Name	Date started	Target end date

GCE A LEVEL — PURE MATHEMATICS B QUESTION PACK

0975-01 (Legacy C3) + 0976-01 (Legacy C4) · New spec Unit 3 Topic 11 · A2 unit, 35% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS — PURE B · DIFFERENTIATION - IMPLICIT & PARAMETRIC

Differentiation (Implicit & Parametric)

Every implicit and parametric differentiation question from the legacy WJEC C3 + C4 papers (June 2011 – June 2017) for new-spec A2 Unit 3 applications

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~3 hours 38 minutes

*Derived from the legacy C3/C4 paper's pace of **~1.25 min/mark** (174 marks over 23 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C3 and C4 papers (2008 modular spec) that maps onto new-spec A2 Unit 3 Topic 11 (2.3.9).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q3	11	
2	Jan 12 Q3	12	
3	Jan 12 Q4	4	
4	Jun 12 Q3	11	
5	Jan 13 Q3	13	
6	Jun 13 Q3	7	
7	Jun 13 Q4	8	
8	Jan 14 Q3	4	
9	Jan 14 Q4	10	
10	Jun 14 Q3	8	
11	Jun 14 Q4	7	
12	Jun 15 Q3	7	

Q	Source	Max	Mark
13	Jun 15 Q4	9	
14	Jun 16 Q3	4	
15	Jun 16 Q4	8	
16	Jun 17 Q3	12	
17	Jun 11 Q2	5	
18	Jun 12 Q2	4	
19	Jun 13 Q2	5	
20	Jun 14 Q1	5	
21	Jun 15 Q2	7	
22	Jun 16 Q3	7	
23	Jun 17 Q2	6	
Total		174	

Differentiation (Implicit & Parametric) — what the new spec asks

WJEC GCE A Level Mathematics (from 2017) · Unit 3: Pure Mathematics B · Topic 2.3.9.

Implicit differentiation 2.3.9

- Differentiate both sides with respect to x; treat y as a function of x.
- $\frac{d}{dx}(y^n) = ny^{n-1} \cdot \frac{dy}{dx}$.
- Solve for $\frac{dy}{dx}$ at the end.

Parametric differentiation 2.3.9

- $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.
- Second derivative: $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d/dt(dy/dx)}{dx/dt}$.
- Use to find tangent lines / stationary points on parametric curves.

Differentiation - Implicit & Parametric in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Implicit differentiation

Differentiate both sides w.r.t. x.

Use chain rule on terms containing y: $\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$.

Collect $\frac{dy}{dx}$ terms and solve.

Product terms involving y

$\frac{d}{dx}(xy) = y + x\frac{dy}{dx}$ (product rule). $\frac{d}{dx}(x^2y) = 2xy + x^2\frac{dy}{dx}$.

Tangent/normal at (a, b)

Differentiate implicitly to get $\frac{dy}{dx}$ as a function of x and y.

Substitute (a, b) to get gradient.

Use point-gradient form for tangent or perpendicular gradient for normal.

Parametric setup

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ (chain rule). $\frac{d^2y}{dx^2} = \frac{d/dt(dy/dx)}{dx/dt}$.

Vertical/horizontal tangents

Horizontal: $dy/dx = 0$ (numerator zero).

Vertical: dy/dx undefined (denominator zero).

Watch the algebra

Be tidy when collecting dy/dx terms.

Factor out cleanly before dividing.

SECTION T11

Differentiation (Implicit & Parametric)

Questions 1–23 · 174 marks

3. *(a)* Given that

$$2x^3 + x^2 \cos y + y^4 + 2x - 25 = 0,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that

$$x = t^3, \quad y = 2t^2 + 5t^4,$$

(i) find and simplify an expression for $\frac{dy}{dx}$ in terms of t,

(ii) show that there is no real value of t for which $\frac{dy}{dx} = 5$. [7]

3. *(a)* A function is defined parametrically by

$$x = 3t^2,\ y = t^6 - 4t^3.$$

(i) Find $\dfrac{dy}{dx}$ in terms of t.

(ii) Given that $\dfrac{dy}{dx} = \dfrac{7}{2}$, show that $2t^4 - 4t - 7 = 0$. [5]

(b) Show that the equation

$$2t^4 - 4t - 7 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$t_{n+1} = \left(\frac{4t_n + 7}{2}\right)^{\frac{1}{4}}$$

with $t_0 = 1{\cdot}6$ can be used to find α. Find and record the values of t_1, t_2, t_3, t_4. Write down the value of t_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

4. Given that $x^2y^2 + x^4 + 6 = 2y^3 + 2x$, find the value of $\frac{dy}{dx}$ at the point (2, 3). [4]

3. *(a)* The curve C is defined by

$$x^3 - 4x^2y = 2y^3 - 3x - 2.$$

Find the value of $\frac{dy}{dx}$ at the point (3, 1). [4]

(b) Given that

$$x = \sin at,\ y = \cos at,$$

where a is a constant, find and simplify

(i) an expression for $\frac{dy}{dx}$ in terms of a and t,

(ii) an expression for $\frac{d^2y}{dx^2}$ in terms of a and t. [7]

3. *(a)* Given that

$$x^3 + 5x^4y - 2y^3 + 7 = 0,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = t^3 - 5$, $y = t^4 + 7t^5$,

(i) find an expression for $\frac{dy}{dx}$ in terms of t,

(ii) find an expression for $\frac{d^2y}{dx^2}$ in terms of t,

(iii) find the value of $\frac{d^2y}{dx^2}$ when $x = 3$. [9]

3. The curve C is defined by

$$x^3y^2 = 128.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y. [3]

The point P lies on C and has coordinates (a, b).

(b) Given that the value of $\frac{dy}{dx}$ at the point P is 3,

(i) show that $b = -2a$,

(ii) find the value of a and the value of b. [4]

4. Given that, for $t > 0$,

$$x = \ln t,\ y = 5t^4,$$

(a) find and simplify an expression for $\dfrac{dy}{dx}$ in terms of t, [4]

(b) find the value of t for which $\dfrac{d^2y}{dx^2} = 0{\cdot}648$. [4]

3. The curve C is defined by

$$x^3 - 2x^2y + 3y^2 = 3.$$

Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point $(-2, -1)$. [4]

4. The variables x and y are defined parametrically in terms of the variable t. It is known that $x = 2t^3$ and that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2t + 4t^3$.

(a) Find an expression for $\frac{\mathrm{d}x}{\mathrm{d}t}$ in terms of t. [1]

(b) Find an expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of t and hence show there is no value of t for which $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2$. [4]

(c) Given that $y = 10$ when $t = 1$, find an expression for y in terms of t. [5]

3. The curve C is defined by

$$y^4 - 2x^2 + 8xy^2 + 9 = 0.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{x - 2y^2}{y^3 + 4xy}$. [4]

(b) Show that there is no point on C at which $\dfrac{dy}{dx} = 0$. [4]

4. Given that $x = 2e^{t} - 5$, $y = 8e^{-t} + 3e^{t} - 4$, find the value of t when $\frac{dy}{dx} = -1$.

Give your answer correct to three decimal places. [7]

3. *(a)* The curve C_1 is defined by

$$x^3 + 2x\cos y + y^2 = 1 + \frac{\pi}{4}^2.$$

Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point $(1, \frac{\pi}{2})$. [4]

(b) The curve C_2 is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 y.$$

Find an expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of x and y. Simplify your answer. [3]

4. Given that $x = \tan^{-1} t$, $y = \ln t$, where $t > 0$,

(a) find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, [4]

(b) find the value of x for which $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$. [5]

3. The curve C is defined by

$$x^2 + 3xy + 2y^3 - 2x = 21.$$

The point P has coordinates $(-5, 2)$ and lies on C.

Find the value of $\frac{dy}{dx}$ at P. [4]

4. A function is defined parametrically by

$$x = 4\sin 3t,\ y = 2\cos 3t.$$

(a) Find and simplify an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [4]

(b) Find and simplify an expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(i) in terms of t,

(ii) in terms of y. [4]

3. *(a)* Given that

$$x^4 - 3x^2y + 2y^3 - 4x = 7,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = 7t + 2t^2$, $y = \frac{4 + 3t}{7 + 4t}$,

(i) show that $\frac{dy}{dx} = \frac{k}{(7 + 4t)^n}$,

where the values of the constants k and n are to be found,

(ii) find a similar expression for $\frac{d^2y}{dx^2}$. [8]

2. Find the equation of the normal to the curve

$$x^4 - 2x^2y + y^2 = 4$$

at the point (1, 3). [5]

2. Find the equation of the tangent to the curve

$$y^3 - 4x^2 - 3xy + 25 = 0$$

at the point $(2, -3)$. [4]

2. Find the equation of the normal to the curve

$$x^3 - 2xy^2 + y^3 = 5$$

at the point (2, 1). [5]

1. The curve *C* is defined by

$$3x^3 - 5xy^2 + 2y^4 = 15.$$

The point *P* has coordinates (1, 2) and lies on *C*.
Find the equation of the **normal** to *C* at *P*. [5]

2. The curve C has equation

$$x^4 + 3x^2y - 2y^2 = 34.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{4x^3 + 6xy}{4y - 3x^2}$. [3]

(b) Find the coordinates of each of the points on C where the tangent is parallel to the y-axis. [4]

3. The curve C has equation

$$x^4 + 2x^3y - 3y^4 = 16.$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x^3 + 3x^2y}{6y^3 - x^3}$. [3]

(b) Show that there are only two points on C where the gradient of the tangent is -2. Find the coordinates of each of these two points. [4]

2. The curve *C* has equation

$$y^6 - 3x^4 - 9x^2y + 48 = 0.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{6xy + 4x^3}{2y^5 - 3x^2}$. [3]

(b) Find the gradient of the tangent to *C* at each of the points where *C* crosses the x-axis. [3]

END OF DIFFERENTIATION - IMPLICIT & PARAMETRIC PACK

Source: WJEC C3 + C4 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec A2 Unit 3 — Topic 11 (2.3.9)