Name	Date started	Target end date

GCE AS / A LEVEL — PURE MATHEMATICS A QUESTION PACK

0974-01 (Legacy C2) · New spec Unit 1 Topic 4 · AS unit, 25% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS — PURE A · GEOMETRIC SEQUENCES & SERIES

Geometric Sequences & Series

Every geometric-series question from the legacy WJEC C2 papers (June 2011 - June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours

*Derived from the legacy C1/C2 paper's pace of **~1.25 min/mark** (96 marks over 12 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C1 and C2 papers (2008 modular spec) that maps onto new-spec AS Unit 1 Topic 4 (2.1.4).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q5	9	
2	Jan 12 Q5	9	
3	Jun 12 Q5	9	
4	Jan 13 Q5	9	
5	Jun 13 Q5	9	
6	Jan 14 Q4	9	

Q	Source	Max	Mark
7	Jan 14 Q10	4	
8	Jun 14 Q5	8	
9	Jun 15 Q5	11	
10	Jun 16 Q5	9	
11	Jun 17 Q5	7	
12	Jun 17 Q10	3	
Total		**96**	

Geometric Sequences & Series — what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 1: Pure Mathematics A · Topic 2.1.4.

Geometric series basics 2.1.4

- $u_n = ar^{n-1}$ — first term a, common ratio r.
- $S_n = \dfrac{a(1-r^n)}{1-r}$ (for $r \neq 1$).
- Prove S_n formula by computing $S_n - rS_n = a(1-r^n)$ and dividing.

Sum to infinity 2.1.4

- $S_\infty = \dfrac{a}{1-r}$ converges iff $|r| < 1$.
- Given S_∞ and one term, set up simultaneous equations for a, r.
- Always check the $|r| < 1$ condition before invoking S_∞.

Finding the common ratio 2.1.4

- Ratio of consecutive terms gives r directly: $r = u_{n+1}/u_n$.
- Two non-adjacent terms: $u_q/u_p = r^{q-p}$.
- Watch sign: r can be negative, e.g. alternating series.

Recursive sequences 2.1.4

- $u_{n+1} = f(u_n)$ — each term depends on the previous.
- Compute the first few terms to spot patterns, fixed points or periodicity.
- Common forms: $u_{n+1} = 1 - 1/u_n$ (periodic), $u_{n+1} = 3u_n + 1$ (linear recurrence).

Geometric Sequences & Series in one page

Quick-reference notes — revisit before each section. Don't use during questions.

GP definition

A *geometric progression*: first term a, common ratio r.

$u_1 = a, u_2 = ar, u_3 = ar^2, \ldots$

$u_n = ar^{n-1}$.

Sum of a GP

$$S_n = \frac{a(1-r^n)}{1-r} \text{ for } r \neq 1.$$

If $r = 1$, $S_n = na$ (all terms equal a).

Proving the sum formula

Compute $S_n - rS_n$:

$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$.

$rS_n = ar + ar^2 + \cdots + ar^n$.

Subtract: $(1-r)S_n = a(1-r^n)$.

Divide.

Sum to infinity

$S_\infty = \dfrac{a}{1-r}$ — converges iff $|r| < 1$.

For $|r| \geq 1$: the series diverges.

Always state the $|r| < 1$ condition when claiming S_∞.

Finding the common ratio

Consecutive terms: $r = u_{n+1}/u_n$.

Two terms p apart: $u_{n+p}/u_n = r^p$.

Solve $r^p =$ value; remember r can be negative if p is even.

GP simultaneous equations

Two facts ⇒ two equations in a and r.

Common: $u_p =$ value, $S_q =$ value, $S_\infty =$ value.

Divide equations to eliminate a and isolate r.

Recursive sequences

$u_{n+1} = f(u_n)$ — not always a GP.

Compute $u_2, u_3, u_4, \ldots$ to spot patterns: fixed point, periodicity, divergence.

Example: $u_{n+1} = 1 - 1/u_n$ is periodic with period 3.

GP modelling

Annual percentage growth: $r = 1 + \text{rate}$.

Annual percentage decay: $r = 1 - \text{rate}$.

Total accumulated value over n years: S_n.

Strategy

1. Identify a and r from the data.
2. For sums use the right formula: S_n or S_∞.
3. For S_∞: verify $|r| < 1$.
4. For recursive: compute several terms, look for pattern.

SECTION T4

Geometric Sequences & Series

Questions 1–12 · 96 marks

5. *(a)* A geometric series has first term a and common ratio r. Prove that the sum of the first n terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}\ .$$

[3]

(b) The sum to infinity of a geometric series is equal to four times the first term of the series.

(i) Find the value of the common ratio of the series.

(ii) Given that the sum of the first two terms of the series is 35, find the sum of the first nine terms of the series. Give your answer correct to the nearest whole number.

[6]

5. *(a)* A geometric series has first term a and common ratio r. Prove that the sum of the first n terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$ [3]

(b) The sum of the first two terms of a geometric series is 25·2. The sum to infinity of the series is 30. Given that the common ratio is positive, find the common ratio and first term of this geometric series. [6]

5. A geometric series has first term a and common ratio r. The sum of the first and second terms of the series is 72. The sum of the first and third terms of the series is 120.

(a) Show that r satisfies the equation

$$3r^2 - 5r - 2 = 0.$$ [4]

(b) Given that $|r| < 1$, find the value of r and the sum to infinity of the series. [5]

5. *(a)* The pth term of a geometric series is 16. The $(p + 1)$th term of this series is 24. Find the $(p + 4)$th term of the series. [3]

(b) The sum of the first three terms of another geometric series is 22·8. The sum to infinity of the series is 18·75. Find the common ratio and the first term of this geometric series. [6]

5. *(a)* Find the sum of the first eighteen terms of the geometric series

$$100 + 80 + 64 + ...$$

Give your answer correct to the nearest whole number. [3]

(b) The second term of a geometric series is -20. The sum to infinity of the series is 64.

(i) Show that r, the common ratio of the series, satisfies the equation

$$16r^2 - 16r - 5 = 0.$$

(ii) Find the value of r, giving a reason for your answer. [6]

4. *(a)* A geometric series has first term a and common ratio r. Prove that the sum of the first n terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$ [3]

(b) The fourth term of a geometric series is -108 and the seventh term is 4.

(i) Find the common ratio of the series.

(ii) Find the sum to infinity of the series. [6]

10. The nth term of a number sequence is denoted by t_n. The $(n + 1)$th term of the sequence satisfies

$$t_{n+1} = 1 - \frac{1}{t_n},$$

for all positive integers n. Given that $t_1 = 4$,

(a) evaluate t_2, t_3, and t_4, [2]

(b) describe the behaviour of the sequence and hence, without carrying out any further calculation, write down the value of t_{50}. [2]

5. A geometric series has first term a and common ratio r. The sum of the second and third terms of the series is -216. The sum of the fifth and sixth terms of the series is 8.

(a) Prove that $r = -\frac{1}{3}$. [5]

(b) Find the sum to infinity of the series. [3]

5. *(a)* The eighth and ninth terms of a geometric series are 576 and 2304 respectively. Find the fifth term of the geometric series. [3]

(b) Another geometric series has first term a and common ratio r. The third term of this geometric series is 24. The sum of the second, third and fourth terms of the series is -56.

(i) Show that r satisfies the equation

$$3r^2 + 10r + 3 = 0.$$

(ii) Given that $|r| < 1$, find the value of r and the sum to infinity of the series. [8]

5. *(a)* A geometric series has first term a and common ratio r. Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{a(1-r^n)}{1-r} \, .$$ [3]

(b) The sum of the first five terms of a geometric series is 275. The sum to infinity of the series is 243. Find the common ratio and the first term of the geometric series. [6]

5. A rich businessman makes one donation per year to a certain charity. He starts by donating £100 in the first year. In each subsequent year, the value of the donation is 1·2 times the value of the previous year's donation.

(a) Find the value of the businessman's donation in the 12^{th} year. Give your answer correct to the nearest pound. [2]

(b) After receiving the nth donation, the charity's treasurer calculates that over the years, the businessman has donated a **total** of £15 474, correct to the nearest pound. Find the value of n. [5]

10. The nth term of a number sequence is denoted by t_n. The $(n + 1)$th term of the sequence satisfies

$$t_{n+1} = 3t_n + 1,$$

for all positive integers n. Given that $t_4 = 202$,

(a) evaluate t_1, [2]

(b) explain why 29 999 999 cannot be one of the terms of this number sequence. [1]

END OF PAPER

END OF GEOMETRIC SEQUENCES & SERIES PACK

Source: WJEC C1 + C2 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec AS Unit 1 — Topic 4 (2.1.4)