Name	Date started	Target end date

GCE AS / A LEVEL — APPLIED MATHEMATICS A QUESTION PACK

0983-01 (Legacy S1) · New spec Unit 2 Topic 3 · AS unit, 25% of A-level, 75 marks, 1h 45min paper

REVISE
.wales

MATHEMATICS — APPLIED A · BINOMIAL DISTRIBUTION

Binomial Distribution

Every Binomial distribution question from the legacy WJEC S1 papers (2011-2017). Standard tabulated probabilities, modelling and B(n, p) calculations

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 52 minutes

*Derived from the legacy S1 paper's pace of **~1.25 min/mark** (138 marks over 16 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC S1 papers (2008 modular spec) that map onto new-spec AS Unit 2 Topic 3 (2.2.4).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet and statistical tables may be referred to. Take $g = 9.8\,\mathrm{ms}^{-2}$ for mechanics.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q7	10	
2	Jan 12 Q3	8	
3	Jan 12 Q8	7	
4	Jun 12 Q4	8	
5	Jan 13 Q2	9	
6	Jan 13 Q5	9	
7	Jun 13 Q3	6	
8	Jun 13 Q4	12	

Q	Source	Max	Mark
9	Jan 14 Q4	8	
10	Jan 14 Q6	11	
11	Jun 14 Q2	5	
12	Jun 14 Q5	11	
13	Jun 15 Q1	8	
14	Jun 15 Q6	9	
15	Jun 16 Q6	7	
16	Jun 17 Q5	10	
Total		138	

Binomial Distribution — what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 2: Applied Mathematics A · Topic 2.2.4.

Binomial model 2.2.4

- $X \sim B(n, p)$ counts successes in n independent trials with success probability p.
- $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ for $k = 0, 1, \ldots, n$.
- Mean $E(X) = np$; variance $\mathrm{Var}(X) = np(1-p)$ (not assessed in U2 but used implicitly).

Calculating probabilities 2.2.4

- Use binomial tables (or calculator) for cumulative $P(X \leq k)$.
- $P(X \geq k) = 1 - P(X \leq k-1)$.
- Modelling: trials must be independent with constant p - check assumptions in context.

Binomial Distribution in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Setup

$X \sim B(n, p)$: n independent trials, each success with probability p.

Trials independent, p constant - check before modelling.

pmf

$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ for $k = 0, 1, \ldots, n$.

$\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$.

Tables

WJEC stats tables give cumulative $P(X \le k)$ for selected (n, p).

$P(X \ge k) = 1 - P(X \le k-1)$.

$P(X = k) = P(X \le k) - P(X \le k-1)$.

Mean and variance

$E(X) = np$, $\mathrm{Var}(X) = np(1-p)$.

Useful as a sanity check (not the main focus in U2).

Modelling assumptions

Independent trials. Constant p. Two outcomes only (success / failure).

State these explicitly when justifying a binomial model in context.

Common pitfalls

'At least one' means $1 - P(X = 0)$.

'More than k' means $P(X > k) = P(X \ge k+1)$, not $P(X \ge k)$.

SECTION T3

Binomial Distribution

Questions 1–16 · 138 marks

7. *(a)* A series of trials is carried out, each resulting in either success or failure.
State **two** conditions that have to be satisfied in order for the total number of successes to be modelled by the binomial distribution. [2]

(b) Each time Ann shoots an arrow at a target, she hits it with probability 0·4.
She shoots 20 arrows at the target. Determine the probability that she hits it

(i) exactly 8 times,

(ii) between 6 and 10 times (both inclusive). [5]

(c) Each time she shoots an arrow, she hits the centre of the target with probability 0·04. She shoots 100 arrows at the target. Use a Poisson approximation to find the probability that she hits the centre of the target less than 5 times. [3]

3. Alun and Ben are snooker players. When they play a game against each other, Alun wins with probability 0·6 and successive games are independent.

(a) One evening they play 10 games against each other. Determine the probability that Alun wins

(i) exactly 7 games,

(ii) at least 6 games. [5]

(b) On another evening, find the probability that Alun wins for the first time on the fourth game. [3]

8. The random variable X has the binomial distribution $B(16, p)$, where $p < 0{\cdot}5$.
Given that the variance of X is 2·56,

(a) calculate the value of p, [4]

(b) for this value of p, calculate $E(X^2)$. [3]

TURN OVER

4. Charlie and Dave regularly play chess against each other. When they play each other, Charlie wins with probability 0·75 and successive games are independent.

(a) One weekend they play 10 games against each other. Determine the probability that Charlie wins

(i) exactly 4 games,

(ii) more than 5 games. [5]

(b) The probability that a game lasts for less than one hour is 0·08.
They play 45 games against each other over a holiday period. Use a Poisson approximation to determine the probability that more than 6 of these games last for less than one hour. [3]

2. The random variable X has the binomial distribution B(16, 0·2). The random variable Y is defined by

$$Y = 2X + 5.$$

(a) Find the mean and variance of Y. [6]

(b) Evaluate $P(Y = 11)$. [3]

5. *(a)* When a certain type of seed is planted, there is a probability of 0·7 that it produces red flowers. A gardener plants 20 of these seeds. Calculate the probability that

(i) exactly 15 seeds produce red flowers,

(ii) more than 12 seeds produce red flowers. [6]

(b) When a different type of seed is planted, there is a probability of 0·09 that it produces white flowers. The gardener plants 150 of these seeds. Use an appropriate Poisson distribution to determine, approximately, the probability that exactly 10 seeds produce white flowers. [3]

3. The random variable X has a binomial distribution with parameters $n = 25$, $p = 0{\cdot}8$. The random variable Y is defined by $Y = aX + b$, where a, $b > 0$.
Given that the mean and standard deviation of Y are 65 and 6 respectively, find the values of a and b. [6]

4. Bethan has two fair dice, each in the shape of a regular tetrahedron. The four faces of each dice are numbered 1, 2, 3, 4 respectively.

(a) She throws one of the dice 20 times and her score on each throw is defined as the number appearing on the face in contact with the table. Let X denote the number of throws resulting in a score of 4.

(i) Write down the distribution of X.

(ii) Determine $P(3 \leqslant X \leqslant 9)$.

(iii) **Without the use of tables**, calculate $P(X = 6)$. [6]

(b) She now throws the two dice simultaneously 160 times and her score on each throw is defined as the sum of the numbers on the two faces in contact with the table. Use a Poisson approximation to determine the probability that the number of throws resulting in a score of 8 is

(i) equal to 12,

(ii) between 6 and 14 (both inclusive). [6]

4. *(a)* The random variable X has the binomial distribution B(20, 0·2).

(i) Without the use of tables, calculate $P(X = 6)$,

(ii) Determine $P(2 \leqslant X \leqslant 8)$. [5]

(b) The random variable Y has the binomial distribution B(200, 0·0123).
Use the Poisson distribution to determine the approximate value of $P(Y = 3)$. [3]

6. Jim takes part in a quiz in which he has to answer 10 questions on his chosen topic. You may assume that he answers each question correctly with probability 0·75 and that answers to successive questions are independent.
Let X denote the number of questions that he answers correctly.

(a) (i) Find the mean and the variance of X.

(ii) Find the most likely value of X. [7]

(b) Jim wins £10 for each question answered correctly but loses £2 for each question not answered correctly. Let £W denote the total amount that Jim wins.

(i) Show that $W = aX - b$, where a, b are positive integers whose values are to be found.

(ii) Find the mean and the variance of W. [4]

2. The random variable X has the binomial distribution $B(n, p)$. Given that the mean and the standard deviation of X are both equal to 0·9, find the value of n and the value of p. [5]

5. A zoologist is studying a certain breed of dog.

(a) He knows from past experience that the probability of a newly born puppy being female is 0·55. He selects a random sample of 20 newly born puppies. Calculate the probability that the number of females in the sample is

(i) exactly 12,

(ii) between 8 and 16 (both inclusive). [8]

(b) The probability of a newly born puppy being yellow is 0·05. Use an approximating distribution to find the probability that less than 5 out of a random sample of 60 newly born puppies are yellow. [3]

1. The random variable X has the binomial distribution B(10, 0·3) and $Y = 2X + 1$. Calculate

(a) the mean and the variance of Y, [5]

(b) $P(Y = 7)$. [3]

6. *(a)* A factory manufactures cups. The manager knows from past experience that 5% of the cups produced are defective. Given a random sample of 50 of these cups, determine the probability that the number of defective cups in this sample is

(i) exactly 2,

(ii) between 3 and 8 (both inclusive). [6]

(b) The factory also manufactures plates. The manager knows that 1·5% of the plates produced are defective. Use an appropriate Poisson distribution to find, approximately, the probability that a random sample of 250 of these plates contains exactly 4 defective plates. [3]

6. In a shooting range at a country fair, customers pay £5 to fire 8 shots at a target. Let X denote the number of shots which hit the target. Prizes are awarded according to the following rules.

If $X < 2$, no prize is awarded.
If $X = 2$, a prize of £10 is awarded.
If $X > 2$, a prize of £25 is awarded.

Jim decides to spend £5 to fire 8 shots. You may assume that the probability of one of his shots hitting the target is 0·12 and that successive shots are independent.

(a) Calculate the probability that he wins

(i) no prize,

(ii) a £10 prize,

(iii) a £25 prize. [5]

(b) Calculate his expected profit, giving your answer correct to two decimal places. [2]

5. Anne and Brian play a board game against each other regularly.

(a) The probability that Anne wins a game is 0·7 and the probability that Brian wins a game is 0·3, independently of all other games. One day, they play 10 games. Let X denote the number of games won by Anne on that day.

(i) State the distribution of X, including any parameters.

(ii) Determine the mean and the standard deviation of X.

(iii) Find the probability that Anne wins more games than Brian. [7]

(b) The probability that one of their games takes more than 1 hour to complete is 0·06. During a school holiday, they play 44 games. Use a Poisson approximation to find the probability that more than 2 of these games take more than 1 hour to complete. [3]

END OF BINOMIAL DISTRIBUTION PACK

Source: WJEC S1 (2008 modular spec) · 2011–2017

Curated for WJEC Maths 2017 spec AS Unit 2 — Topic 3 (2.2.4)