

Name	Date started	Target end date
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GCE A LEVEL – PURE MATHEMATICS B QUESTION PACK

0975-01 (Legacy C3) + 0976-01 (Legacy C4) · New spec Unit 3 Topic 3 · A2 unit, 35% of A-level, 120 marks, 2h 30min paper

REVISE

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MATHEMATICS – PURE B · PROOF - COUNTER-EXAMPLE & CONTRADICTION

Proof (Counter-example & Contradiction)

Every proof question from the legacy WJEC C3 + C4 papers (June 2011 – June 2017) that maps onto new-spec A2 Unit 3 proof methods

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~0 hours 46 minutes

Derived from the legacy C3/C4 paper's pace of ~1.25 min/mark (37 marks over 9 questions).

You are advised to **not** attempt to complete all of this in one sitting.

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C3 and C4 papers (2008 modular spec) that maps onto new-spec A2 Unit 3 Topic 3 (2.3.1).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

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Q	Source	Max	Mark
1	Jun 11 Q7	5	
2	Jun 15 Q10	11	
3	Jun 11 Q10	3	
4	Jun 12 Q10	3	
5	Jun 13 Q10	3	
6	Jun 14 Q10	3	
7	Jun 15 Q10	3	
8	Jun 16 Q10	3	
9	Jun 17 Q10	3	
Total		37	

Proof (Counter-example & Contradiction) – what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 3: Pure Mathematics B · Topic 2.3.1.

Counter-example 2.3.1

- Disprove a general statement by finding a single counter-example.
- Pick simple values (often integer or trigonometric special angles).
- Show the conclusion fails with that example.

Proof by contradiction 2.3.1

- Assume the negation of the proposition.
- Derive a contradiction from the assumption.
- Conclude the original proposition must be true.

Proof - Counter-example & Contradiction in one page

Quick-reference notes – revisit before each section. Don't use during questions.

Counter-example method

To disprove $\forall x, P(x)$, find one x_0 with $\neg P(x_0)$.

Pick simple values: 0, 1, $\pi/2$, integers, etc.

State the value and show the conclusion fails.

Proof by contradiction

To prove P : assume $\neg P$.

Derive a contradiction (a false statement).

Conclude P must hold.

Classic examples

$\sqrt{2}$ irrational (assume rational a/b in lowest terms, derive both even).

\sqrt{n} irrational for non-square n .

Infinitely many primes (Euclid's argument).

Worked structure

1. Start: "Assume for contradiction..."
2. Manipulate algebraically.
3. Reach a contradiction.
4. Conclude.

Pitfalls

- Don't prove the converse by mistake.
- Be careful with assumed structure (lowest terms, even/odd).

WJEC C4 Q10 style

Most C4 papers end with a 3-mark proof by contradiction.

The first line is usually given – you must complete the logic.

SECTION T3

Proof (Counter-example & Contradiction)

Questions 1-9 · 37 marks

7. (a) Show, by counter-example, that the statement

$$|a + b| \equiv |a| + |b|$$

is false.

[2]

- (b) Solve the equation

$$|2x + 1| = |3x - 4|$$

[3]

10. (a) Show, by counter-example, that the following statement is false.

'If two functions h and k are such that their derivatives h' and k' are equal, then the functions h and k must themselves be equal.'

[2]

- (b) The functions f and g have domains $[7, 60]$ and $[9, \infty)$ respectively and are defined by

$$f(x) = 2\ln(4x + 5) + 3,$$

$$g(x) = e^x.$$

- (i) Find an expression for $f^{-1}(x)$.
- (ii) Write down the domain of f^{-1} , giving the end-points of your domain correct to the nearest integer.
- (iii) Write down an expression for $gf(x)$ and simplify your answer. [9]

END OF PAPER

10. Prove by contradiction the following proposition.

When x is real and positive,

$$4x + \frac{9}{x} \geq 12.$$

The first line of the proof is given below.

Assume that there is a positive and real value of x such that

$$4x + \frac{9}{x} < 12. \quad [3]$$

10. Complete the following proof by contradiction to show that $\sqrt{5}$ is irrational.

Assume that $\sqrt{5}$ is rational. Then $\sqrt{5}$ may be written in the form $\frac{a}{b}$, where a, b are integers having no common factors.

$$\therefore a^2 = 5b^2.$$

$\therefore a^2$ has a factor 5.

$\therefore a$ has a factor 5 so that $a = 5k$, where k is an integer.

[3]

10. Prove by contradiction the following proposition.

When x is real,

$$(5x - 3)^2 + 1 \geq (3x - 1)^2.$$

The first line of the proof is given below.

Assume that there is a real value of x such that

$$(5x - 3)^2 + 1 < (3x - 1)^2. \quad [3]$$

10. Complete the following proof by contradiction to show that

$$\sin \theta + \cos \theta \leq \sqrt{2}$$

for all values of θ .

*Assume that there is a value of θ for which $\sin \theta + \cos \theta > \sqrt{2}$.
Then squaring both sides, we have:*

[3]

END OF PAPER

10. Prove by contradiction the following proposition.

If a and b are odd integers such that 4 is a factor of $a - b$,
then 4 is **not** a factor of $a + b$.

The first lines of the proof are given below.

Assume that 4 is a factor of $a + b$.

Then there exists an integer c such that $a + b = 4c$.

[3]

END OF PAPER

10. Prove by contradiction the following proposition.

When x is real and $x \neq 0$,

$$\left| x + \frac{1}{x} \right| \geq 2.$$

The first two lines of the proof are given below.

Assume that there is a real value of x such that

$$\left| x + \frac{1}{x} \right| < 2.$$

Then squaring both sides, we have:

[3]

END OF PAPER

10. Complete the following proof by contradiction to show that $\sqrt{7}$ is irrational.

Assume that $\sqrt{7}$ is rational. Then $\sqrt{7}$ may be written in the form $\frac{a}{b}$,
where a, b are integers having no factors in common.

$$\therefore a^2 = 7b^2.$$

$\therefore a^2$ has a factor 7.

$\therefore a$ has a factor 7 so that $a = 7k$, where k is an integer.

[3]

END OF PAPER

END OF PROOF - COUNTER-EXAMPLE & CONTRADICTION PACK

Source: WJEC C3 + C4 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec A2 Unit 3 – Topic 3 (2.3.1)

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