Name	Date started	Target end date

GCE A LEVEL — PURE MATHEMATICS B QUESTION PACK

0976-01 (Legacy C4) · New spec Unit 3 Topic 15 · A2 unit, 35% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS — PURE B · DIFFERENTIAL EQUATIONS

Differential Equations (Separation of Variables)

Every first-order differential equation / rate question from the legacy WJEC C4 papers (June 2011 – June 2017) for new-spec A2 Unit 3

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 16 minutes

*Derived from the legacy C3/C4 paper's pace of **~1.25 min/mark** (61 marks over 7 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C3 and C4 papers (2008 modular spec) that maps onto new-spec A2 Unit 3 Topic 15 (2.3.14).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q8	11	
2	Jun 12 Q8	9	
3	Jun 13 Q8	7	
4	Jun 14 Q8	10	
5	Jun 15 Q9	8	
6	Jun 16 Q7	9	
7	Jun 17 Q8	7	
	Total	61	

Differential Equations (Separation of Variables) — what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 3: Pure Mathematics B · Topic 2.3.14.

Separation of variables 2.3.14

- For $\frac{dy}{dx} = g(x)h(y)$, separate into $\frac{1}{h(y)}\,dy = g(x)\,dx$ and integrate.
- Add a single constant of integration after one integral.
- Use boundary conditions to find the constant.

Rate problems 2.3.14

- “Rate proportional to X” means $\frac{dX}{dt} = kX$ (positive = growth, negative = decay).
- “Rate proportional to X^n” means $\frac{dX}{dt} = kX^n$.
- After solving, find k from given data.

Differential Equations in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Separable equations

$\frac{dy}{dx} = g(x)h(y) \Rightarrow \frac{1}{h(y)}\,dy = g(x)\,dx$.

Integrate both sides.

Single constant $+c$.

Rate problems

"Rate $\propto X$": $\frac{dX}{dt} = kX$.

Sign: $k > 0$ for growth, $k < 0$ for decay.

Solution: $X = X_0 e^{kt}$.

$\frac{dV}{dt} \propto V^n$

$\frac{dV}{dt} = kV^n$.

Separable: $\int V^{-n}\,dV = \int k\,dt$.

Using boundary conditions

After integrating, use the given initial value to find c.

Then use a second condition to find k.

Newton's law of cooling

$\frac{d\theta}{dt} = -k(\theta - \theta_s)$.

Rearranges to $\theta - \theta_s = Ce^{-kt}$.

Check at boundary

Always verify your solution by substituting $t = 0$ and any given times.

SECTION T15

Differential Equations (Separation of Variables)

Questions 1–7 · 61 marks

8. The size N of the population of a small island may be modelled as a continuous variable. At time t, the rate of increase of N is directly proportional to the value of N.

(a) Write down the differential equation that is satisfied by N. [1]

(b) Show that $N = Ae^{kt}$, where A and k are constants. [3]

(c) Given that $N = 100$ when $t = 2$ and that $N = 160$ when $t = 12$,

(i) show that $k = 0{\cdot}047$, correct to three decimal places,

(ii) find the size of the population when $t = 20$. [7]

8. Water is leaking from a hole at the bottom of a large tank. The volume of the water in the tank at time t hours is $V\,\text{m}^3$. The rate of decrease of V is directly proportional to V^3.

(a) Write down a differential equation satisfied by V. [1]

(b) Given that $V = 60$ when $t = 0$, show that

$$V^2 = \frac{3600}{at + 1},$$

where a is a constant. [4]

(c) When $t = 2$, the volume of the water in the tank is $50\,\text{m}^3$. Find the value of t when the volume of the water in the tank is $27\,\text{m}^3$. Give your answer correct to one decimal place. [4]

TURN OVER

(0976-01)

8. Part of the surface of a small lake is covered by green algae. The area of the lake covered by the algae at time t years is $A\,\text{m}^2$. The rate of increase of A is directly proportional to $\sqrt{A}$.

(a) Write down a differential equation satisfied by A. [1]

(b) The area of the lake covered by the algae at time $t = 3$ is $64\,\text{m}^2$ and the area covered at time $t = 5{\cdot}5$ is $196\,\text{m}^2$. Find an expression for A in terms of t. [6]

TURN OVER

8. The value £V of a long term investment may be modelled as a continuous variable. At time t years, the rate of increase of V is directly proportional to the value of V.

(a) Write down a differential equation satisfied by V. [1]

(b) Show that $V = Ae^{kt}$, where A and k are constants. [3]

(c) The value of the investment after 2 years is £292 and its value after 28 years is £637.

(i) Show that $k = 0{\cdot}03$, correct to two decimal places.

(ii) Find the value of A correct to the nearest integer.

(iii) Find the initial value of the investment. Give your answer correct to the nearest pound. [6]

TURN OVER

 (0976-01)

9. A bookseller values a rare book at £A on August 1st 2010. The value, £P, of the book t years after this date may be modelled as a continuous variable. The rate of increase of P may be assumed to be directly proportional to P^2.

(a) Write down a differential equation satisfied by P. [1]

(b) Show that

$$\frac{1}{k}\left(\frac{P-A}{PA}\right) = t,$$

where k is a constant. [4]

(c) The value of the book is £800 on August 1st 2013 and £900 on August 1st 2014. Find the value of A. [3]

7. The value, £V, of a particular car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^3.

(a) Write down a differential equation satisfied by V. [1]

(b) Given that the initial value of the car is £A, show that

$$V^2 = \frac{A^2}{bt+1},$$

where b is a constant. [4]

(c) When $t = 2$, the value of the car has fallen to a half of its initial value. Find the value of t when the value of the car will have fallen to a quarter of its initial value. [4]

TURN OVER

8. The size N of the population of a small island may be modelled as a continuous variable. At time t years, the rate of increase of N is assumed to be directly proportional to the value of $\sqrt{N}$.

(a) Write down a differential equation satisfied by N. [1]

(b) When t = 5, the size of the population was 256. When t = 7, the size of the population was 400. Find an expression for N in terms of t. [6]

TURN OVER

END OF DIFFERENTIAL EQUATIONS PACK

Source: WJEC C3 + C4 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec A2 Unit 3 — Topic 15 (2.3.14)