

Name	Date started	Target end date
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GCE A LEVEL – PURE MATHEMATICS B QUESTION PACK

0976-01 (Legacy C4) · New spec Unit 3 Topic 7 · A2 unit, 35% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS – PURE B · TRIGONOMETRY - R-FORM & DOUBLE ANGLE

Trigonometry (R-form, Double Angle & Compound Angle)

Every R sin/cos transformation + double-angle question from the legacy WJEC C4 papers (June 2011 – June 2017) for new-spec A2 Unit 3

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 42 minutes

Derived from the legacy C3/C4 paper's pace of ~1.25 min/mark (82 marks over 7 questions).

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C3 and C4 papers (2008 modular spec) that maps onto new-spec A2 Unit 3 Topic 7 (2.3.6).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

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Q	Source	Max	Mark
1	Jun 11 Q3	11	
2	Jun 12 Q3	13	
3	Jun 13 Q3	12	
4	Jun 14 Q3	10	
5	Jun 15 Q3	11	
6	Jun 16 Q4	13	
7	Jun 17 Q3	12	
Total		82	

Trigonometry (R-form, Double Angle & Compound Angle) – what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 3: Pure Mathematics B · Topic 2.3.6.

Double-angle formulae 2.3.6

- $\sin 2\theta = 2 \sin \theta \cos \theta$.
- $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1$.
- $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$.

R-form: $a \sin \theta + b \cos \theta$ 2.3.6

- Write as $R \sin(\theta + \alpha)$ or $R \cos(\theta - \alpha)$ where $R = \sqrt{a^2 + b^2}$.
- Find α from $\tan \alpha = \frac{b}{a}$ (matching coefficients).
- Used to find maximum/minimum values: $-R \leq a \sin \theta + b \cos \theta \leq R$.

Trigonometry - R-form & Double Angle in one page

Quick-reference notes – revisit before each section. Don't use during questions.

Compound angles

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B.$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}.$$

Double-angle formulae

$$\sin 2A = 2 \sin A \cos A.$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 -$$

$$2 \sin^2 A = 2 \cos^2 A - 1.$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}.$$

R-form (sine version)

$$a \sin \theta + b \cos \theta = R \sin(\theta + \alpha) \text{ where}$$

$$R = \sqrt{a^2 + b^2} \text{ and } \tan \alpha = b/a.$$

R-form (cosine version)

$$a \cos \theta + b \sin \theta = R \cos(\theta - \alpha) \text{ where}$$

$$R = \sqrt{a^2 + b^2} \text{ and } \tan \alpha = b/a.$$

Max / min applications

Max value of $a \sin \theta + b \cos \theta$ is $+R$, min is $-R$.

Max value of $\frac{1}{a \sin \theta + b \cos \theta}$ is $1/(-R)$ etc.

Solving in R-form

To solve $a \sin \theta + b \cos \theta = c$: rewrite as

$$R \sin(\theta + \alpha) = c, \text{ then } \theta + \alpha =$$

$$\sin^{-1}(c/R).$$

SECTION T7

Trigonometry (R-form, Double Angle & Compound Angle)

Questions 1-7 · 82 marks

3. (a) Find all values of x in the range $0^\circ \leq x \leq 180^\circ$ satisfying

$$\tan 2x = 4 \tan x. \quad [5]$$

- (b) Express $7 \cos \theta + 24 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.
Hence, find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$7 \cos \theta + 24 \sin \theta = 16. \quad [6]$$

3. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$4\cos 2\theta = 1 - 2\sin\theta. \quad [6]$$

- (b) (i) Express $8\sin x + 15\cos x$ in the form $R\sin(x + \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.

- (ii) Find all values of x in the range $0^\circ \leq x \leq 360^\circ$ satisfying

$$8\sin x + 15\cos x = 11.$$

- (iii) Find the greatest possible value for k so that

$$8\sin x + 15\cos x = k$$

has solutions. Give a reason for your answer. [7]

3. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$8 \cos 2\theta + 6 = \cos^2 \theta + \cos \theta. \quad [6]$$

- (b) Express $\sqrt{15} \cos \theta - \sin \theta$ in the form $R \cos(\theta + \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.
Hence find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$\sqrt{15} \cos \theta - \sin \theta = 3. \quad [6]$$

3. (a) Find all values of x in the range $0^\circ \leq x \leq 180^\circ$ satisfying

$$\tan 2x = 3 \cot x. \quad [4]$$

- (b) (i) Express $21 \sin \theta - 20 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.

- (ii) Use your results to part (i) to find the greatest value of

$$\frac{1}{21 \sin \theta - 20 \cos \theta + 31}.$$

Write down a value for θ for which this greatest value occurs. [6]

3. (a) Find all values of x in the range $0^\circ \leq x \leq 180^\circ$ satisfying

$$\tan(x + 45^\circ) = 8 \tan x. \quad [5]$$

- (b) (i) Express $\sqrt{13} \sin \theta - 6 \cos \theta$ in the form $R \sin(\theta - \alpha)$,
where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.
- (ii) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$\sqrt{13} \sin \theta - 6 \cos \theta = -4. \quad [6]$$

4. (a) The angle x is such that $0^\circ \leq x \leq 180^\circ$, $x \neq 90^\circ$.

Given that x satisfies the equation $3 \tan 2x + 16 \cot^2 x = 0$,

- (i) show that $3 \tan^3 x - 8 \tan^2 x + 8 = 0$,
- (ii) find all possible values of x , giving each answer in degrees, correct to one decimal place. [8]
- (b) Express $24 \cos \theta - 7 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.

Hence, find the range of values of k for which the equation

$$24 \cos \theta - 7 \sin \theta = k$$

has no solutions.

[5]

3. (a) Show that the equation

$$5 \cos^2 \theta + 7 \sin 2\theta = 3 \sin^2 \theta$$

may be rewritten in the form

$$a \tan^2 \theta + b \tan \theta + c = 0,$$

where a, b, c are non-zero constants whose values are to be found.

Hence, find all values of θ in the range $0^\circ \leq \theta \leq 180^\circ$ satisfying the equation

$$5 \cos^2 \theta + 7 \sin 2\theta = 3 \sin^2 \theta. \quad [6]$$

- (b) (i) Express $\sqrt{5} \cos \phi + \sqrt{11} \sin \phi$ in the form $R \cos(\phi - \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.

- (ii) Use your result to part (i) to find the least value of

$$\frac{1}{\sqrt{5} \cos \phi + \sqrt{11} \sin \phi + 6}.$$

Write down a value for ϕ for which this least value occurs.

[6]

END OF TRIGONOMETRY - R-FORM & DOUBLE ANGLE PACK

Source: WJEC C3 + C4 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec A2 Unit 3 – Topic 7 (2.3.6)

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