Name	Date started	Target end date

GCE A LEVEL — PURE MATHEMATICS B QUESTION PACK

0976-01 (Legacy C4) · New spec Unit 3 Topic 8 · A2 unit, 35% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS — PURE B · PARAMETRIC EQUATIONS

Parametric Equations

Every parametric equation question from the legacy WJEC C4 papers (June 2011 – June 2017) for new-spec A2 Unit 3

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 22 minutes

*Derived from the legacy C3/C4 paper's pace of **~1.25 min/mark** (66 marks over 7 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C3 and C4 papers (2008 modular spec) that maps onto new-spec A2 Unit 3 Topic 8 (2.3.10).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q4	9	
2	Jun 12 Q6	11	
3	Jun 13 Q6	11	
4	Jun 14 Q6	9	
5	Jun 15 Q6	8	
6	Jun 16 Q5	8	
7	Jun 17 Q6	10	
	Total	**66**	

Parametric Equations — what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 3: Pure Mathematics B · Topic 2.3.10.

Parametric form 2.3.10

- Curve given by $x = f(t), y = g(t)$.
- $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.
- Eliminate t when possible to get a Cartesian equation.

Tangents at parametric points 2.3.10

- Find $\frac{dy}{dx}$ in terms of t.
- Evaluate at the parameter value p to get the gradient at the point.
- Use point-gradient form: $y - g(p) = m(x - f(p))$.

Parametric Equations in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Setup

Curve given parametrically: $x = f(t), y = g(t)$.

Gradient: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

Tangent line equation

At parameter $t = p$: point is $(f(p), g(p))$, gradient $m = g'(p)/f'(p)$.

Tangent: $y - g(p) = m(x - f(p))$.

Normal line equation

Normal gradient $= -1/m$ at the same point.

$y - g(p) = -\frac{1}{m}(x - f(p))$.

Second derivative

$\frac{d^2y}{dx^2} = \frac{d/dt(dy/dx)}{dx/dt}$.

Not $\frac{d^2y/dt^2}{d^2x/dt^2}$!

Eliminating parameter

Use trig identities (e.g. $\sin^2 t + \cos^2 t = 1$) or algebra to remove t.

Often gives a recognisable Cartesian curve (ellipse, parabola).

Common form: $x = at^2, y = 2at$

Parabola $y^2 = 4ax$.

Tangent at $(at^2, 2at)$: $ty = x + at^2$.

SECTION T8

Parametric Equations

Questions 1–7 · 66 marks

4. The curve C has the parametric equations

$$x = 3\cos t,\ y = 4\sin t.$$

The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

$$(3\sin p)\,y + (4\cos p)\,x - 12 = 0.$$ [5]

(b) The tangent to C at the point P meets the x-axis at the point A and the y-axis at the point B. Given that $p = \dfrac{\pi}{6}$,

(i) find the coordinates of A and B,

(ii) show that the exact length of AB is $2\sqrt{19}$. [4]

6. The parametric equations of the curve C are $x = t^2$, $y = 2t$.

(a) Show that the normal to C at the point P with parameter p has equation

$$y + px = p^3 + 2p.$$

[5]

(b) The normal to C at the point P intersects C again at the point with parameter 3.

(i) Show that $p^3 - 7p - 6 = 0$.

(ii) Hence show that P can be one of two points. Find the coordinates of each of these two points. [6]

6. The curve C has the parametric equations

$$x = at,\ y = \frac{b}{t},$$

where a, b are positive constants.
The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

$$ap^2y + bx - 2abp = 0.$$ [5]

(b) The tangent to C at the point P meets the x-axis at the point A and the y-axis at the point B. Find the area of triangle AOB, where O denotes the origin. Give your answer in its simplest form. [3]

(c) The point D has coordinates $(2a, b)$. Show that there is no point P on C such that the tangent to C at the point P passes through D. [3]

6. The curve C has the parametric equations $x = 2t$, $y = 5t^3$. The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

$$2y = 15p^2x - 20p^3.$$

[4]

(b) The tangent to C at the point P intersects C again at the point $Q(2q, 5q^3)$. Given that $p = 1$, show that q satisfies the equation

$$q^3 - 3q + 2 = 0.$$

Hence find the value of q. [5]

6. The parametric equations of the curve *C* are $x = at^2$, $y = 2at$, where a is a positive constant. The points *P* and *Q* lie on *C* and have parameters p and q respectively.

(a) Simplifying your answer in each case, find

(i) the gradient of the tangent to *C* at the point *P*,

(ii) the equation of the tangent to *C* at the point *P*. [4]

(b) (i) Find an expression, in its simplest form, for the gradient of the line *PQ*.

(ii) Explain how you could use the answer of *(b)*(i) to derive the gradient of the tangent to *C* at the point *P*. [4]

5. The parametric equations of the curve *C* are

$$x = \frac{3}{t}, \; y = 4t.$$

(a) Show that the tangent to *C* at the point *P* with parameter p has equation

$$3y = -4p^2x + 24p.$$ [4]

(b) The tangent to *C* at the point *P* passes through the point (1, 9). Show that *P* can be one of two points. Find the coordinates of each of these two points. [4]

6. The curve *C* has the parametric equations $x = at^2$, $y = bt^3$, where a, b are positive constants. The point *P* lies on *C* and has parameter p.

(a) Show that the equation of the tangent to *C* at the point *P* is

$$2ay = 3bpx - abp^3.$$ [5]

(b) The tangent to *C* at the point *P* intersects *C* again at the point with coordinates $(4a, 8b)$. Show that p satisfies the equation

$$p^3 - 12p + 16 = 0.$$

Hence find the value of p. [5]

END OF PARAMETRIC EQUATIONS PACK

Source: WJEC C3 + C4 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec A2 Unit 3 — Topic 8 (2.3.10)