Name	Date started	Target end date

GCE A LEVEL — APPLIED MATHEMATICS B QUESTION PACK

0983-01 (Legacy S1) · New spec Unit 4 Topic 1 · A2 unit, 15% of A-level, 80 marks, 1h 45min paper

REVISE
.wales

MATHEMATICS — APPLIED B · CONDITIONAL PROBABILITY

Conditional Probability

Every conditional probability and Bayes-style question from the legacy WJEC S1 papers (2011-2017). Covers P(A|B), tree diagrams and conditional Venn arguments

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 48 minutes

*Derived from the legacy S1 paper's pace of **~1.25 min/mark** (134 marks over 19 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC S1 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 1 (2.4.1).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet and statistical tables may be referred to. Take $g = 9.8\,\text{ms}^{-2}$ for mechanics.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q3	7	
2	Jun 11 Q6	7	
3	Jan 12 Q1	7	
4	Jan 12 Q5	6	
5	Jun 12 Q1	8	
6	Jun 12 Q5	6	
7	Jan 13 Q1	5	
8	Jun 13 Q1	5	
9	Jun 13 Q5	6	
10	Jan 14 Q1	5	

Q	Source	Max	Mark
11	Jan 14 Q5	8	
12	Jun 14 Q1	6	
13	Jun 14 Q6	9	
14	Jun 15 Q2	7	
15	Jun 15 Q5	8	
16	Jun 16 Q1	9	
17	Jun 16 Q2	6	
18	Jun 17 Q1	9	
19	Jun 17 Q7	10	
Total		**134**	

Conditional Probability — what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 4: Applied Mathematics B · Topic 2.4.1.

Conditional probability formula 2.4.1

- $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ provided $P(B) > 0$.
- Rearranged: $P(A \cap B) = P(A \mid B)\,P(B) = P(B \mid A)\,P(A)$.
- Independence: $P(A \mid B) = P(A)$, equivalently $P(A \cap B) = P(A)P(B)$.

Tree diagrams and total probability 2.4.1

- Law of total probability: $P(A) = P(A \mid B)P(B) + P(A \mid B')P(B')$.
- Bayes: $P(B \mid A) = \dfrac{P(A \mid B)P(B)}{P(A)}$.
- Tree diagrams: multiply along branches, add between branches.

Crib sheet »

Conditional Probability in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Definition

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, for $P(B) > 0$.

Read aloud as 'probability of A given B'.

Multiplication rule

$P(A \cap B) = P(A \mid B)\,P(B) = P(B \mid A)\,P(A)$.

Use this to swap which side of the bar carries the unknown.

Independence

A, B independent $\iff P(A \cap B) = P(A)P(B)$.

Equivalently $P(A \mid B) = P(A)$.

Mutually exclusive $\Rightarrow$ *not* independent (unless one has $P = 0$).

Total probability and Bayes

$P(A) = P(A \mid B)P(B) + P(A \mid B')P(B')$.

$P(B \mid A) = \dfrac{P(A \mid B)P(B)}{P(A)}$ — flip the conditioning.

Tree-diagram approach

Branches: $P(B), P(B')$. Sub-branches: $P(A \mid B), P(A \mid B')$.

Joint probabilities multiply along a path.

Marginal $P(A)$ is the sum of branch products ending in A.

Common pitfalls

$P(A \cup B) \neq P(A) + P(B)$ unless mutually exclusive.

Don't confuse $P(A \mid B)$ with $P(B \mid A)$.

'A given B' restricts the sample space to B only.

SECTION T1

Conditional Probability

Questions 1–19 · 134 marks

3. The events A and B are such that

$$P(A) = 0{\cdot}25,\ P(B) = 0{\cdot}4 \text{ and } P(A' \cap B') = 0{\cdot}45.$$

Determine whether

(a) A and B are mutually exclusive, [3]

(b) A and B are independent. [4]

6. A box contains three coins. Two of these coins are fair and the third coin is double-headed so that when tossed a head is always obtained. One of these coins is selected at random and tossed three times.

(a) Find the probability that three heads are obtained. [4]

(b) Given that three heads are obtained, find the probability that the double-headed coin was selected. [3]

(c) The selected coin is tossed a fourth time. Find the probability that a head is obtained. [2]

1. A class contains 8 girls and 6 boys. A sub-committee of 3 members of the class is to be formed and it is decided to select its members at random from the class. Calculate the probability that the sub-committee will contain

(a) 3 boys, [2]

(b) more boys than girls. [4]

5. Each of three boxes contains 3 cards. Box A contains 1 red card, Box B contains 2 red cards and Box C contains 3 red cards. One of the boxes is selected at random and a card is chosen at random from that box.

(a) Find the probability that a red card is chosen. [3]

(b) Given that a red card is chosen, find the probability that Box A was selected. [3]

1. The events A and B are such that

$$P(A) = 0{\cdot}5,\ P(B) = 0{\cdot}3.$$

(a) Evaluate $P(A \cup B)$ when

(i) A, B are mutually exclusive,
(ii) A, B are independent. [5]

(b) Given that $P(A \cup B) = 0{\cdot}7$, find the value of $P(B|A)$. [3]

5. In a certain population, 60% are male and 40% are female. It is known that 8% of males are colour-blind and 3% of females are colour-blind. A member of the population is selected at random.

(a) Find the probability that this person is colour-blind. [3]

(b) Given that this person is colour-blind, find the probability that the person is female. [3]

1. The independent events A, B are such that

$$P(A) = 0{\cdot}2,\ P(A \cup B) = 0{\cdot}4.$$

(a) Determine the value of $P(B)$. [4]

(b) Calculate the probability that exactly one of the events A, B occurs. [3]

1. The events A and B are such that

$$P(A) = 0{\cdot}25,\ P(A \cup B) = 0{\cdot}4.$$

Evaluate $P(B)$ when

(a) A, B are mutually exclusive, [2]

(b) A, B are independent. [3]

5. Box A contains four balls numbered 1, 2, 3, 4 respectively, Box B contains three balls numbered 1, 2, 3 respectively and Box C contains two balls numbered 1, 2 respectively. Gwen selects one of these boxes at random and then selects a ball at random from that box.

(a) Determine the probability that a ball numbered 1 is selected. [3]

(b) Given that a ball numbered 1 is selected, determine the probability that Box A was selected. [3]

1. The events A and B are such that

$$P(A) = 0{\cdot}5,\ P(B) = 0{\cdot}2,\ P(A|B) = 0{\cdot}4.$$

(a) Calculate

(i) $P(A \cap B)$,

(ii) $P(B|A)$. [4]

(b) Giving a reason, state whether or not A and B are mutually exclusive. [1]

5. Three drawers each contain 4 coins. Drawer A contains 4 gold coins. Drawer B contains 3 gold coins and 1 silver coin. Drawer C contains 2 gold coins and 2 silver coins. David selects one of these drawers at random and then selects 2 coins at random from that drawer without replacement.

(a) Determine the probability that he selects 2 gold coins. [5]

(b) Given that he selects 2 gold coins, determine the probability that Drawer A was selected. [3]

1. The events A and B are such that

$$P(A) = 0{\cdot}3,\ P(B) = 0{\cdot}4,\ P(A \cup B) = 0{\cdot}5.$$

(a) Determine whether or not A and B are independent. [3]

(b) Evaluate $P(A|B')$. [3]

6. A purse contains three fair coins and one double-headed coin. A coin is selected at random from the purse and tossed.

(a) Find the probability that a head is obtained. [3]

(b) Given that a head is obtained,

(i) determine the probability that the double-headed coin was selected,

(ii) find the probability that a head will be obtained if the selected coin is tossed a second time. [6]

2. The events A and B are such that

$$P(A) = 0{\cdot}4,\ P(B) = 0{\cdot}5 \text{ and } P(A \cup B) = 2 \times P(A \cap B).$$

(a) Show that $P(A \cap B) = 0{\cdot}3$. [2]

(b) Evaluate $P(A|B)$. [2]

(c) Evaluate $P(B|A')$. [3]

5. At a certain university, 60% of the students are male and 40% are female. It is known that 75% of the male students own a bicycle and 30% of the female students own a bicycle. One of the students is selected at random.

(a) Calculate the probability that the selected student

(i) is a male student who owns a bicycle,

(ii) owns a bicycle. [5]

(b) Given that the selected student owns a bicycle, calculate the probability that this student is female. [3]

1. The events A and B are such that

$$P(A) = 0{\cdot}3,\ P(B) = 0{\cdot}4.$$

Evaluate $P(A \cup B)$ in each of the following cases.

(a) A and B are mutually exclusive. [2]

(b) A and B are independent. [3]

(c) $P(A|B) = 0{\cdot}25$. [4]

2. In a certain population, 45% are male and 55% are female. It is known that 3% of the males have red hair while 5% of the females have red hair. One of the members of the population is selected at random.

(a) Calculate the probability that the selected person has red hair. [3]

(b) Given that the selected person has red hair, calculate the probability that this person is female. [3]

1. The events A and B are such that

$$P(A) = 0{\cdot}2,\ P(B) = 0{\cdot}3,\ P(A \cup B) = 0{\cdot}4.$$

(a) Show that A and B are not independent. [3]

(b) Determine the value of

(i) $P(A'|B)$,

(ii) $P(A \cup B')$. [6]

7. It is known that 5% of animals of a certain species have a particular disease. A diagnostic test can be applied to animals of this species to indicate whether or not they have this disease. When applied to an animal which has this disease, the test gives a positive response with probability 0·96. When applied to an animal which does not have this disease, the test gives a positive response with probability 0·02.

(a) The test is given to a randomly chosen animal.

(i) Calculate the probability that a positive response is obtained. [3]

(ii) Given that a positive response is obtained, find the probability that this animal has the disease. [3]

(b) A randomly chosen animal gave a positive response when tested. It is tested again.

(i) Find the probability that it gives a second positive response.

(ii) Given that this second response is positive, calculate the probability that this animal has the disease. [4]

END OF CONDITIONAL PROBABILITY PACK

Source: WJEC S1 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec A2 Unit 4 — Topic 1 (2.4.1)