

Name	Date started	Target end date
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GCE AS / A LEVEL – PURE MATHEMATICS A QUESTION PACK

0974-01 (Legacy C2) · New spec Unit 1 Topic 8 · AS unit, 25% of A-level, 120 marks, 2h 30min paper

REVISE

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MATHEMATICS – PURE A · TRIG EQUATIONS & IDENTITIES

Trigonometric Equations & Identities

Trig equations using Pythagorean and quotient identities from the legacy WJEC C2 papers (June 2011 – June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 10 minutes

Derived from the legacy C1/C2 paper's pace of ~1.25 min/mark (104 marks over 9 questions).

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **single-topic practice question pack**, drilling one narrow new-spec sub-topic. It contains questions from the legacy WJEC C1 and C2 papers (2008 modular spec) that map onto new-spec AS Unit 1 Topic 8 (2.1.5). Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

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Q	Source	Max	Mark	Q	Source	Max	Mark	
1	Jun 11 Q2	19		6	Jun 14 Q2	12		
2	Jan 12 Q2	10		7	Jun 15 Q2	11		
3	Jun 12 Q2	12		8	Jun 16 Q2	11		
4	Jun 13 Q2	10		9	Jun 17 Q2	9		
5	Jan 14 Q2	10						
						Total	104	

Trigonometric Equations & Identities – what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 1: Pure Mathematics A · Topic 2.1.5.

Pythagorean identity 2.1.5

- $\sin^2 \theta + \cos^2 \theta = 1$ for all θ .
- Rearrange: $\sin^2 \theta = 1 - \cos^2 \theta$ and vice versa.
- Use to convert mixed \sin^2 / \cos^2 equations into a single trig function.

Quotient identity 2.1.5

- $\tan \theta = \frac{\sin \theta}{\cos \theta}$ for $\cos \theta \neq 0$.
- Use to combine \sin and \tan terms (or to eliminate \tan).
- Multiply through by $\cos \theta$ when needed – watch for $\cos \theta = 0$ as a separate case.

Solving in a range 2.1.5

- For $\sin \theta = k$ in $[0, 360^\circ]$: principal $\arcsin k$ then $180^\circ -$ principal.
- For $\cos \theta = k$: principal $\arccos k$ then $360^\circ -$ principal.
- For $\tan \theta = k$: principal $\arctan k$ then add 180° .

Quadratic in \sin / \cos 2.1.5

- Reduce equation to a single trig function using Pythagorean identity.
- Substitute $u = \sin \theta$ (or $\cos \theta$), solve quadratic in u .
- Reject any $|u| > 1$, then find all θ in the range.

Trig Equations & Identities in one page

Quick-reference notes – revisit before each section. Don't use during questions.

Pythagorean identity

$$\sin^2 \theta + \cos^2 \theta = 1.$$

Rearrange: $\sin^2 \theta = 1 - \cos^2 \theta$ or

$$\cos^2 \theta = 1 - \sin^2 \theta.$$

Use to eliminate one trig function from an equation.

Quotient identity

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \text{ when } \cos \theta \neq 0.$$

Sometimes useful to multiply through by $\cos \theta$.

Watch separately for $\cos \theta = 0$ if that produces a solution.

Solving $\sin \theta = k$

Principal value: $\theta_1 = \arcsin k$.

Second solution in $[0, 360^\circ]$: $\theta_2 = 180^\circ - \theta_1$.

Need $|k| \leq 1$ for real solutions.

Solving $\cos \theta = k$

Principal value: $\theta_1 = \arccos k$.

Second solution: $\theta_2 = 360^\circ - \theta_1$.

Both in $[0, 360^\circ]$.

Solving $\tan \theta = k$

Principal value: $\theta_1 = \arctan k$.

Period is 180° : add 180° for the next solution.

All real k give a solution.

Quadratic in $\sin \theta$ or $\cos \theta$

Reduce to single trig: $u = \sin \theta$ or $u = \cos \theta$.

Solve quadratic in u (formula or factor).

Reject $|u| > 1$; find all θ from each valid u .

Equations of form $\sin(n\theta) = k$

Let $\phi = n\theta$. Solve $\sin \phi = k$ over the extended range $[0, 360n^\circ]$.

Divide each ϕ by n to get θ .

No-solution arguments

If the equation reduces to a quadratic in u with $\Delta < 0$, no real $u \Rightarrow$ no θ .

If all u have $|u| > 1$, no θ either.

Cite either reason explicitly in your answer.

Strategy

1. Use Pythagorean / quotient to reduce to one trig function.
2. Solve the resulting equation in u .
3. Reject invalid u .
4. Find all θ in the stated range.

SECTION T8

Trigonometric Equations & Identities

Questions 1-9 · 104 marks

2. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$\sin \theta + 12 \cos^2 \theta = 6. \quad [6]$$

- (b) Find all values of x in the range $0^\circ \leq x \leq 180^\circ$ satisfying

$$\cos(2x - 35^\circ) = 0.891. \quad [3]$$

- (c) Find all values of ϕ in the range $0^\circ \leq \phi \leq 360^\circ$ satisfying

$$\sin \phi + \cos \phi = 0. \quad [3]$$

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2. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$10\sin^2\theta + 7\cos\theta = 5\cos^2\theta + 8. \quad [6]$$

- (b) Find all values of x in the range $0^\circ \leq x \leq 360^\circ$ satisfying

$$\sin(x - 50^\circ) = -0.682. \quad [3]$$

- (c) Without carrying out any calculations, explain why there are no values of ϕ which satisfy the equation

$$\sin\phi + \cos\phi = 3. \quad [1]$$

2. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$10\cos^2 \theta + 3\cos \theta = 4\sin^2 \theta - 2. \quad [6]$$

- (b) Find all values of x in the range $0^\circ \leq x \leq 180^\circ$ satisfying

$$\sin (3x - 21^\circ) = -0.809. \quad [3]$$

- (c) Find all values of ϕ in the range $0^\circ \leq \phi \leq 360^\circ$ satisfying

$$\cos \phi - 5 \sin \phi = 0. \quad [3]$$

2. (a) (i) Show that the equation

$$6\cos\theta + 5\tan\theta = 0$$

may be rewritten in the form

$$6\sin^2\theta - 5\sin\theta - 6 = 0.$$

- (ii) Hence find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying the equation

$$6\cos\theta + 5\tan\theta = 0. \quad [7]$$

- (b) Find all values of x in the range $0^\circ \leq x \leq 180^\circ$ satisfying

$$\cos(2x - 60^\circ) = 0.788. \quad [3]$$

2. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$8\cos^2\theta - 7\sin^2\theta = 4\cos\theta - 3. \quad [6]$$

- (b) The angles X , Y and Z are the three angles of a triangle. Given that $\tan X = -2.246$ and that $\tan(Y - Z) = 0.364$, find the values of X , Y and Z . Give each angle correct to the nearest degree. [4]

2. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$4\cos^2\theta + 1 = 4\sin^2\theta - 2\cos\theta. \quad [6]$$

- (b) The angle α satisfies

$$\sin(\alpha + 40^\circ) = \frac{1}{\sqrt{2}}$$

$$\text{and } \sin(\alpha - 35^\circ) = \frac{\sqrt{3}}{2}.$$

Given that $0^\circ < \alpha < 180^\circ$, find the value of α . [3]

- (c) Find all values of ϕ in the range $0^\circ \leq \phi \leq 360^\circ$ satisfying

$$\frac{7}{\cos\phi} - \frac{10}{\sin\phi} = 0. \quad [3]$$

2. (a) Show that there is no angle θ which satisfies the equation

$$4\cos^2\theta - 2\sin^2\theta - \sin\theta + 8 = 0,$$

giving a mathematical reason to explain how you came to your conclusion. [4]

- (b) Find all values of x in the range $0^\circ \leq x \leq 180^\circ$ satisfying

$$\sin(2x - 75^\circ) = -0.515. \quad [3]$$

- (c) Find all values of ϕ in the range $0^\circ \leq \phi \leq 180^\circ$ satisfying

$$4\tan\phi + 7\sin\phi = 0. \quad [4]$$

2. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$6\sin^2\theta + 1 = 2(\cos^2\theta - \sin\theta). \quad [6]$$

- (b) Find all values of x in the range $0^\circ \leq x \leq 180^\circ$ satisfying

$$\tan(3x - 57^\circ) = -0.81. \quad [4]$$

- (c) Without carrying out any calculations, explain why there are no values of ϕ which satisfy the equation

$$2\sin\phi + 4\cos\phi = -7. \quad [1]$$

2. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$\sin^2\theta + 6\cos^2\theta + 13\sin\theta = 0. \quad [5]$$

- (b) The angles A , B and C are the three angles of a triangle. Given that $\cos A = -0.342$ and that $\tan(B - C) = 0.404$, find the values of A , B and C . Give each angle correct to the nearest degree. [4]

END OF TRIG EQUATIONS & IDENTITIES PACK

Source: WJEC C1 + C2 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec AS Unit 1 – Topic 8 (2.1.5)

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