

Name	Date started	Target end date
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GCE A LEVEL – PURE MATHEMATICS B QUESTION PACK

0976-01 (Legacy C4) · New spec Unit 3 Topic 16 · A2 unit, 35% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS – PURE B · VECTORS

Vectors (3D, Lines, Scalar Product)

Every 3D vector / line / scalar product question from the legacy WJEC C4 papers (June 2011 – June 2017) for new-spec A2 Unit 3

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 25 minutes

Derived from the legacy C3/C4 paper's pace of ~1.25 min/mark (68 marks over 7 questions).

You are advised to **not** attempt to complete all of this in one sitting.

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C3 and C4 papers (2008 modular spec) that maps onto new-spec A2 Unit 3 Topic 16 (2.3.15).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

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Q	Source	Max	Mark
1	Jun 11 Q9	9	
2	Jun 12 Q9	10	
3	Jun 13 Q9	11	
4	Jun 14 Q9	11	
5	Jun 15 Q8	9	
6	Jun 16 Q8	10	
7	Jun 17 Q9	8	
Total		68	

Vectors (3D, Lines, Scalar Product) – what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 3: Pure Mathematics B · Topic 2.3.15.

Position vectors & lines 2.3.15

- Vector equation of a line: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$ (\mathbf{a} = position vector of a point on the line, \mathbf{d} = direction).
- $\vec{AB} = \mathbf{b} - \mathbf{a}$.
- Convert to Cartesian by eliminating λ .

Scalar product 2.3.15

- $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}| \cos \theta$.
- Perpendicular: $\mathbf{a} \cdot \mathbf{b} = 0$.
- Used to find angle between vectors: $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$.

Vectors in one page

Quick-reference notes – revisit before each section. Don't use during questions.

Position vectors

$$\vec{AB} = \mathbf{b} - \mathbf{a}.$$

$$\text{Distance: } |\vec{AB}| = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2}.$$

Vector equation of a line

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}.$$

\mathbf{a} = position vector of a known point.

\mathbf{d} = direction vector.

Scalar product

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3.$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta.$$

Perpendicular check

$\mathbf{a} \cdot \mathbf{b} = 0$ iff $\mathbf{a} \perp \mathbf{b}$ (or one is zero).

Angle between vectors

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}.$$

Use $\theta = \cos^{-1}(\dots)$ to find the angle.

Intersection of lines

Two lines $\mathbf{r}_1 = \mathbf{a}_1 + \lambda \mathbf{d}_1$, $\mathbf{r}_2 = \mathbf{a}_2 + \mu \mathbf{d}_2$ intersect if you can find λ, μ satisfying all three components.

SECTION T16

Vectors (3D, Lines, Scalar Product)

Questions 1-7 · 68 marks

9. (a) Given that the vectors $5\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}$ and $4\mathbf{i} + 6\mathbf{j} + a\mathbf{k}$ are perpendicular, find the value of the constant a . [3]

(b) The line L_1 passes through the point with position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$ and is parallel to the vector $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

(i) Write down the vector equation of the line L_1 .

(ii) The line L_2 has vector equation

$$\mathbf{r} = 4\mathbf{i} + 7\mathbf{j} + 5\mathbf{k} + \mu(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}).$$

Show that L_1 and L_2 do not intersect.

[6]

9. The position vectors of the points A and B are given by

$$\mathbf{a} = 4\mathbf{i} + \mathbf{j} - 6\mathbf{k},$$
$$\mathbf{b} = 6\mathbf{i} + 2\mathbf{j} - 4\mathbf{k},$$

respectively.

- (a) Determine whether or not the vectors \mathbf{a} and \mathbf{b} are perpendicular, giving a reason for your answer. [2]
- (b) (i) Write down the vector \mathbf{AB} .
(ii) Find the vector equation of the line AB . [3]
- (c) The vector equation of the line L is given by

$$\mathbf{r} = 2\mathbf{i} + 6\mathbf{j} + p\mathbf{k} + \mu(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}),$$

where p is a constant.

Given that the lines AB and L intersect, find the value of p . [5]

9. The position vectors of the points A and B are given by

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} - 7\mathbf{k},$$

$$\mathbf{b} = 7\mathbf{i} - \mathbf{j} + 5\mathbf{k},$$

respectively.

- (a) Write down the vector \mathbf{AB} . [1]

- (b) The point C lies on the line AB and is such that $AC : CB = 3 : 1$.
Find the position vector of C . [2]

- (c) The vector equation of the line L is given by

$$\mathbf{r} = -\mathbf{i} + \mathbf{j} + 11\mathbf{k} + \lambda(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k}).$$

- (i) Find the vector equation of the line parallel to L which passes through A .
- (ii) **Verify** that B is in fact the foot of the perpendicular from A to L . [8]

9. (a) The vectors \mathbf{p} and \mathbf{q} are given by

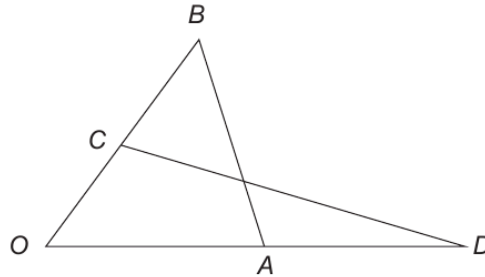
$$\mathbf{p} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$$

$$\text{and } \mathbf{q} = 5\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}.$$

Find the angle between \mathbf{p} and \mathbf{q} .

[4]

- (b) In the diagram below, the points O , A , B , C and D are such that A is the mid-point of OD and C is the mid-point of OB .



Taking O as the origin, the position vectors of A and B are denoted by \mathbf{a} and \mathbf{b} respectively.

- (i) Show that $\mathbf{CD} = 2\mathbf{a} - \frac{1}{2}\mathbf{b}$.

Hence show that the vector equation of the line CD may be expressed in the form

$$\mathbf{r} = 2\lambda\mathbf{a} + \frac{1}{2}(1 - \lambda)\mathbf{b}.$$

The vector equation of the line L may be expressed in the form

$$\mathbf{r} = \frac{1}{3}\mu\mathbf{a} + \frac{1}{3}(\mu - 1)\mathbf{b}.$$

The lines CD and L intersect at the point E .

- (ii) By giving λ and μ appropriate values, or otherwise, show that E has position vector $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$.

- (iii) Give a geometrical interpretation of the fact that E has position vector $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$. [7]

8. The position vectors of the points A and B are given by

$$\mathbf{a} = 5\mathbf{i} - \mathbf{j} - \mathbf{k},$$
$$\mathbf{b} = 4\mathbf{i} - 3\mathbf{j} + 6\mathbf{k},$$

respectively.

- (a) (i) Write down the vector \mathbf{AB} .
- (ii) Find the vector equation of the line AB . [3]

- (b) The vector equation of the line L is given by

$$\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k}).$$

Show that the lines AB and L intersect and find the position vector of the point of intersection. [6]

TURN OVER

8. The position vectors of the points A and B are given by

$$\begin{aligned}\mathbf{a} &= \mathbf{i} + 3\mathbf{j} - 3\mathbf{k}, \\ \mathbf{b} &= 3\mathbf{i} + 4\mathbf{j} - \mathbf{k},\end{aligned}$$

respectively.

- (a) (i) Write down the vector \mathbf{AB} .
(ii) Find the vector equation of the line AB . [3]

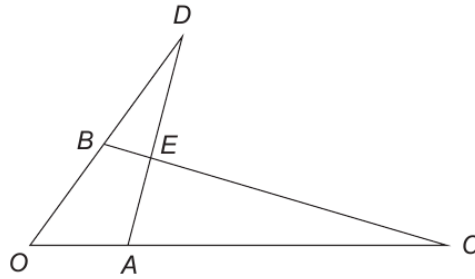
- (b) The vector equation of the line L is given by

$$\mathbf{r} = -\mathbf{i} + 8\mathbf{j} + p\mathbf{k} + \mu(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}),$$

where p is a constant.

- (i) Given that the lines AB and L intersect, find the value of p .
(ii) Determine whether or not the line L is perpendicular to the vector $6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$, giving a reason for your answer. [7]

9. In the diagram below, the points O , A , B , C and D are as follows. A lies on OC and $OC = 5OA$. B lies on OD and $OD = 2OB$.



Taking O as origin, the position vectors of A and B are denoted by \mathbf{a} and \mathbf{b} respectively.

- (a) Write down the vector \mathbf{AD} in terms of \mathbf{a} and \mathbf{b} .
Hence show that the vector equation of the line AD may be expressed in the form

$$\mathbf{r} = (1 - \lambda)\mathbf{a} + 2\lambda\mathbf{b}. \quad [3]$$

- (b) Find a similar expression for the vector equation of the line BC . [2]

- (c) The lines AD and BC intersect at the point E . Find the position vector of E in terms of \mathbf{a} and \mathbf{b} . [3]

END OF VECTORS PACK

Source: WJEC C3 + C4 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec A2 Unit 3 – Topic 16 (2.3.15)

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