Name	Date started	Target end date

GCE AS / A LEVEL — PURE MATHEMATICS A QUESTION PACK

0973-01 (Legacy C1) · New spec Unit 1 Topic 5 · AS unit, 25% of A-level, 120 marks, 2h 30min paper

REVISE
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MATHEMATICS — PURE A · BINOMIAL THEOREM

Binomial Theorem

Every binomial-expansion question from the legacy WJEC C1 papers (June 2011 - June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hour 19 minutes

*Derived from the legacy C1/C2 paper's pace of **~1.25 min/mark** (63 marks over 10 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C1 and C2 papers (2008 modular spec) that maps onto new-spec AS Unit 1 Topic 5 (2.1.5).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

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Q	Source	Max	Mark
1	Jun 11 Q7	8	
2	Jan 12 Q4	7	
3	Jun 12 Q4	4	
4	Jan 13 Q7	4	
5	Jun 13 Q5	6	

Q	Source	Max	Mark
6	Jan 14 Q5	8	
7	Jun 14 Q4	5	
8	Jun 15 Q6	8	
9	Jun 16 Q4	5	
10	Jun 17 Q5	8	
	Total	63	

Binomial Theorem — what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 1: Pure Mathematics A · Topic 2.1.4.

Binomial expansion 2.1.4

- $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$ for positive integer n.
- $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$ — binomial coefficient.
- Pascal's triangle gives coefficients for small n.

Term containing x^r 2.1.4

- In $(a+bx)^n$, the term in x^r is $\binom{n}{r} a^{n-r} b^r x^r$.
- Coefficient of x^r is therefore $\binom{n}{r} a^{n-r} b^r$.
- Common: relate two coefficients (e.g. x^2 coefficient = $5\times$ coefficient of x) and solve for n.

Approximations 2.1.4

- $(1+x)^n \approx 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$ for small x.
- Substitute a small numeric value of x to estimate $(1.05)^6$ etc.
- Verify which terms are needed for the desired accuracy.

Coefficient of unknown a 2.1.4

- Expand $(a+bx)^n$ symbolically and equate the coefficient of a specific power to a target.
- Solve the resulting polynomial equation in a.
- Reject the trivial $a = 0$ when stated.

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Binomial Theorem in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Binomial coefficients

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Symmetry: $\binom{n}{k} = \binom{n}{n-k}$.

$\binom{n}{0} = \binom{n}{n} = 1$, $\binom{n}{1} = n$.

Pascal's triangle

Row n: $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$.

Each entry is the sum of the two above it.

Use for small n — faster than factorials.

Binomial expansion

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$.

First few terms: $a^n + na^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots$

Term in x^r of $(a+bx)^n$

Term: $\binom{n}{r}a^{n-r}(bx)^r = \binom{n}{r}a^{n-r}b^r \cdot x^r$.

Coefficient of x^r: $\binom{n}{r}a^{n-r}b^r$.

Expansion of $(1+x)^n$

$(1+x)^n = 1 + nx + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots$

Useful for approximations when x is small.

Relating two coefficients

Equate, e.g. $\text{coeff}(x^2) = 5 \cdot \text{coeff}(x)$.

This gives an equation in n (often quadratic).

Solve for n; reject non-positive integers.

Finding a from a coefficient

Expand $(a+bx)^n$ symbolically.

Set the relevant coefficient equal to the target.

Solve the resulting polynomial in a.

Approximations

For $(1+x)^n$ with small x:

$(1+x)^n \approx 1 + nx + \frac{n(n-1)}{2}x^2$.

Substitute a small numeric x to estimate $(1.05)^n$ etc.

Strategy

1. Identify n, a, b.
2. Pick the term you need: $\binom{n}{r}a^{n-r}b^r$.
3. Equate coefficients if related to another term.
4. Solve and verify.

SECTION T5

Binomial Theorem

Questions 1–10 · 63 marks

7. *(a)* Use the binomial theorem to expand $(3+2x)^4$, simplifying each term of the expansion. [4]

(b) In the binomial expansion of $\left(1+\frac{x}{4}\right)^n$, the coefficient of x^2 is five times the coefficient of x.
Given that n is a positive integer, find the value of n. [4]

4. *(a)* Use the binomial theorem to expand $\left(x + \frac{3}{x}\right)^4$, simplifying each term of the expansion. [4]

(b) The coefficient of x^2 in the expansion of $(1 + 2x)^n$ is 760. Given that n is a positive integer, find the value of n. [3]

4. Using the binomial theorem, write down and simplify the first four terms in the expansion of $(1-2x)^6$ in ascending powers of x. [4]

7. In the binomial expansion of $(a + 4x)^6$, where $a \neq 0$, the coefficient of the term in x^2 is twice the coefficient of the term in x. Find the value of a. [4]

5. *(a)* Using the binomial theorem, write down and simplify the first three terms in the expansion of $(1 + 2x)^7$ in ascending powers of x. [3]

(b) Use your answer to part *(a)* to find the first three terms in the expansion of $(1 - 4x)(1 + 2x)^7$ in ascending powers of x. [3]

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5. *(a)* **Use the binomial theorem** to express $(1+\sqrt{6})^5$ in the form $a+b\sqrt{6}$, where a, b are integers whose values are to be found. [5]

(b) The coefficient of x^2 in the expansion of $(1+3x)^n$ is 495. Given that n is a positive integer, find the value of n. [3]

4. *(a)* Write down the expansion of $(1 + x)^6$ in ascending powers of x up to and including the term in x^3. [2]

(b) **Showing all your working**, substitute an appropriate value for x in your expansion in part *(a)* to find an approximate value for $1{\cdot}1^6$. [3]

6. *(a)* Using the binomial theorem, write down and simplify the first four terms in the expansion of $\left(1-\frac{x}{2}\right)^8$ in ascending powers of x. [4]

(b) The first two terms in the expansion of $(2+ax)^n$ in ascending powers of x are 32 and $-240x$ respectively. Find the value of n and the value of a. [4]

4. **Use the binomial theorem** to express $\left(\sqrt{3}-1\right)^5$ in the form $a+b\sqrt{3}$, where a, b are integers whose values are to be found. [5]

5. *(a)* Use the binomial theorem to expand $\left(x + \frac{2}{x}\right)^4$, simplifying each term of the expansion. [4]

(b) In the binomial expansion of $(a + 2x)^6$, where $a \neq 0$, the coefficient of the term in x^2 is equal to the coefficient of the term in x. Find the value of a. [4]

END OF BINOMIAL THEOREM PACK

Source: WJEC C1 + C2 (2008 modular spec) · 2011–2017

Curated for WJEC Maths 2017 spec AS Unit 1 — Topic 5 (2.1.5)

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