

Name	Date started	Target end date
------	--------------	-----------------

## GCE A LEVEL – PURE MATHEMATICS B QUESTION PACK

0975-01 (Legacy C3) + 0976-01 (Legacy C4) · New spec Unit 3 Topic 12 · A2 unit, 35% of A-level, 120 marks, 2h 30min paper

**REVISE**  
.wales

# MATHEMATICS – PURE B · INTEGRATION - SUBSTITUTION & PARTS

## *Integration (Standard Integrals, Substitution & Parts)*

*Every integration technique question (standard, by substitution, by parts) from the legacy WJEC C3 + C4 papers (June 2011 – June 2017) for new-spec A2 Unit 3*

LEGACY 2008 SPECIFICATION

### Estimated time for entire question pack: ~3 hours 26 minutes

*Derived from the legacy C3/C4 paper's pace of ~1.25 min/mark (165 marks over 17 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

### ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C3 and C4 papers (2008 modular spec) that maps onto new-spec A2 Unit 3 Topic 12 (2.3.11).

Questions are ordered chronologically.

### INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

*A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.*

*All question content is © WJEC CBAC Ltd. and reproduced for revision purposes.*

*For Examiner's use only*

Q	Source	Max	Mark	Q	Source	Max	Mark
1	Jun 11 Q6	10		10	Jun 17 Q6	12	
2	Jan 12 Q6	10		11	Jun 11 Q7	8	
3	Jun 12 Q6	11		12	Jun 12 Q7	8	
4	Jan 13 Q6	10		13	Jun 13 Q7	9	
5	Jun 13 Q6	10		14	Jun 14 Q7	8	
6	Jan 14 Q7	11		15	Jun 15 Q7	11	
7	Jun 14 Q7	10		16	Jun 16 Q6	8	
8	Jun 15 Q7	9		17	Jun 17 Q7	9	
9	Jun 16 Q7	11		<b>Total</b>		<b>165</b>	

# Integration (Standard Integrals, Substitution & Parts) – what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 3: Pure Mathematics B · Topic 2.3.11.

## Standard integrals 2.3.11

- $\int e^{ax+b} dx = \frac{1}{a}e^{ax+b} + c.$
- $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax + b| + c.$
- $\int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + c.$

## By substitution 2.3.11

- Let  $u = f(x)$ , then  $du = f'(x) dx.$
- Swap variable, including the limits if definite.
- Useful when the integrand contains  $f(x)$  and a factor of  $f'(x).$

## By parts 2.3.11

- $\int u dv = uv - \int v du.$
- Choose  $u$  so that  $\frac{du}{dx}$  is simpler (LIATE: log/inv/alg/trig/exp).
- Sometimes repeat parts (e.g.  $\int x^2 e^x dx$ ).

# Integration - Substitution & Parts in one page

Quick-reference notes – revisit before each section. Don't use during questions.

## Standard integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1).$$

$$\int \frac{1}{x} dx = \ln|x| + c.$$

$$\int e^{ax+b} dx = \frac{1}{a}e^{ax+b} + c.$$

## Trig integrals

$$\int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + c.$$

$$\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + c.$$

## By substitution

$$\text{Let } u = f(x), du = f'(x) dx.$$

Change variable AND limits (if definite).

$$\text{Look for } \int g(f(x)) \cdot f'(x) dx.$$

## By parts

$$\int u dv = uv - \int v du.$$

LIATE rule for choosing  $u$ : log, inverse trig, algebraic, trig, exponential.

## Repeated parts

For  $\int x^2 e^x dx$ : apply parts twice.

For  $\int e^x \sin x dx$ : apply parts twice, solve for the original integral.

## Definite integral check

Don't forget to apply limits and not add  $+c$ .

Carefully evaluate at upper and subtract lower.

# SECTION T12

*Integration (Standard Integrals, Substitution & Parts)*

Questions 1-17 · 165 marks

6. (a) Find

(i)  $\int \frac{9}{4x+3} dx$ ,      (ii)  $\int 3e^{5-2x} dx$ ,      (iii)  $\int \frac{5}{(7x-1)^3} dx$ .      [6]

(b) Evaluate  $\int_0^{\frac{\pi}{3}} \cos\left(3x - \frac{\pi}{6}\right) dx$ .      [4]

6. (a) Find each of the following, simplifying your answer wherever possible.

(i)  $\int \sin\left(\frac{x}{4}\right) dx$ ,      (ii)  $\int e^{\frac{2x}{3}} dx$ ,      (iii)  $\int \frac{7}{8x-2} dx$ .      [6]

(b) Evaluate  $\int_1^9 \frac{3}{\sqrt{5x+4}} dx$ .      [4]

6. Find

(a) (i)  $\int 3e^{2-\frac{x}{4}} dx$       (ii)  $\int \frac{9}{(2x-3)^6} dx$       (iii)  $\int \frac{7}{3x+1} dx$       [6]

(b) Given that  $0 < a < \frac{\pi}{2}$  and that

$$\int_0^a \sin 2x dx = \frac{1}{4},$$

find the value of the constant  $a$ .      [5]

6. (a) Find

(i)  $\int \cos\left(\frac{4x+5}{3}\right) dx$ , (ii)  $\int e^{2x+9} dx$ , (iii)  $\int \frac{3}{(7-2x)^6} dx$ . [6]

(b) Express  $\int_2^{44} \frac{1}{3x-4} dx$

in the form  $\ln k$ , where  $k$  is an integer whose value is to be found. [4]

6. (a) Find

(i)  $\int \cos\left(3x + \frac{\pi}{2}\right) dx,$       (ii)  $\int e^{3-4x} dx,$

(iii)  $\int \frac{7}{8x+5} dx.$  [6]

(b) Evaluate  $\int_1^2 \frac{9}{(2x-1)^4} dx.$  [4]

7. (a) Find each of the following, simplifying your answer wherever possible.

(i)  $\int e^{\frac{5x}{6}} dx$ ,      (ii)  $\int \sqrt[3]{8x+1} dx$ ,      (iii)  $\int \sin\left(1 - \frac{x}{3}\right) dx$ .      [6]

- (b) Given that  $a > 2$ , and that

$$\int_2^a \frac{1}{4x-1} dx = 0.284,$$

find the value of the constant  $a$ . Give your answer correct to one decimal place.      [5]

7. (a) Find each of the following, simplifying your answer wherever possible.

(i)  $\int \cos(2 - 5x) dx,$       (ii)  $\int \frac{4}{e^{3x-2}} dx,$       (iii)  $\int \frac{5}{\frac{1}{6}x - 3} dx.$  [6]

(b) Evaluate  $\int_2^6 \sqrt{4x+1} dx.$  [4]

7. (a) Find each of the following integrals, simplifying your answer wherever possible.

(i)  $\int \frac{(7x^2 - 2)}{x} dx$

(ii)  $\int \sin\left(\frac{2x}{3} - \pi\right) dx$  [5]

(b) Evaluate  $\int_3^6 \frac{1}{\sqrt[4]{(5x-14)}} dx$ . [4]

7. (a) Find each of the following, simplifying your answer wherever possible.

(i)  $\int 7e^{5-\frac{3}{4}x} dx$       (ii)  $\int \sin\left(\frac{2x}{3} + 5\right) dx$       (iii)  $\int \frac{8}{(9-10x)^3} dx$       [6]

- (b) Given that  $a > 0$  and that

$$\int_a^6 \frac{1}{4x+3} dx = 0.1986,$$

find the value of the constant  $a$ . Give your answer correct to one decimal place.      [5]

**TURN OVER**

6. (a) Find each of the following integrals, simplifying your answer wherever possible.

(i)  $\int 8e^{2-5x} dx$

(ii)  $\int \frac{6}{\sqrt[3]{4x-7}} dx$

(iii)  $\int \cos\left(\frac{7x-9}{3}\right) dx$

[6]

- (b) (i) Differentiate  $\ln(3x^2 - 8)$  with respect to  $x$ .

- (ii) Use your answer to (b)(i) to evaluate

$$\int_2^6 \frac{3x}{3x^2 - 8} dx.$$

Give your answer in the form  $\ln k$ , where  $k$  is an integer whose value is to be found. [6]

# TURN OVER

7. (a) Find  $\int x \sin 2x \, dx$ . [4]

(b) Use the substitution  $u = 5 - x^2$  to evaluate

$$\int_0^2 \frac{x}{(5-x^2)^3} \, dx. \quad [4]$$

**TURN OVER**

7. (a) Find  $\int xe^{-2x} dx$ . [4]

(b) Use the substitution  $u = 1 + 3\ln x$  to evaluate

$$\int_1^e \frac{1}{x(1+3\ln x)} dx.$$

Give your answer correct to four decimal places. [4]

7. (a) Find  $\int (3x - 1)\cos 2x \, dx$ . [4]

(b) Use the substitution  $u = 2x + 1$  to evaluate

$$\int_0^1 \frac{x}{(2x + 1)^3} \, dx. \quad [5]$$

7. (a) Find  $\int x^4 \ln 2x \, dx$ . [4]

(b) Use the substitution  $u = 10 \cos x - 1$  to evaluate

$$\int_0^{\frac{\pi}{3}} \sqrt{(10 \cos x - 1)} \sin x \, dx. \quad [4]$$

7. (a) Use the substitution  $u = 12 - x^3$  to evaluate

$$\int_0^2 \frac{x^2}{(12 - x^3)^2} dx. \quad [4]$$

- (b) (i) Find  $\int x \cos 2x dx$ .

- (ii) Use the result of (b)(i) to find

$$\int x \sin^2 x dx. \quad [7]$$

6. (a) Find  $\int (2x+1)e^{-3x} dx$ . [4]

(b) Use the substitution  $u = 4 + 5 \tan x$  to evaluate

$$\int_0^{\frac{\pi}{4}} \frac{\sqrt{4+5\tan x}}{\cos^2 x} dx. \quad [4]$$

7. (a) Find  $\int \frac{\ln x}{x^4} dx$ . [4]

(b) Use the substitution  $u = x^2 + 1$  to evaluate

$$\int_0^1 x^3(x^2 + 1)^4 dx. \quad [5]$$

## **END OF INTEGRATION - SUBSTITUTION & PARTS PACK**

Source: WJEC C3 + C4 (2008 modular spec) · 2011–2017  
Curated for WJEC Maths 2017 spec A2 Unit 3 – Topic 12 (2.3.11)

© WJEC CBAC Ltd. Pack layout © revise.wales for revision purposes only.