Name	Date started	Target end date

GCE AS / A LEVEL — PURE MATHEMATICS A QUESTION PACK

0974-01 (Legacy C2) · New spec Unit 1 Topic 7 · AS unit, 25% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS — PURE A · CIRCLES

Circles

Circle equation, tangents, normals and two-circle intersection questions from the legacy WJEC C2 papers (June 2011 – June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 9 minutes

*Derived from the legacy C1/C2 paper's pace of **~1.25 min/mark** (103 marks over 8 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **single-topic practice question pack**, drilling one narrow new-spec sub-topic. It contains questions from the legacy WJEC C1 and C2 papers (2008 modular spec) that map onto new-spec AS Unit 1 Topic 7 (2.1.3).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 12 Q8	6	
2	Jan 13 Q8	19	
3	Jun 13 Q8	11	
4	Jan 14 Q8	15	

Q	Source	Max	Mark
5	Jun 14 Q8	16	
6	Jun 15 Q8	9	
7	Jun 16 Q8	12	
8	Jun 17 Q8	15	
Total		103	

Circles — what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 1: Pure Mathematics A · Topic 2.1.3.

Circle equation forms 2.1.3

- Centre-radius form: $(x-a)^2+(y-b)^2=r^2$.
- General form: $x^2+y^2+2gx+2fy+c=0$ — centre $(-g,-f)$, radius $\sqrt{g^2+f^2-c}$.
- Complete the square to convert between the two forms.

Tangents and normals 2.1.3

- Tangent at P is perpendicular to the radius AP (from centre A).
- Gradient of tangent at P: $-1/m_{AP}$.
- Line through P with that gradient is the tangent.

Intersections with lines 2.1.3

- Substitute the line equation into the circle, get a quadratic in x.
- $\Delta > 0$: two intersection points; $\Delta = 0$: tangent; $\Delta < 0$: no intersection.
- Verify by computing the perpendicular distance from centre to line vs radius.

Two circles 2.1.3

- Distance d between centres governs configuration.
- Touch externally: $d = r_1 + r_2$; touch internally: $d = |r_1 - r_2|$.
- Intersect at two points iff $|r_1 - r_2| < d < r_1 + r_2$.

Circles in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Circle equation (centre-radius)

$(x-a)^2+(y-b)^2=r^2$.

Centre (a,b), radius r.

Useful when centre is obvious from the question.

Circle equation (general form)

$x^2+y^2+2gx+2fy+c=0$.

Centre $(-g,-f)$, radius $\sqrt{g^2+f^2-c}$.

Complete the square to switch between forms.

Point inside, on or outside?

Compute distance from point to centre.

Less than r: inside; equal: on; greater: outside.

Equivalently, plug coordinates into general form: <0 inside, $=0$ on, >0 outside.

Tangent to a circle at a point P

Gradient of radius AP: m_{AP}.

Gradient of tangent at P: $-1/m_{AP}$.

Tangent line: $y-y_P=(-1/m_{AP})(x-x_P)$.

Tangent from external point T

Distance from T to centre: $d=\sqrt{(x_T-a)^2+(y_T-b)^2}$.

Tangent length from T to circle: $L=\sqrt{d^2-r^2}$.

Line meeting a circle

Substitute the line equation into the circle.

Resulting quadratic: $\Delta>0$ two points, $\Delta=0$ tangent, $\Delta<0$ no meeting.

Alternative: compare perpendicular distance to radius.

Two circles meeting

d = distance between centres.

External touch: $d=r_1+r_2$.

Internal touch: $d=|r_1-r_2|$.

Two intersection points iff $|r_1-r_2|<d<r_1+r_2$.

Common tangent at touching circles

At the touching point P, both circles share the tangent.

Tangent is perpendicular to the line A_1A_2 joining centres.

Pass through P.

Strategy

1. Convert to centre-radius form if needed.
2. For tangents: perpendicular to radius.
3. For intersections: substitute line into circle.
4. Always check by plugging coordinates back in.

SECTION T7

Circles

Questions 1–8 · 103 marks

8. The circle C has centre A and equation

$$x^2 + y^2 - 4x + 6y + 1 = 0.$$

(a) Find the coordinates of A and the radius of C. [3]

(b) The point R lies on the circle C. The tangent to the circle at R passes through the point $T(8, 2)$. Find the length of RT. [3]

8. The circle C has centre A and equation

$$x^2 + y^2 + 6x - 10y + 14 = 0.$$

(a) (i) Find the coordinates of A and the radius of C.

(ii) The point P has coordinates $(-6, 2)$. Determine whether P lies inside C, on C or outside C. [5]

(b) The line L has equation

$$y = 2x + 1.$$

(i) Show that L is a tangent to the circle C and find the coordinates of Q, the point of contact of L and C.

(ii) The point R has coordinates $(4, 9)$ and R lies on L. Find $A\hat{R}Q$. [8]

8. The circle C_1 has centre A and equation

$$x^2 + y^2 + 2x - 6y - 15 = 0.$$

(a) Find the coordinates of A and the radius of C_1. [3]

(b) The line L has equation $y = -x + 9$.

(i) Show that L is not a diameter of C_1.

(ii) Find the coordinates of the points of intersection of L and C_1. [5]

(c) The circle C_2 has centre $B(11, 8)$ and radius 6. Find the shortest distance between the circles C_1 and C_2. [3]

8. The circle C has centre A and equation

$$x^2 + y^2 - 4x + 8y - 5 = 0.$$

(a) (i) Write down the coordinates of A.

(ii) The point P has coordinates $(6, -7)$ and lies on C. Find the equation of the tangent to C at P. [5]

(b) The line L has equation $y = x + 3$. Show that L and C do not intersect. [4]

8. *(a)* The circle C_1 has centre $A(-2, 9)$ and radius 5. The circle C_2 has centre $B(10, -7)$ and radius 15.

(i) Show that C_1 and C_2 touch, justifying your answer.

(ii) Given that the circles touch at the point $P(1, 5)$, find the equation of the common tangent at P. [7]

(b) Gareth, who has been asked by his teacher to investigate the properties of another circle C_3, claims that the equation of this circle C_3 is given by

$$x^2 + y^2 + 4x - 6y + 20 = 0.$$

Show that Gareth cannot possibly be correct. [3]

8. The circle C has centre A and radius r. The points $P(-2, -3)$ and $Q(8, 1)$ are at opposite ends of a diameter of C.

(a) (i) Write down the coordinates of A.

(ii) Show that $r = \sqrt{29}$. [3]

(b) Given that the point $R(5, 4)$ lies on the circle C, find $P\hat{Q}R$. Give your answer in degrees, correct to one decimal place. [3]

(c) The point S lies on the circle C. The tangent to the circle at S passes through the point $T(11, 0)$. Find the length of ST. [3]

8. The circle C_1 has centre A and equation

$$x^2 + y^2 + 6x - 20y + 59 = 0.$$

(a) (i) Find the coordinates of A and the radius of C_1.

(ii) Find the shortest distance from the origin to the circle C_1. Give your answer correct to two decimal places. [5]

(b) The line L has equation $y = 3x - 1$. The line L and the circle C_1 intersect at the points P and Q.

(i) Find the coordinates of P and Q.

(ii) The circle C_2 has centre $B(6, 7)$ and is such that PQ is the common chord of C_1 and C_2. Find the equation of C_2. [7]

8. The circle C has centre A and equation

$$x^2 + y^2 + 10x - 8y + 21 = 0.$$

(a) (i) Find the coordinates of A and the radius of C.

(ii) The point P has coordinates $(-2, 0)$. Determine whether P lies inside C, on C or outside C. [5]

(b) The line L has equation $y = 2x + 4$. Show that L is a tangent to the circle C and find the coordinates of the point of contact of L and C. [5]

END OF CIRCLES PACK

Source: WJEC C1 + C2 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec AS Unit 1 — Topic 7 (2.1.3)