Name	Date started	Target end date

GCE AS / A LEVEL — PURE MATHEMATICS A QUESTION PACK

0974-01 (Legacy C2) · New spec Unit 1 Topic 3 · AS unit, 25% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS — PURE A · ARITHMETIC SEQUENCES & SERIES

Arithmetic Sequences & Series

Every arithmetic-series question from the legacy WJEC C2 papers (June 2011 - June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hour 54 minutes

*Derived from the legacy C1/C2 paper's pace of **~1.25 min/mark** (91 marks over 10 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C1 and C2 papers (2008 modular spec) that maps onto new-spec AS Unit 1 Topic 3 (2.1.4).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q4	8	
2	Jan 12 Q4	8	
3	Jun 12 Q4	10	
4	Jan 13 Q4	9	
5	Jun 13 Q4	8	

Q	Source	Max	Mark
6	Jan 14 Q3	9	
7	Jun 14 Q4	10	
8	Jun 15 Q4	10	
9	Jun 16 Q4	9	
10	Jun 17 Q4	10	
Total		**91**	

Arithmetic Sequences & Series — what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 1: Pure Mathematics A · Topic 2.1.4.

Arithmetic series basics 2.1.4

- $u_n = a + (n-1)d$ — n-th term given first term a and common difference d.
- $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$ where l is the last term.
- Prove S_n formula by writing S_n forwards and backwards then adding.

Solving for a, d 2.1.4

- Two facts about the series typically give two simultaneous equations in a and d.
- Example: third + eighth = 0, plus fifth + seventh + tenth = 22.
- Watch for terms expressed as u_p or S_p — translate to a and d.

Finding n 2.1.4

- Given S_n value, solve a quadratic in n from $S_n = \frac{n}{2}(2a + (n-1)d)$.
- Given u_n value, solve a linear equation in n for $u_n = a + (n-1)d$.
- Reject any non-integer or negative root.

Word problems 2.1.4

- Identify a, d, then translate the situation into series terms.
- Common context: training, savings, rent, repayment plans.
- Final answer often a count of weeks / days / payments — keep units.

Crib sheet »

Arithmetic Sequences & Series in one page

Quick-reference notes — revisit before each section. Don't use during questions.

AP definition

An *arithmetic progression*: first term a, common difference d.

$u_1 = a, u_2 = a + d, u_3 = a + 2d, \ldots$

$u_n = a + (n-1)d$.

Sum of an AP

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$$

where $l = u_n$ is the last term.

Two equivalent forms — pick whichever fits the data.

Proving the sum formula

Write $S_n = a + (a + d) + \cdots + l$.

Reverse: $S_n = l + (l - d) + \cdots + a$.

Add: $2S_n = n(a + l)$, so $S_n = \frac{n}{2}(a + l)$.

Finding a and d

Two facts give two equations:

$u_p = a + (p-1)d$, $S_q = \frac{q}{2}[2a + (q-1)d]$.

Solve simultaneously.

Finding n

If S_n is given: solve a quadratic in n.

If u_n is given: solve a linear equation in n.

Reject non-integer or non-positive solutions.

Sigma notation

$$\sum_{r=1}^{n} u_r = u_1 + u_2 + \cdots + u_n = S_n.$$

$$\sum(\alpha u_r + \beta v_r) = \alpha \sum u_r + \beta \sum v_r.$$

Index shifting: $\sum_{r=2}^{n} u_r = \sum_{r=1}^{n} u_r - u_1$.

Word problems

Identify a and d in context.

Be careful with the question's n — days, weeks, payments etc.

Distinguish u_n (a single term) from S_n (cumulative).

Proving statements from first principles

Show $u_n =$ given formula by substituting n into $a + (n-1)d$.

Show $S_n =$ given formula by adding forwards + backwards and simplifying.

Avoid jumping to the result.

Strategy

1. Write down a, d.
2. Pick u_n or S_n formula.
3. Form simultaneous equations from problem data.
4. Solve, check n is a positive integer.

SECTION T3

Arithmetic Sequences & Series

Questions 1–10 · 91 marks

4. *(a)* The sum of the first fifteen terms of an arithmetic series is 780. The sum of the second, fourth and tenth terms of the series is 100. Find the first term and the common difference of the series. [5]

(b) The pth term of another arithmetic series is 1023. The $(p + 4)$th term of this series is 1059. Find the $(p + 7)$th term of the series. [3]

4. The fifteenth term of an arithmetic series is seven times the fifth term.
The sum of the first eleven terms of the series is 88.

(a) Find the first term and common difference of the arithmetic series. [6]

(b) Given that the nth term of the series is 143, find the value of n. [2]

4. *(a)* An arithmetic series has first term a and common difference d. Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}[2a + (n-1)d].$$ [3]

(b) The sum of the third, fourth and tenth terms of an arithmetic series is 79. The sum of the sixth and seventh terms of the series is 61. Find the first term and the common difference of the series. [4]

(c) Find an expression, in terms of n, for the sum of the first n terms of the arithmetic series

$$15 + 13 + 11 + 9 + \ldots.$$

Simplify your answer. [3]

4. *(a)* The first term of an arithmetic series is 1 and the common difference is 4.

(i) Show that the nth term of the arithmetic series is $4n - 3$.

(ii) The sum of the first n terms of this series is given by

$$S_n = 1 + 5 + \ldots + (4n - 7) + (4n - 3).$$

Prove from first principles, without using the formula for the sum of the first n terms, that

$$S_n = n(2n - 1).$$ [4]

(b) The sum of the first ten terms of another arithmetic series is 55. The sum of the fourth, seventh and ninth terms of the series is 27. Find the first term and the common difference of this arithmetic series. [5]

4. *(a)* An arithmetic series has first term a and common difference d. Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right].$$ [3]

(b) The sum of the first ten terms of an arithmetic series is 115. The sum of the **next** four terms of this series is 130. Find the first term and the common difference of the arithmetic series. [5]

3. *(a)* The sum of the third and eighth terms of an arithmetic series is zero. The sum of the fifth, seventh and tenth terms of the series is 22. Find the first term and the common difference of the series. [4]

(b) The first term of another arithmetic series is 9 and the common difference is 2. The sum of the first $2n$ terms of this arithmetic series is 3 times the sum of the first n terms of the series. Find the value of n. [5]

4. *(a)* An arithmetic series has first term a and common difference d.
Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}[2a + (n-1)d].$$ [3]

(b) The first term of an arithmetic series is 3 and the common difference is 2. The sum of the first n terms of the series is 360.
Write down an equation satisfied by n. Hence find the value of n. [3]

(c) The tenth term of another arithmetic series is seven times the third term. The sum of the eighth and ninth terms of the series is 80. Find the first term and common difference of this arithmetic series. [4]

4. *(a)* The first term of an arithmetic series is 4 and the common difference is 6.

(i) Show that the nth term of the arithmetic series is $6n - 2$.

(ii) The sum of the first n terms of this series is given by

$$S_n = 4 + 10 + \ldots + (6n - 8) + (6n - 2).$$

Without using the formula for the sum of the first n terms of an arithmetic series, **prove** that

$$S_n = n(3n + 1).$$ [4]

(b) The tenth term of another arithmetic series is four times the fifth term.
The sum of the first fifteen terms of the series is 210.

(i) Find the first term and common difference of this arithmetic series.

(ii) Given that the kth term of the series is 200, find the value of k.

[6]

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4. *(a)* Gwilym has decided to run in a half marathon race. In order to get himself fit, he devises a training programme whereby he runs around his local track each day, gradually increasing the distance he runs from day to day. On the first day, he runs 6 laps of the track and subsequently, on any given day, he runs 2 laps further than he did on the day before.

(i) How many laps does he run on the 20th day of his programme?

(ii) After how many days will the total number of laps he has run since the beginning of his training programme be equal to 750? [6]

(b) The nth term of an **arithmetic** series is denoted by t_n. It is known that

$$t_{12} + t_{13} = 50.$$

(i) **Write down** the value of $t_{11} + t_{14}$. [1]

(ii) Find the sum of the first twenty-four terms of this arithmetic series. [2]

4. *(a)* An arithmetic series has first term a and common difference d. Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right].$$ [3]

(b) The sum of the first eight terms of an arithmetic series is 156 and the sum of the first sixteen terms of the series is 760. Find the first term and common difference of this series. [4]

(c) The pth term of another arithmetic series is 2057. The $(p + 5)$th term of this series is 2102. Find the $(p + 8)$th term of the series. [3]

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END OF ARITHMETIC SEQUENCES & SERIES PACK

Source: WJEC C1 + C2 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec AS Unit 1 — Topic 3 (2.1.4)