Name	Date started	Target end date

GCE AS / A LEVEL — PURE MATHEMATICS A QUESTION PACK

0973-01 (Legacy C1) · New spec Unit 1 Topic 2 · AS unit, 25% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS — PURE A · POLYNOMIALS & FUNCTION SKETCHING

Polynomials & Function Sketching

Factor / remainder theorem on cubics and y=f(x) transformations from the legacy WJEC C1 papers (June 2011 - June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 40 minutes

*Derived from the legacy C1/C2 paper's pace of **~1.25 min/mark** (128 marks over 21 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C1 and C2 papers (2008 modular spec) that maps onto new-spec AS Unit 1 Topic 2 (2.1.2 + 2.1.3).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q8	9	
2	Jun 11 Q9	5	
3	Jan 12 Q8	7	
4	Jan 12 Q9	6	
5	Jun 12 Q8	8	
6	Jun 12 Q9	6	
7	Jan 13 Q8	6	
8	Jan 13 Q9	6	
9	Jun 13 Q8	6	
10	Jun 13 Q9	6	
11	Jan 14 Q7	5	

Q	Source	Max	Mark
12	Jan 14 Q9	8	
13	Jun 14 Q8	6	
14	Jun 14 Q9	7	
15	Jun 15 Q8	6	
16	Jun 15 Q9	4	
17	Jun 16 Q7	5	
18	Jun 16 Q9	7	
19	Jun 17 Q6	3	
20	Jun 17 Q7	7	
21	Jun 17 Q8	5	
	Total	128	

Polynomials & Function Sketching — what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 1: Pure Mathematics A · Topic 2.1.2.

Factor & remainder theorems 2.1.2

- Factor theorem: $(x-a)$ is a factor of $p(x)$ iff $p(a)=0$.
- Remainder theorem: when $p(x)$ is divided by $(x-a)$, the remainder is $p(a)$.
- Use to find unknown coefficients before solving cubics.

Polynomial division 2.1.2

- Long division of $p(x)$ by $(x-a)$ produces a quadratic quotient.
- Synthetic division (a shortcut) works for linear divisors of the form $(x-a)$.
- After finding one root, factor and solve the resulting quadratic.

Graph transformations 2.1.2

- $y=f(x)+a$: vertical shift by a.
- $y=f(x+a)$: horizontal shift by $-a$.
- $y=kf(x)$: vertical stretch by k; $y=f(kx)$: horizontal stretch by $1/k$.

Quadratic inequalities 2.1.2

- Solve $ax^2+bx+c\leq 0$ by finding the roots and sketching the parabola.
- Values between the roots if $a<0$; outside the roots if $a>0$.
- Cubic inequalities: factor and use sign-table reasoning.

Polynomials & Function Sketching in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Factor theorem

$(x - a)$ is a factor of $p(x)$ iff $p(a) = 0$.

$(x + a)$ is a factor iff $p(-a) = 0$.

To check whether $(ax - b)$ is a factor: test $p(b/a) = 0$.

Remainder theorem

When $p(x)$ is divided by $(x - a)$, the remainder is $p(a)$.

When divided by $(ax - b)$, the remainder is $p(b/a)$.

Use to find an unknown coefficient: set $p(a) =$ given remainder.

Polynomial long division

Divide $p(x)$ by a linear factor $(x - a)$ to get a quadratic quotient.

Result: $p(x) = (x - a) \cdot Q(x) + R$ where $R = p(a)$.

Solve $Q(x) = 0$ to find the remaining roots.

Solving cubic equations

1. Spot a rational root using the factor theorem.
2. Divide $p(x)$ by $(x - \text{root})$ to get a quadratic.
3. Solve the quadratic using formula or factorisation.

Graph transformations (vertical)

$y = f(x) + a$: shift up by a.

$y = kf(x)$: stretch vertically by k (reflection in x-axis if $k < 0$).

Stationary points and x-intercepts move accordingly.

Graph transformations (horizontal)

$y = f(x + a)$: shift left by a.

$y = f(kx)$: stretch horizontally by $1/k$ (reflection in y-axis if $k < 0$).

y-coordinates of stationary points stay the same.

Quadratic inequalities

Solve $ax^2 + bx + c \leq 0$:

1. Find the roots of $ax^2 + bx + c = 0$.
2. Sketch the parabola.
3. Read off values where the curve is below the x-axis.

Polynomial inequalities

Factor the polynomial into linear factors.

Use a sign table: test sign in each interval between roots.

Combine into the answer in interval notation.

Strategy

1. Factor theorem ⇒ find a root.
2. Long division ⇒ quadratic factor.
3. Quadratic formula ⇒ remaining roots.
4. Sketches: roots, y-intercept, end behaviour.

SECTION T2

Polynomials & Function Sketching

Questions 1–21 · 128 marks

8. The polynomial $px^3 - x^2 - 31x + q$ has $x + 2$ as a factor. When the polynomial is divided by $x - 1$, the remainder is -36.

(a) Show that $p = 6$ and $q = -10$. [6]

(b) Factorise $6x^3 - x^2 - 31x - 10$. [3]

TURN OVER

9. Figure 1 shows a sketch of the graph of $y = f(x)$. The graph has a minimum point at $(-3, -4)$ and intersects the x-axis at the points $(-8, 0)$ and $(2, 0)$.

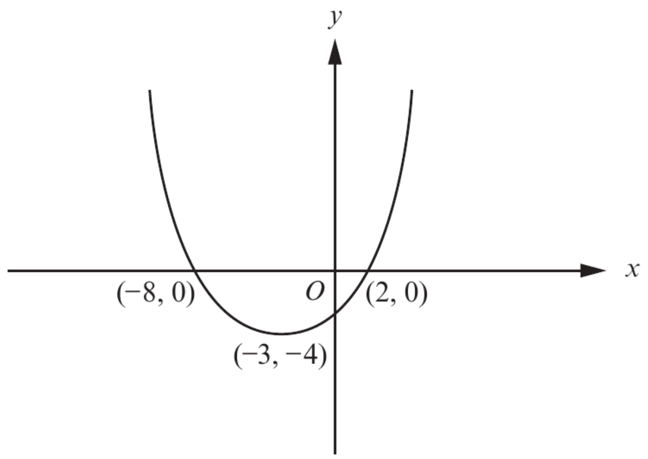


Figure 1

(a) Sketch the graph of $y = f(x + 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(b) Figure 2 shows a sketch of the graph having **one** of the following equations with an appropriate value of either p, q or r.

$y = f(px)$, where p is a constant
$y = f(x) + q$, where q is a constant
$y = rf(x)$, where r is a constant.

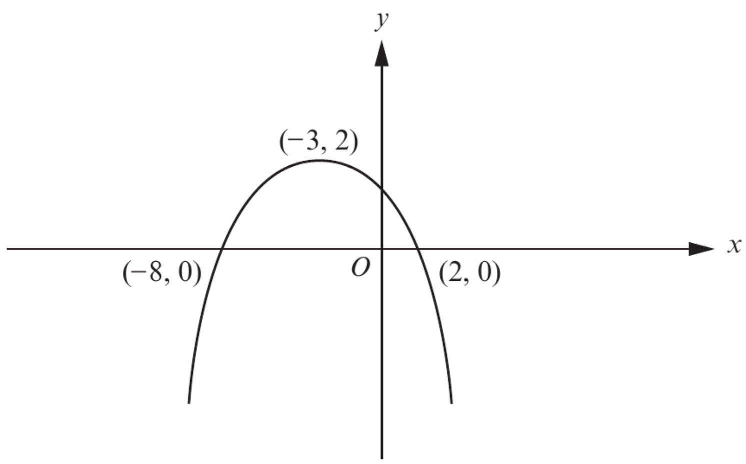


Figure 2

Write down the equation of the graph sketched in Figure 2, together with the value of the corresponding constant. [2]

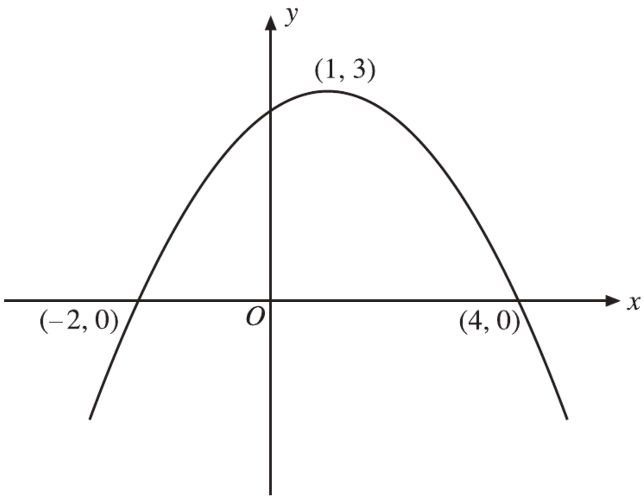
5

8. *(a)* When $ax^3 - 21x - 10$ is divided by $x - 3$, the remainder is 35.
Write down an equation satisfied by a and hence show that $a = 4$. [2]

(b) Factorise $4x^3 - 21x - 10$. [5]

TURN OVER

9. The diagram shows a sketch of the graph of $y = f(x)$. The graph has a maximum point at (1, 3) and intersects the x-axis at the points (–2, 0) and (4, 0).



(a) Sketch the graph of $y = f(2x)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(b) (i) Sketch the graph of $y = f(x) – 5$, indicating the coordinates of the stationary point.

(ii) Given that f is a quadratic function, use the graph you have drawn in part (i) to write down the number of real roots of the equation

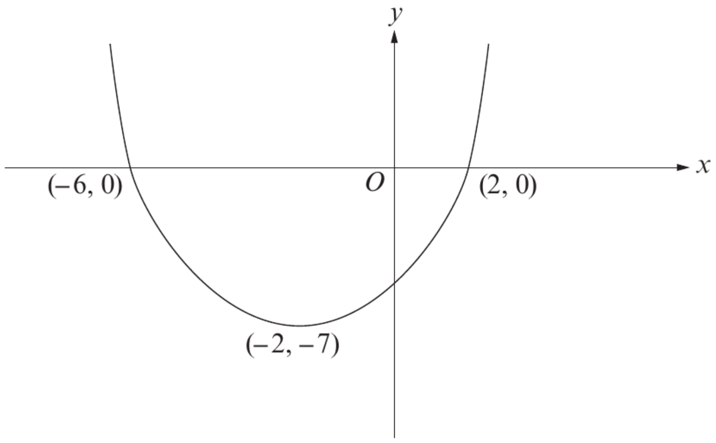
$$f(x) - 5 = 0.$$ [3]

8. *(a)* Solve the equation $6x^3 - 19x^2 + 11x + 6 = 0$. [6]

(b) When $x^3 - 53$ is divided by $x - a$, the remainder is 11. Find the value of the constant a. [2]

TURN OVER

9. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-6, 0)$ and $(2, 0)$ and has a minimum point at $(-2, -7)$.



Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

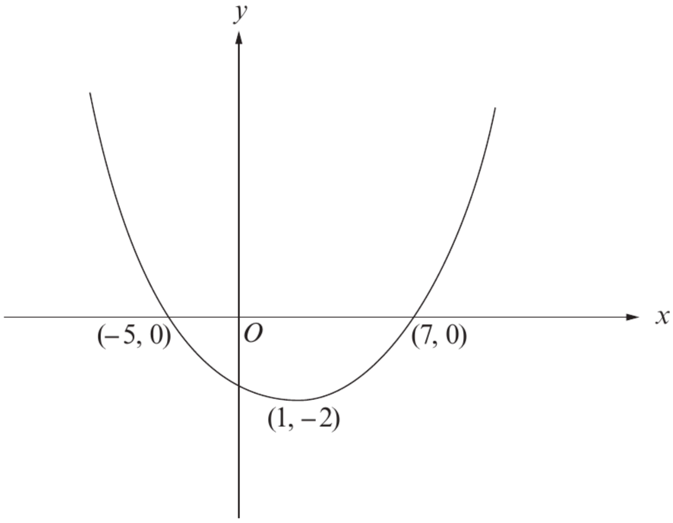
(a) $y = f(x - 5)$ [3]

(b) $y = f\left(\frac{x}{2}\right)$ [3]

8. *(a)* Given that $x + 2$ is a factor of $px^3 + 18x^2 - 4x - 8$, write down an equation satisfied by p. Hence show that $p = 9$. [2]

(b) Solve the equation $9x^3 + 18x^2 - 4x - 8 = 0$. [4]

9. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-5, 0)$ and $(7, 0)$ and has a minimum point at $(1, -2)$.



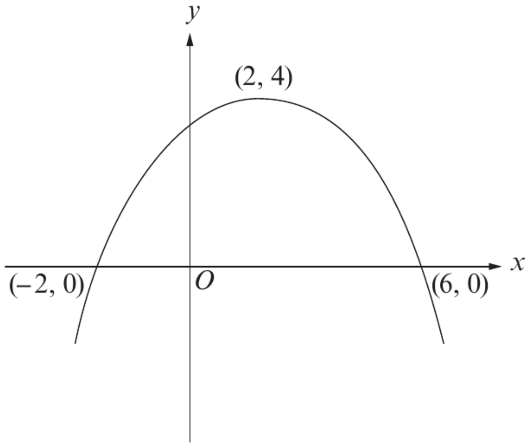
Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

(a) $y = 3f(x)$ [3]

(b) $y = f(-x)$ [3]

8. Solve the equation $8x^3 - 2x^2 - 7x + 3 = 0$. [6]

9. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-2, 0)$ and $(6, 0)$ and has a maximum point at $(2, 4)$.



Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

(a) $y = f(x + 5)$ [3]

(b) $y = f(-2x)$ [3]

TURN OVER

7. **Figure 1** shows a sketch of the graph of $y = f(x)$. The graph has a maximum point at (2, 6) and intersects the x-axis at the points (–4, 0) and (8, 0).

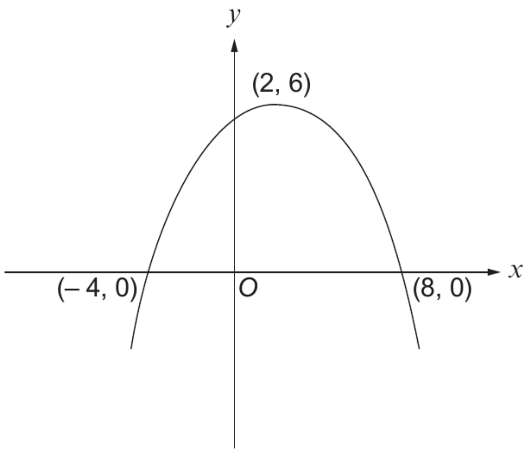


Figure 1

(a) Sketch the graph of $y = f(x - 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(b) **Figure 2** shows a sketch of the graph having **one** of the following equations with an appropriate value of p, q or r.

$y = f(x) + p$, where p is a constant
$y = f(qx)$, where q is a constant
$y = rf(x)$, where r is a constant

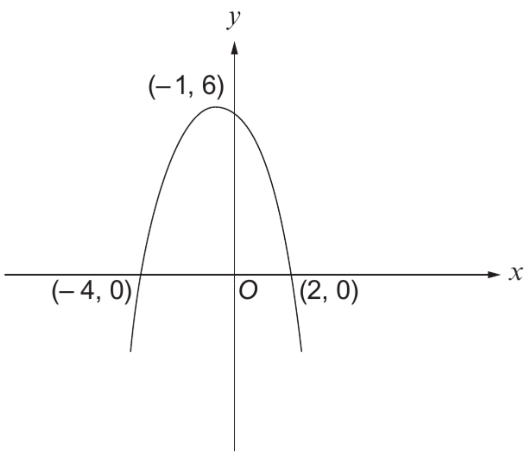


Figure 2

Write down the equation of the graph sketched in **Figure 2**, together with the value of the corresponding constant. [2]

TURN OVER

4

1

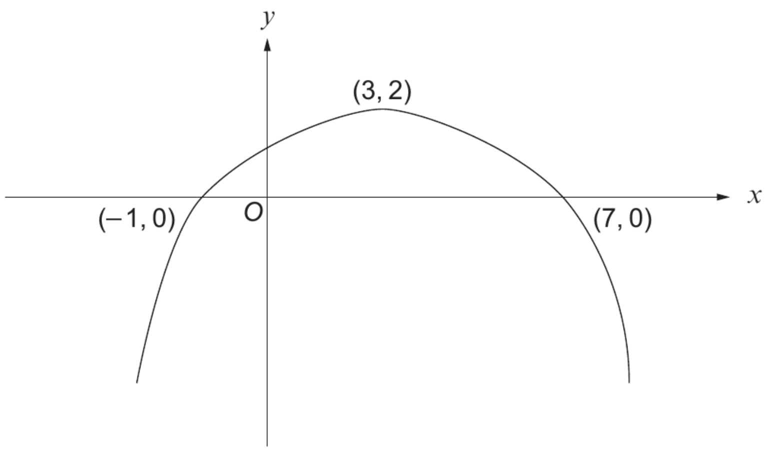
9. *(a)* When $ax^3 + 13x^2 - 10x - 24$ is divided by $x + 3$, the remainder is -39.

Write down an equation satisfied by a and hence show that $a = 6$. [2]

(b) Solve the equation $6x^3 + 13x^2 - 10x - 24 = 0$. [6]

8. Solve the equation $6x^3 - 13x^2 + 4 = 0$. [6]

9. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-1, 0)$ and $(7, 0)$ and has a maximum point at $(3, 2)$.



(a) Sketch the following graphs, using a separate set of axes for each graph. In each case, you should indicate the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis.

(i) $y = f(x + 4)$

(ii) $y = -2f(x)$ [6]

(b) Hence write down one root of the equation

$$f(x + 4) = -2f(x) + 4.$$ [1]

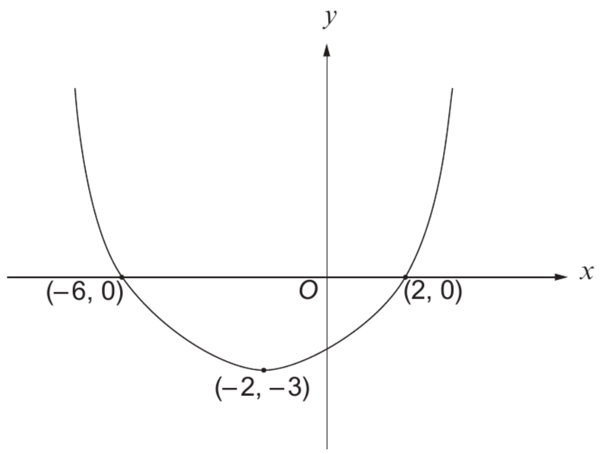
TURN OVER

8. *(a)* Given that $x - 3$ is a factor of $px^3 - 13x^2 - 19x + 12$, write down an equation satisfied by p. Hence show that $p = 6$. [2]

(b) Solve the equation $6x^3 - 13x^2 - 19x + 12 = 0$. [4]

TURN OVER

9. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-6, 0)$ and $(2, 0)$ and has a minimum point at $(-2, -3)$.



(a) Sketch the graph of $y = f(\frac{1}{2}x)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(b) Angharad is asked by her teacher to draw the graph of $y = af(x)$ for various non-zero values of the constant a. One of Angharad's graphs passes through the origin *O*. Explain why this cannot possibly be correct. [1]

7. Figure 1 shows a sketch of the graph of $y = f(x)$. The graph has a minimum point at (1, –3) and intersects the x-axis at the points (– 4, 0) and (6, 0).

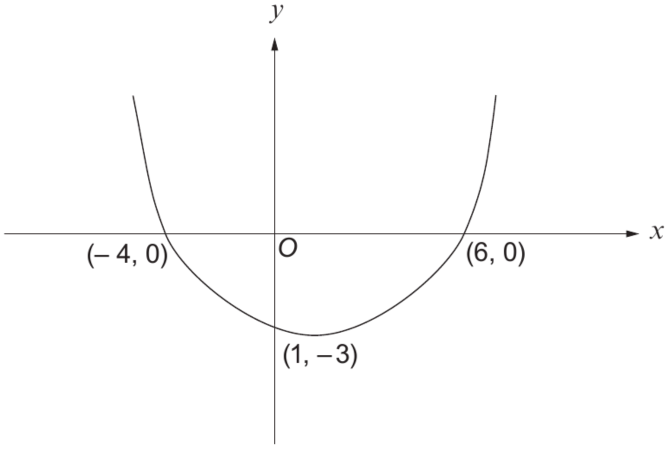


Figure 1

(a) Sketch the graph of $y = -3f(x)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(b) Figure 2 shows a sketch of the graph of $y = g(x)$, where
$g(x) = f(x) + p$, where p is a constant,
or $g(x) = f(qx)$, where q is a constant,
or $g(x) = rf(x)$, where r is a constant,
or $g(x) = f(x + s)$, where s is a constant.

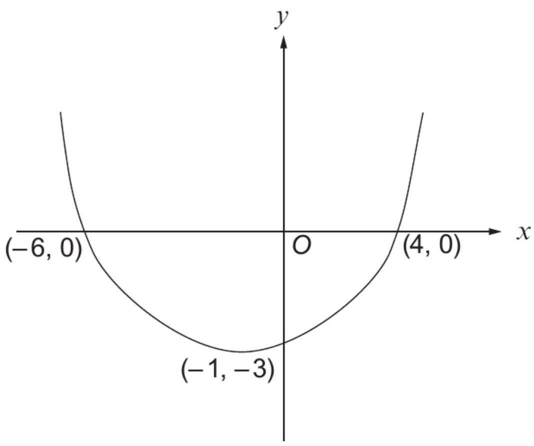


Figure 2

The function g can in fact be any one of **two** of the above functions. In each of these two cases, write down the expression for $g(x)$, including the value of the corresponding constant. [2]

TURN OVER

4

dy

9. The polynomial $f(x)$ is given by

$$f(x) = 8x^3 + 2x^2 - 41x + 10.$$

(a) Factorise $f(x)$. [5]

(b) Hence or otherwise, evaluate $f(2{\cdot}25)$. [2]

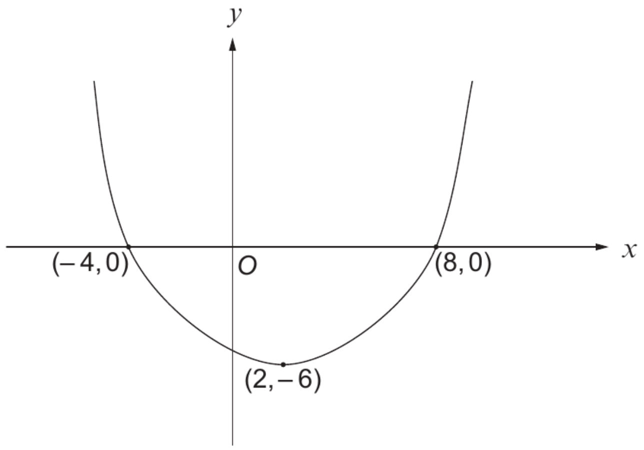
6. Solve the inequality $2x^2 + 11x + 12 \geqslant 0$. [3]

7. *(a)* Given that $x - 2$ is a factor of $kx^3 + 2x^2 - 41x + 10$, write down an equation satisfied by k. Hence show that $k = 8$. [2]

(b) Factorise $8x^3 + 2x^2 - 41x + 10$. [3]

(c) Find the remainder when $8x^3 + 2x^2 - 41x + 10$ is divided by $2x + 1$. [2]

8. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points (–4, 0) and (8, 0) and has a minimum point at (2, –6).



(a) Sketch the graph of $y = -\frac{1}{2}f(x)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(b) Siân is asked by her teacher to draw the graph of $y = f(ax)$ for various non-zero values of the constant a. Write down two facts about the stationary point on Siân's graph which will always be true whatever her choice of a. [2]

TURN OVER

4

END OF POLYNOMIALS & FUNCTION SKETCHING PACK

Source: WJEC C1 + C2 (2008 modular spec) · 2011–2017

Curated for WJEC Maths 2017 spec AS Unit 1 — Topic 2 (2.1.2 + 2.1.3)