) Determine the *p*-value of these results and state your conclusion in context. [9]

1. Jim is a tennis player. His serve has a mean speed of 120 miles per hour (mph). He buys a new racket and he wishes to investigate whether or not using this racket changes the mean speed of his serve. He therefore goes to a tennis centre where he hits 10 serves and the measured speeds are as follows (mph).

121·2 119·1 118·3 120·1 117·9 118·3 119·4 119·6 120·3 117·8

You may assume that this is a random sample from a normal distribution with a standard deviation of 1·2.

(a) State suitable hypotheses for his investigation. [1]

(b) Determine the *p*-value of these results and state your conclusion in context. [8]

4. The independent random variables X, Y are such that X is N(μ_x, $1{\cdot}5^2$) and Y is N(μ_y, $2{\cdot}5^2$). In order to test the hypotheses

$$H_0 : \mu_x = \mu_y \quad ; \quad H_1 : \mu_x \neq \mu_y$$

a random sample of size 8 is taken from the distribution of X and a random sample of size 12 is taken from the distribution of Y. The means of these two samples are denoted by $\bar{x}$ and $\bar{y}$ respectively. The significance level is to be 10%.

(a) Determine the critical region in the form $|\bar{x} - \bar{y}| > k$, where the value of k is to be found. [5]

(b) (i) If, in fact, $\mu_x - \mu_y = 0{\cdot}5$, find the probability of incorrectly accepting H_0.

(ii) Comment on your result in (i). [9]

END OF HYPOTHESIS TESTING - NORMAL (SAMPLE MEAN) PACK

Source: WJEC S2 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec A2 Unit 4 — Topic 3 (2.4.4)