Name	Date started	Target end date

GCE AS / A LEVEL — APPLIED MATHEMATICS A QUESTION PACK

0984-01 (Legacy S2) · New spec Unit 2 Topic 4 · AS unit, 25% of A-level, 75 marks, 1h 45min paper

REVISE
.wales

MATHEMATICS — APPLIED A · HYPOTHESIS TESTING - BINOMIAL PROPORTION

Hypothesis Testing - Binomial Parameter p

Hypothesis tests on a Binomial parameter p from the legacy WJEC S2 papers (2011-2017). Small-sample exact tests, critical regions and normal-approximation extensions

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 40 minutes

*Derived from the legacy S2 paper's pace of **~1.25 min/mark** (80 marks over 6 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC S2 papers (2008 modular spec) that map onto new-spec AS Unit 2 Topic 4 (2.2.5).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet and statistical tables may be referred to. Take $g = 9.8\,\text{ms}^{-2}$ for mechanics.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q6	16	
2	Jun 13 Q4	12	
3	Jun 14 Q4	14	
4	Jun 15 Q4	10	
5	Jun 16 Q5	14	
6	Jun 17 Q5	14	
Total		**80**	

Hypothesis Testing - Binomial Parameter p — what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 2: Applied Mathematics A · Topic 2.2.5.

Setting up the test 2.2.5

- Let X = number of successes in n trials; under H_0, $X \sim B(n, p_0)$.
- One-tailed: $H_1 : p < p_0$ or $H_1 : p > p_0$.
- Two-tailed: $H_1 : p \neq p_0$.

Critical region and p-value 2.2.5

- p-value (one-tailed): $P(X \leq x_{\text{obs}})$ or $P(X \geq x_{\text{obs}})$ under H_0.
- Two-tailed: double the one-tailed p-value (or use symmetry / nearest tail).
- Reject H_0 if p-value $< \alpha$; conclude in context, not just statistical jargon.

Crib sheet »

Hypothesis Testing - Binomial Proportion in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Hypotheses

$H_0 : p = p_0$ vs $H_1 : p \neq p_0$ (two-tailed) or $H_1 : p < p_0$ / $p > p_0$ (one-tailed).

State in context of the question.

Test statistic

Under H_0, $X \sim B(n, p_0)$. Compute the tail probability of the observed x.

$X \leq x$ for lower-tail evidence; $X \geq x$ for upper-tail.

p-value

One-tailed: directly $P(X \leq x)$ or $P(X \geq x)$ from tables.

Two-tailed: double the nearer tail probability (or sum both tails $\leq$ symmetric).

Critical region

Choose C so that $P(X \in C \mid H_0) \leq \alpha$.

For small n, α may not be attainable exactly - get closest below.

Normal approximation

If n large, $np > 5$, $n(1-p) > 5$: $X \approx N(np, np(1-p))$.

Use continuity correction: $P(X \leq k) \approx \Phi\left(\dfrac{k + 0.5 - np}{\sqrt{np(1-p)}}\right)$.

Conclusion

If p-value $< \alpha$: reject H_0.

Always state the conclusion in context of the original claim.

SECTION T4

Hypothesis Testing - Binomial Parameter p

Questions 1–6 · 80 marks

6. David is given a biased coin and is told that the probability of obtaining a head when the coin is tossed is either 0·3 or 0·6. To determine which, he defines the following hypotheses.

$$H_0 : p = 0{\cdot}3; \quad H_1 : p = 0{\cdot}6.$$

(a) He tosses the coin 20 times and denotes the number of heads obtained by x.
He will accept H_1 if $x \geqslant 9$ and he will accept H_0 if $x \leqslant 8$.
Calculate the probability of

(i) accepting H_1 when H_0 is true,

(ii) accepting H_0 when H_1 is true. [8]

(b) He now tosses the coin 80 times and denotes the number of heads obtained by y.
He will accept H_1 if $y \geqslant 36$ and he will accept H_0 if $y \leqslant 35$.
Using a normal approximation, calculate the probability of

(i) accepting H_1 when H_0 is true,

(ii) accepting H_0 when H_1 is true. [8]

4. A politician believes that 40% of the voters support him but his agent believes that his support is greater than this. They decide to question a random sample of voters to try to resolve the situation.

(a) State suitable hypotheses. [1]

(b) They question a random sample of 50 voters, of whom 25 state that they support the politician. Determine the p-value and state your conclusion in context at the 5% significance level. [4]

(c) They now decide to question a random sample of 400 voters, of whom 181 state that they support the politician. Determine an approximate p-value and state your conclusion in context. [7]

4. Gwilym buys a new computer game. He claims that he wins, on average, 60% of games played. His friend Huw believes that Gwilym wins less than 60% of games played.

(a) To investigate these conflicting claims, Gwilym plays the game 20 times and wins 7 of them.

(i) State suitable hypotheses for testing these claims.

(ii) Determine the *p*-value of the above result and state your conclusion in context. [7]

(b) During the following week, Gwilym plays the game 80 times and wins 37 of them. Use a suitable approximation to determine the *p*-value and state your conclusion in context. [7]

4. When Edwin shoots an arrow at a target, he hits it with probability $p = 0{\cdot}4$.
Successive shots are independent. He attends a course to try to improve his technique and he wishes to test whether or not the course has succeeded in doing that. He defines the following hypotheses.

$$H_0 : p = 0{\cdot}4; \qquad H_1 : p > 0{\cdot}4$$

(a) He decides initially to shoot 20 arrows at the target and X denotes the number of these shots which hit the target. Determine the critical region having a significance level closest to 1%. [3]

(b) He decides to carry out a further test in which he shoots 120 arrows at the target. He finds that 55 of these shots hit the target. Calculate an approximate p-value and state your conclusion in context. [7]

5. A seed manufacturer claims that 70% of seeds of a certain variety will germinate but the manager of a garden centre claims that the germination rate is less than this.

(a) A trial is therefore conducted in which 50 seeds of this variety are planted. It is found that 32 of these seeds germinate.

(i) State suitable hypotheses to test these claims.

(ii) Calculate the *p*-value of this result and state your conclusion in context. [7]

(b) A further trial is conducted in which 500 seeds of this variety are planted and it is found that 329 of these seeds germinate. Carry out a hypothesis test using a normal approximation and state your conclusion in context. [7]

5. Charlie is given a coin and he is told that it is biased so that the probability, p, of obtaining a head when tossed is 0·75. To test this, he defines the following hypotheses.

$$H_0: p = 0{\cdot}75; \quad H_1: p \neq 0{\cdot}75$$

(a) He decides to toss the coin 50 times and he denotes the number of heads obtained by x. He defines the following critical region.

$$(x \leqslant 31) \cup (x \geqslant 44)$$

(i) Determine the significance level of this test.

(ii) Find the probability of accepting H_0 if the value of p is actually 0·5. [7]

(b) In a further attempt to test whether or not the value of p is 0·75, he decides to toss the coin 200 times. He obtains 139 heads.

(i) Calculate the approximate p-value of this result.

(ii) Interpret the p-value. [7]

END OF HYPOTHESIS TESTING - BINOMIAL PROPORTION PACK

Source: WJEC S2 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec AS Unit 2 — Topic 4 (2.2.5)