

Name	Date started	Target end date
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GCE A LEVEL – PURE MATHEMATICS B QUESTION PACK

0975-01 (Legacy C3) · New spec Unit 3 Topic 17 · A2 unit, 35% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS – PURE B · FUNCTIONS - INVERSE, COMPOSITE, DOMAIN & RANGE

Functions (Inverse, Composite, Domain & Range)

Every function (inverse, composite, domain/range) question from the legacy WJEC C3 papers (June 2011 – June 2017) for new-spec A2 Unit 3 algebra & functions

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 38 minutes

Derived from the legacy C3/C4 paper's pace of ~1.25 min/mark (126 marks over 18 questions).

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C3 and C4 papers (2008 modular spec) that maps onto new-spec A2 Unit 3 Topic 17 (2.3.2).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

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Q	Source	Max	Mark	Q	Source	Max	Mark
1	Jun 11 Q9	6		10	Jun 13 Q11	6	
2	Jun 11 Q10	8		11	Jan 14 Q9	6	
3	Jan 12 Q9	6		12	Jan 14 Q10	11	
4	Jan 12 Q10	8		13	Jun 14 Q9	5	
5	Jun 12 Q9	11		14	Jun 14 Q10	8	
6	Jun 12 Q10	5		15	Jun 16 Q9	6	
7	Jan 13 Q8	5		16	Jun 16 Q10	5	
8	Jan 13 Q9	10		17	Jun 17 Q8	6	
9	Jun 13 Q10	6		18	Jun 17 Q9	8	
				Total		126	

Functions (Inverse, Composite, Domain & Range) – what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 3: Pure Mathematics B · Topic 2.3.2.

Inverse function f^{-1} 2.3.2

- Exists only if f is one-to-one on its domain.
- To find: write $y = f(x)$ then swap x and y ; solve for y .
- Domain of f^{-1} = range of f (and vice versa).

Composite function fg 2.3.2

- $fg(x) = f(g(x))$ – apply g first, then f .
- Domain of fg : subset of domain of g such that $g(x) \in$ domain of f .
- Range of fg : range of f restricted to the values $g(x)$ takes.

Functions - Inverse, Composite, Domain & Range in one page

Quick-reference notes – revisit before each section. Don't use during questions.

Inverse function

Exists if f is one-to-one on its domain.
To find f^{-1} : write $y = f(x)$, swap x and y , solve for y .

Domain & range swap

Domain of f^{-1} = range of f .
Range of f^{-1} = domain of f .

Graph of f^{-1}

Reflection of $y = f(x)$ in the line $y = x$.

Composite function

$(fg)(x) = f(g(x))$ – apply g first, then f .
 $fg \neq gf$ in general.

Domain of fg

$x \in \text{dom}(g)$ AND $g(x) \in \text{dom}(f)$.

Range/restriction problems

Typical WJEC: find least value of k so that fg can be formed.
Set up range of g contained within domain of f and solve for k .

SECTION T17

Functions (Inverse, Composite, Domain & Range)

Questions 1-18 · 126 marks

9. The function f has domain $(-\infty, -\frac{1}{2})$ and is defined by

$$f(x) = e^{2x+1} - 3.$$

- (a) Find an expression for $f^{-1}(x)$. [4]
- (b) Write down the domain of f^{-1} . [2]

TURN OVER

10. The functions f and g have domains $(-\infty, 0)$ and $(6, \infty)$ respectively and are defined by

$$f(x) = x^2 - 19,$$

$$g(x) = 1 - \frac{1}{2}x.$$

(a) Write down the range of f and the range of g . [2]

(b) Write down the domain and range of fg . [2]

(c) (i) Write down an expression for $fg(x)$.

(ii) Hence, solve the equation

$$fg(x) = 2x - 26. \quad [4]$$

9. The function f has domain $[6, \infty)$ and is defined by

$$f(x) = 3 - \frac{1}{\sqrt{x-2}}.$$

- (a) Find an expression for $f^{-1}(x)$. [4]
- (b) Write down the domain of f^{-1} . [2]

TURN OVER

(0975-01)

10. The function f has domain $[1, \infty)$ and is defined by

$$f(x) = 3x + k,$$

where k is a constant.

(a) Write down, in terms of k , the range of f . [1]

The function g has domain $[-2, \infty)$ and is defined by

$$g(x) = x^2 - 6.$$

(b) Find the least value of k so that the function gf can be formed. [2]

(c) (i) Write down an expression, in terms of k , for $gf(x)$.

(ii) Given that $gf(2) = 3$, find the value of k . [5]

9. The function f has domain $(-\infty, 0)$ and is defined by

$$f(x) = \frac{x^2 + 3}{x^2 + 5}.$$

- (a) (i) Show that $f'(x)$ is always negative.
- (ii) Write down the range of f . [6]
- (b) (i) Find an expression for $f^{-1}(x)$.
- (ii) Write down the range and domain of f^{-1} . [5]

10. The function g has domain $(-\infty, \infty)$ and is defined by

$$g(x) = \sqrt{3x^2 + 7}.$$

Solve the equation

$$gg(x) = 8. \quad [5]$$

8. The function f has domain $[-1, \infty)$ and is defined by

$$f(x) = \ln(4x + 5) - 2.$$

- (a) Find an expression for $f^{-1}(x)$. [4]
- (b) State the domain of f^{-1} . [1]

9. (a) The functions f and g have domains $(-\infty, \infty)$ and $(0, \infty)$ respectively and are defined by

$$f(x) = x^2 - 25,$$

$$g(x) = 2x - 3.$$

- (i) Write down the domain of fg .
- (ii) Write down the range of fg .
- (iii) Write down an expression for $fg(x)$.
- (iv) Solve the equation $fg(x) = 0$. [7]

- (b) The function h is defined by

$$h(x) = \frac{2x + 7}{5x - 2}.$$

- (i) Show that $hh(x) = x$.
- (ii) **Hence** write down an expression for $h^{-1}(x)$. [3]

10. The function f has domain $(-\infty, 10]$ and is defined by

$$f(x) = e^{5-\frac{x}{2}} + 6.$$

- (a) Find an expression for $f^{-1}(x)$. [4]
- (b) Write down the domain of f^{-1} . [2]

11. The functions f and g have domains $(0, \infty)$ and $\left(0, \frac{\pi}{4}\right]$ respectively and are defined by

$$\begin{aligned}f(x) &= \ln x, \\g(x) &= \tan x.\end{aligned}$$

- (a) (i) Write down the domain of fg .
(ii) Write down the range of fg . [3]
- (b) (i) Solve the equation $fg(x) = -0.4$. Give your answer correct to two decimal places.
(ii) Giving a reason, write down a value for k so that $fg(x) = k$ has no solution. [3]

9. The function f has domain $[7, \infty)$ and is defined by

$$f(x) = 1 + \frac{2}{\sqrt{3x-5}}.$$

- (a) Find an expression for $f^{-1}(x)$. [4]
- (b) Write down the domain of f^{-1} . [2]

TURN OVER

10. The functions f and g have domains $(0, \infty)$ and $(-\infty, -2)$ respectively and are defined by

$$f(x) = \sqrt{x^2 + 5},$$

$$g(x) = \frac{-4}{x+1}.$$

- (a) By considering $g'(x)$, show that g is an increasing function. [2]
- (b) Write down the range of g . [2]
- (c) Write down the domain and range of fg . [2]
- (d) (i) Write down an expression for $fg(x)$.
- (ii) Hence, solve the equation

$$fg(x) = 3. \quad [5]$$

END OF PAPER

9. The function f has domain $(-\infty, 4)$ and is defined by

$$f(x) = x^2 - 8x + 7.$$

- (a) Express $f(x)$ in the form

$$f(x) = (x + a)^2 + b,$$

where a, b are constants whose values are to be found.

[1]

- (b) Hence or otherwise, find an expression for $f^{-1}(x)$.

[4]

TURN OVER

10. The functions f and g have domains $[-2, \infty)$ and $[2, \infty)$ respectively and are defined by

$$\begin{aligned}f(x) &= x^2 + kx - 8, \\g(x) &= kx - 4,\end{aligned}$$

where k is a positive constant.

- (a) Write down, in terms of k , the range of g . [1]
- (b) (i) Find the least value of k so that the function fg can be formed.
- (ii) Write down an expression in terms of k for $fg(x)$.
- (iii) Given that $fg(3) = 0$, find the value of k . [7]

END OF PAPER

9. The function f has domain $(-\infty, 12]$ and is defined by

$$f(x) = e^{4 - \frac{x}{3}} + 8.$$

- (a) Find an expression for $f^{-1}(x)$. [4]
- (b) Write down the domain of f^{-1} . [2]

10. The function h is defined by

$$h(x) = \frac{4x+3}{5x-4}.$$

(a) Show that $hh(x) = x$.

[3]

(b) Use the result of part (a) to write down an expression for $h^{-1}(x)$.

Hence evaluate $h^{-1}(-1)$.

[2]

END OF PAPER

8. The function f has domain $[8, \infty)$ and is defined by

$$f(x) = 2 + \frac{3}{\sqrt{5x-4}}.$$

- (a) Find an expression for $f^{-1}(x)$. [4]
- (b) Write down the domain of f^{-1} . [2]

9. The function f has domain $[2, \infty)$ and is defined by

$$f(x) = 4x + k,$$

where k is a constant.

- (a) Write down, in terms of k , the range of f . [1]

The function g has domain $[-3, \infty)$ and is defined by

$$g(x) = x^2 - 9.$$

- (b) Find the least value of k so that the function gf can be formed. [2]

- (c) (i) Write down an expression, in terms of k , for $gf(x)$.

- (ii) Given that $gf(2) = 7$, find the value of k . [5]

END OF PAPER

END OF FUNCTIONS - INVERSE, COMPOSITE, DOMAIN & RANGE PACK

Source: WJEC C3 + C4 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec A2 Unit 3 – Topic 17 (2.3.2)

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