Name	Date started	Target end date

GCE A LEVEL — APPLIED MATHEMATICS B QUESTION PACK

0984-01 (Legacy S2) · New spec Unit 4 Topic 2 · A2 unit, 15% of A-level, 80 marks, 1h 45min paper

REVISE
.wales

MATHEMATICS — APPLIED B · NORMAL DISTRIBUTION

Normal Distribution (standardising, percentiles, inverse normal)

Normal distribution questions from the legacy WJEC S2 papers (2012-2017). Standardising, quartiles, percentiles, inverse normal lookups and sums of i.i.d. normals

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 34 minutes

*Derived from the legacy S2 paper's pace of **~1.25 min/mark** (75 marks over 5 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC S2 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 2 (2.4.2).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet and statistical tables may be referred to. Take $g = 9.8\,\text{ms}^{-2}$ for mechanics.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 12 Q2	18	
2	Jun 14 Q2	14	
3	Jun 15 Q2	14	

Q	Source	Max	Mark
4	Jun 16 Q3	14	
5	Jun 17 Q3	15	
Total		**75**	

Normal Distribution (standardising, percentiles, inverse normal) — what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 4: Applied Mathematics B · Topic 2.4.2.

Standard Normal 2.4.2

- If $X \sim N(\mu, \sigma^2)$ then $Z = \dfrac{X - \mu}{\sigma} \sim N(0, 1)$.
- $\Phi(z) = P(Z \leq z)$ from tables (or calculator).
- Symmetry: $\Phi(-z) = 1 - \Phi(z)$.

Inverse Normal and sums of i.i.d. normals 2.4.2

- Percentiles: solve $\Phi(z) = p$ for z; then $x = \mu + z\sigma$.
- Upper quartile: $\Phi(z) = 0.75$ gives $z \approx 0.6745$.
- Sum of n i.i.d. $X_i \sim N(\mu, \sigma^2)$ has $S \sim N(n\mu, n\sigma^2)$.

Crib sheet »

Normal Distribution in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Standardising

$X \sim N(\mu, \sigma^2) \Rightarrow Z = \dfrac{X - \mu}{\sigma} \sim N(0, 1)$.

$\Phi(z) = P(Z \leq z)$ from tables / calculator.

Symmetry: $\Phi(-z) = 1 - \Phi(z)$.

Between two values

$P(a \leq X \leq b) = \Phi\left(\dfrac{b - \mu}{\sigma}\right) - \Phi\left(\dfrac{a - \mu}{\sigma}\right)$.

Inverse Normal / percentiles

$P(X \leq x_p) = p \Rightarrow z_p = \Phi^{-1}(p)$, then $x_p = \mu + z_p\sigma$.

Upper quartile: $z = 0.6745$. 90th: $z = 1.2816$. 95th: $z = 1.6449$. 99th: $z = 2.3263$.

Sums of i.i.d. normals

$X_1, \ldots, X_n$ i.i.d. $N(\mu, \sigma^2)$: $\sum X_i \sim N(n\mu, n\sigma^2)$.

Sample mean: $\bar{X} \sim N(\mu, \sigma^2/n)$.

This is exact, not approximate.

Useful z-values

$\Phi(1.96) = 0.975$; $\Phi(1.645) = 0.95$; $\Phi(2.576) = 0.995$.

Memorise these — they appear in nearly every Normal question.

Common pitfalls

Tables give $\Phi(z)$ for positive z only — use symmetry for negative z.

σ^2 vs σ: always check whether the question gives variance or s.d.

For $\bar{X}$, divide σ by $\sqrt{n}$, not n.

SECTION T2

Normal Distribution (standardising, percentiles, inverse normal)

Questions 1–5 · 75 marks

2. The weights X kg of male birds of a certain species are normally distributed with mean 4·4 kg and standard deviation 0·2 kg.

(a) (i) Find the probability that the weight of a randomly selected male bird exceeds 4·5 kg.

(ii) Determine the 95th percentile of X. [5]

(b) The weights Y kg of female birds of the same species are normally distributed with mean 2·6 kg and standard deviation 0·15 kg.

(i) Find the mean and variance of $2Y - X$.

(ii) Find the probability that the weight of a randomly chosen male bird is more than twice the weight of a randomly chosen female bird.

(iii) Two male birds and three female birds are placed on a weighing machine whose maximum permissible weight is 16 kg. Find the probability that the maximum weight is exceeded. [13]

2. The weights of the oranges sold on a market stall are normally distributed with mean 248 grams and standard deviation 8 grams. The weights of the lemons sold on the market stall are normally distributed with mean 85 grams and standard deviation 1·5 grams.

(a) Find the upper quartile of the weights of the lemons. [2]

(b) Ann buys 8 oranges. Calculate the probability that the total weight of her oranges is less than 2000 grams. [5]

(c) Bethan buys 1 orange and 1 lemon. Calculate the probability that the weight of her orange is more than three times the weight of her lemon. [7]

2. In a certain population, the weights of the men are normally distributed with mean 82 kg and standard deviation 2·5 kg. The weights of the women are normally distributed with mean 65 kg and standard deviation 2 kg.

(a) Calculate the 95^{th} percentile of the men's weights. [2]

(b) Determine the probability that the weight of a randomly chosen woman lies between 64 kg and 68 kg. [6]

(c) One morning, 3 men and 4 women hire a boat which has a safety limit of 500 kg. Calculate the probability that their combined weight exceeds the safety limit. You may assume that the weights of the 3 men and 4 women are independent. [6]

3. For a certain breed of dog, the weights of the males are normally distributed with mean 40 kg and standard deviation 2·5 kg. The weights of the females are normally distributed with mean 32 kg and standard deviation 1·5 kg.

(a) Calculate the upper quartile of the weights of male dogs of this breed. [2]

(b) A random selection is made of 3 males and 2 females of the breed.
Calculate the probability that

(i) the combined weight of the 5 dogs exceeds 185 kg,

(ii) the combined weight of the 3 males is less than twice the combined weight of the 2 females. [12]

3. A grocer sells apples and pears. The weights of the apples may be assumed to be normally distributed with mean 110 grams and standard deviation 14 grams. The weights of the pears may be assumed to be normally distributed with mean 160 grams and standard deviation 16 grams.

(a) Find the 90^{th} percentile of the weights of the apples. [2]

(b) George buys 10 apples. Find the probability that the total weight of his 10 apples is less than 1000 grams. [6]

(c) Sue buys 3 apples and 2 pears. Find the probability that the combined weight of her 3 apples is more than the combined weight of her 2 pears. [7]

END OF NORMAL DISTRIBUTION PACK

Source: WJEC S2 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec A2 Unit 4 — Topic 2 (2.4.2)