Name	Date started	Target end date

GCE A LEVEL — PURE MATHEMATICS B QUESTION PACK

0975-01 (Legacy C3) · New spec Unit 3 Topic 2 · A2 unit, 35% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS — PURE B · NUMERICAL ITERATION & ROOT-FINDING

Numerical Methods (Iteration & Root-finding)

Every iteration / recurrence relation question from the legacy WJEC C3 papers (June 2011 – June 2017) for new-spec A2 Unit 3 root-finding

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 22 minutes

*Derived from the legacy C3/C4 paper's pace of **~1.25 min/mark** (66 marks over 9 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C3 and C4 papers (2008 modular spec) that maps onto new-spec A2 Unit 3 Topic 2 (2.3.13).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q4	8	
2	Jun 12 Q4	7	
3	Jan 13 Q4	8	
4	Jan 14 Q5	5	
5	Jun 13 Q8	5	
6	Jun 14 Q5	9	
7	Jun 15 Q5	7	
8	Jun 16 Q5	8	
9	Jun 17 Q4	9	
	Total	66	

Numerical Methods (Iteration & Root-finding) — what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 3: Pure Mathematics B · Topic 2.3.13.

Iterative root-finding 2.3.13

- Use a recurrence $x_{n+1} = f(x_n)$ to locate a root.
- Show that values converge to a stated number of decimal places.
- Show $f(\alpha) = 0$ to confirm the root.

Sign change & iteration 2.3.13

- Use the sign change (intermediate-value) method to locate a root.
- Sketch $y = f(x)$ to estimate root location.
- Newton-Raphson: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

Numerical Iteration & Root-finding in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Iteration setup

To solve $f(x) = 0$, rearrange to $x = g(x)$, then iterate $x_{n+1} = g(x_n)$.

Convergence requires $|g'(\alpha)| < 1$ near the root.

Worked steps

1. Start from x_0 (often given).
2. Compute x_1, x_2, x_3, x_4 to required d.p.
3. State convergence to α.
4. Verify $f(\alpha) \approx 0$.

Sign change check

If $f(a)$ and $f(b)$ have opposite signs and f is continuous on $[a, b]$, then a root lies in (a, b).

Sketch on the same axes can help locate roots.

Newton-Raphson

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

Geometrically: tangent at $(x_n, f(x_n))$ crosses x-axis at x_{n+1}.

Failure modes

Iteration can diverge or oscillate.

Newton-Raphson fails if $f'(x_n) = 0$ or starts too far from root.

Common framework

Standard exam structure:

(a) Show root lies between a and b.

(b) Use recurrence to find root to n d.p.

SECTION T2

Numerical Methods (Iteration & Root-finding)

Questions 1–9 · 66 marks

4. *(a)* Show that $f(x) = 11\tan^{-1} 2x - 3x^2$ has a stationary value when x satisfies

$$12x^3 + 3x - 11 = 0 .$$ [3]

(b) **You may assume** that the equation $12x^3 + 3x - 11 = 0$ has a root α between 0 and 1.

The recurrence relation

$$x_{n+1} = \left(\frac{11 - 3x_n}{12}\right)^{\frac{1}{3}}$$

with x_0 = 0·9 can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. [5]

4. Show that the equation

$$\cos x - 5x + 2 = 0$$

has a root α between 0 and $\frac{\pi}{4}$.

The recurrence relation

$$x_{n+1} = \frac{1}{5}(2 + \cos x_n)$$

with $x_0 = 0{\cdot}6$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

4. *(a)* On the same diagram, sketch the graphs of $y = \ln x$ and $y = 11 - 2x$.
Deduce the number of roots of the equation

$$\ln x + 2x - 11 = 0.$$ [3]

(b) **You may assume** that the equation

$$\ln x + 2x - 11 = 0$$

has a root α between 4 and 5.
The recurrence relation

$$x_{n+1} = \frac{11 - \ln x_n}{2},$$

with $x_0 = 4{\cdot}7$, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [5]

5. **You may assume** that the equation $x^3 + 7x^2 - 3 = 0$ has a root α between 0 and 1.
The recurrence relation

$$x_{n+1} = \sqrt{\frac{3}{x_n + 7}}$$

with $x_0 = 1$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4.

Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. [5]

8. **You may assume** that the equation

$$x^2 + e^x - 3 = 0$$

has a root α between -2 and -1.

The recurrence relation

$$x_{n+1} = -\left(3 - e^{x_n}\right)^{\frac{1}{2}}$$

with $x_0 = -1{\cdot}5$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [5]

TURN OVER

5. *(a)* Show that $f(x) = \ln(3x^2 - 2x - 1) - 4x^2$ has a stationary value when x satisfies

$$12x^3 - 8x^2 - 7x + 1 = 0.$$ [4]

(b) **You may assume** that the equation $12x^3 - 8x^2 - 7x + 1 = 0$ has a root α between -1 and 0.
The recurrence relation

$$x_{n+1} = \left(\frac{8x_n^2 + 7x_n - 1}{12}\right)^{\frac{1}{3}}$$

with $x_0 = -0{\cdot}6$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and show this is the value of α correct to four decimal places. [5]

5. *(a)* On the same diagram, sketch the graphs of $y = \cos^{-1} x$ and $y = 5x - 1$. [2]

(b) **You may assume** that the equation

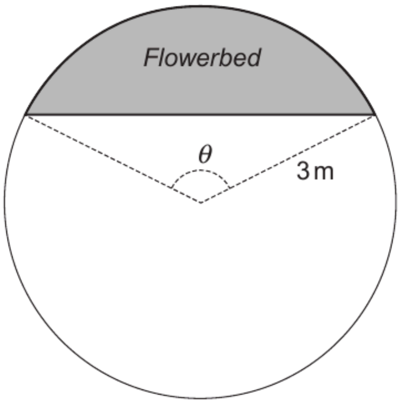
$$\cos^{-1} x - 5x + 1 = 0$$

has a root α between $0{\cdot}4$ and $0{\cdot}5$.
The recurrence relation

$$x_{n+1} = \frac{1}{5}\left(1 + \cos^{-1} x_n\right)$$

with $x_0 = 0{\cdot}4$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and prove that this is the value of α correct to four decimal places. [5]

5. The diagram shows a circular garden plot of radius 3 m. Alun wants to use a minor segment of the plot as a flowerbed and has a 13·5 m length of edging, all of which he intends to use to form the perimeter of the shaded area below. The angle subtended at the centre of the circular plot is denoted by θ radians.



(a) Show that θ satisfies the equation

$$\theta + 2\sin\left(\frac{\theta}{2}\right) = 4{\cdot}5\,.$$ [3]

(b) Alun believes that the value of θ will turn out to be approximately 2·5.
Starting with $\theta_0 = 2{\cdot}5$, use the recurrence relation

$$\theta_{n+1} = 4{\cdot}5 - 2\sin\left(\frac{\theta_n}{2}\right)$$

to find the values of θ_1, θ_2, θ_3. Write down the value of θ_3 correct to two decimal places and prove that this is the value of θ correct to two decimal places. [5]

4. A large tank in the form of a cuboid is used to store water. The width of the tank is denoted by x m. The length of the tank is 4 m **greater** than its width, whilst the height of the tank is 2 m **less** than its width. The volume of the tank is 150 m^3.

(a) (i) Show that $x^3 + 2x^2 - 8x - 150 = 0$.

(ii) Show that $5 < x < 6$. [4]

(b) The recurrence relation

$$x_{n+1} = \left(150 + 8x_n - 2x_n^2\right)^{\frac{1}{3}},$$

with $x_0 = 6$, can be used to find the value of x. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to two decimal places and prove that this is the value of x correct to two decimal places. [5]

END OF NUMERICAL ITERATION & ROOT-FINDING PACK

Source: WJEC C3 + C4 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec A2 Unit 3 — Topic 2 (2.3.13)