Name	Date started	Target end date

GCE AS / A LEVEL — PURE MATHEMATICS A QUESTION PACK

0974-01 (Legacy C2) · New spec Unit 1 Topic 13 · AS unit, 25% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS — PURE A · INTEGRATION

Integration

Indefinite / definite integrals, area under a curve and the trapezium rule from the legacy WJEC C2 papers (June 2011 – June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~3 hours 5 minutes

*Derived from the legacy C1/C2 paper's pace of **~1.25 min/mark** (148 marks over 20 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **single-topic practice question pack**, drilling one narrow new-spec sub-topic. It contains questions from the legacy WJEC C1 and C2 papers (2008 modular spec) that map onto new-spec AS Unit 1 Topic 13 (2.1.8).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q1	4	
2	Jun 11 Q6	13	
3	Jan 12 Q1	4	
4	Jan 12 Q6	10	
5	Jun 12 Q1	4	
6	Jun 12 Q6	12	
7	Jan 13 Q1	4	
8	Jan 13 Q6	12	
9	Jun 13 Q1	4	
10	Jun 13 Q6	11	

Q	Source	Max	Mark
11	Jan 14 Q1	4	
12	Jan 14 Q6	7	
13	Jun 14 Q1	5	
14	Jun 14 Q6	12	
15	Jun 15 Q1	4	
16	Jun 15 Q6	11	
17	Jun 16 Q1	4	
18	Jun 16 Q6	7	
19	Jun 17 Q1	4	
20	Jun 17 Q6	12	
Total		**148**	

Integration — what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 1: Pure Mathematics A · Topic 2.1.8.

Indefinite integration 2.1.8

- $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C$ for $n \neq -1$.
- Linearity: $\int (f+g)\,dx = \int f + \int g$; $\int cf\,dx = c\int f$.
- Always add the constant of integration C on indefinite integrals.

Definite integration 2.1.8

- $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a)$ where F is any antiderivative.
- The constant of integration cancels in the subtraction.
- Substitute top limit first, then bottom — sign matters.

Area under a curve 2.1.8

- Area from $x = a$ to $x = b$ between $y = f(x)$ and the x-axis: $\int_a^b f(x)\,dx$ (for $f \geq 0$).
- If $f < 0$ on part of the interval, split into pieces and take absolute values.
- Area between two curves: $\int_a^b (\text{upper} - \text{lower})\,dx$.

Trapezium rule 2.1.8

- $\int_a^b f(x)\,dx \approx \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$.
- Step width $h = (b-a)/n$ where n is the number of strips.
- Ordinates $y_i = f(a + ih)$ for $i = 0, 1, \ldots, n$. Five ordinates $\Rightarrow n = 4$ strips.

Crib sheet »

Integration in one page

Quick-reference notes — revisit before each section. Don’t use during questions.

Anti-differentiation (power rule)

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$.

$\int (f+g)\,dx = \int f + \int g$; $\int cf\,dx = c\int f$.

Don't forget the constant C.

Integrating fractional / negative powers

Rewrite first:

$1/x^n = x^{-n}$, $\sqrt[k]{x} = x^{1/k}$.

Then apply the power rule. Special: $\int 1/x\,dx$ is not handled here (that's $A2$).

Definite integral

$\int_a^b f(x)\,dx = F(b) - F(a)$ where F is any antiderivative.

The constant of integration cancels.

Substitute the top limit first, then the bottom.

Area under a curve

Area between $y = f(x)$, the x-axis, $x = a$, $x = b$ (when $f \geq 0$): $\int_a^b f(x)\,dx$.

If $f < 0$ on part of the interval, split and take absolute values.

Area between two curves

$\text{Area} = \int_a^b (\text{upper} - \text{lower})\,dx$.

Identify intersection points a and b from the curves' equations.

Sketch first to confirm which curve is on top.

Area bounded by a curve and a line

Find intersection points by setting the curve equal to the line.

Set up $\int (\text{line} - \text{curve})\,dx$ where the line is above.

Evaluate.

Trapezium rule

$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$.

Step width $h = (b-a)/n$; n strips.

Ordinates: $y_i = f(a + ih)$.

Trapezium rule with five ordinates

Five ordinates means $n = 4$ strips.

Compute y_0, y_1, y_2, y_3, y_4 at evenly spaced x values.

Plug into the rule and evaluate.

Strategy

1. Indefinite: anti-differentiate using power rule, add $+C$.
2. Definite: $F(b) - F(a)$.
3. Area: sketch first, set up integral, evaluate.
4. Trapezium rule: tabulate ordinates, apply the formula.

SECTION T13

Integration

Questions 1–20 · 148 marks

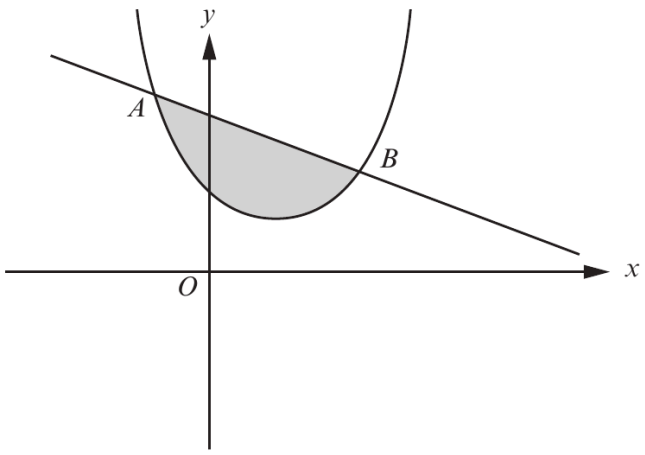
1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_{1\cdot6}^{2} \frac{1}{9-x^3}\,\mathrm{d}x\,.$$

Show your working and give your answer correct to three decimal places. [4]

6. *(a)* Find $\int \left(\sqrt[3]{x} - \frac{2}{x^{\frac{3}{4}}} \right) \mathrm{d}x$. [2]

(b)



The diagram shows a sketch of the curve $y = x^2 - 4x + 6$ and the line $y = -x + 10$.
The curve and the line intersect at the points A and B.

(i) Showing your working, find the coordinates of A and B.

(ii) Find the area of the shaded region.

[11]

TURN OVER

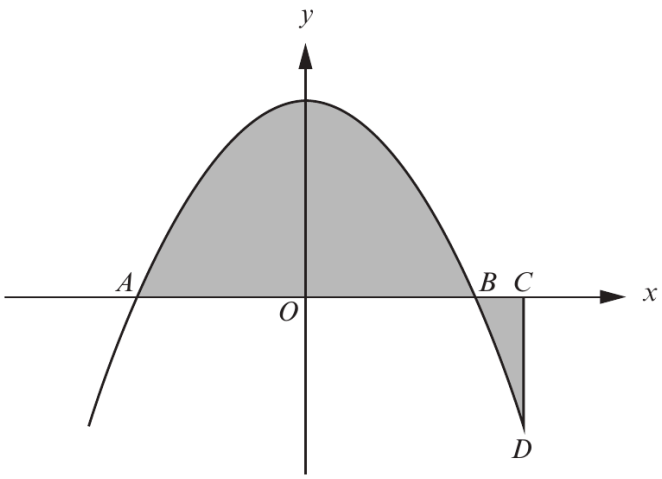
1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_1^3 \frac{x}{1+\sqrt{x}}\,\mathrm{d}x.$$

Show your working and give your answer correct to three decimal places. [4]

6. *(a)* Find $\int \left(\frac{4}{x^3} - 3x^{\frac{1}{4}} \right) \mathrm{d}x.$ [2]

(b)



The diagram shows a sketch of the curve $y = 4 - x^2$.
The curve intersects the x-axis at the points A and B. The point C has coordinates (3, 0).
The point D lies on the curve and CD is parallel to the y-axis.

(i) Showing your working, find the x-coordinates of the points A and B. [2]

(ii) Find the **total** area of the shaded regions. [6]

TURN OVER

4

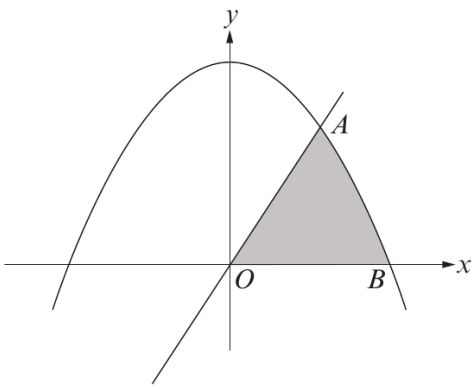
1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_1^2 \frac{1}{\sqrt{5-x^2}}\,\mathrm{d}x.$$

Show your working and give your answer correct to four decimal places. [4]

6. *(a)* Find $\int \left(3\sqrt{x} - \frac{2}{x^{\frac{5}{3}}}\right) \mathrm{d}x$. [2]

(b)



The diagram shows a sketch of the curve $y = 36 - x^2$ and the line $y = 5x$. The curve and the line intersect at the point A in the first quadrant and the curve intersects the positive x-axis at the point B.

(i) Showing your working, find the coordinates of A and the coordinates of B.

(ii) Find the area of the shaded region. [10]

TURN OVER

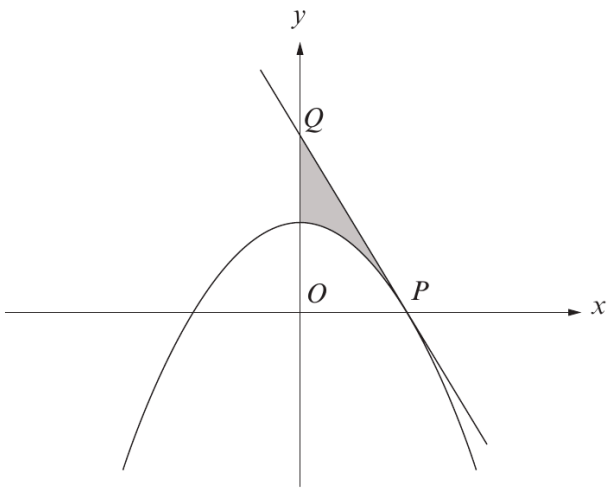
1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_0^2 \sqrt{10 - x^3}\, \mathrm{d}x.$$

Show your working and give your answer correct to four decimal places. [4]

6. *(a)* Find $\int\left(\frac{5}{x^4} - 7x^{\frac{2}{3}}\right)\mathrm{d}x$. [2]

(b)



The diagram shows a sketch of the curve $y = 9 - x^2$ which intersects the positive x-axis at the point $P(a, 0)$.

(i) Find the value of a.

The tangent to the curve at P intersects the y-axis at the point $Q(0, b)$.

(ii) Show that $b = 18$.

(iii) Find the area of the shaded region. [10]

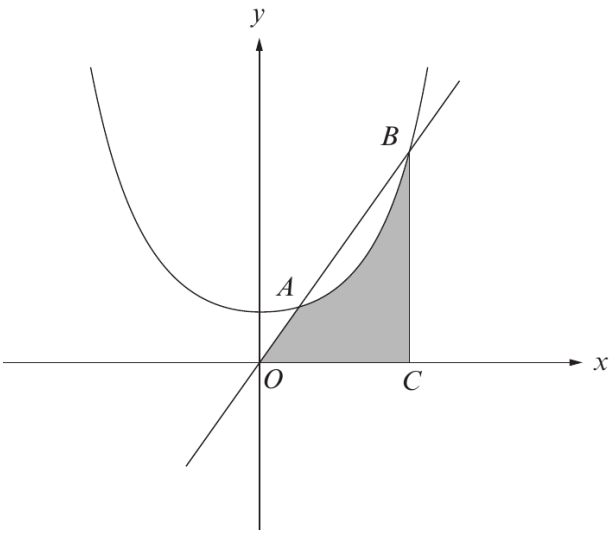
1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_0^2 \frac{1}{2+x^3}\,\mathrm{d}x.$$

Show your working and give your answer correct to three decimal places. [4]

6. *(a)* Find $\int \left(\sqrt[4]{x} + \frac{2}{x^5} \right) \mathrm{d}x$. [2]

(b)



The diagram shows a sketch of the curve $y = x^2 + 3$ and the line $y = 4x$. The curve and the line intersect at the points A and B.
The line BC is parallel to the y-axis.

(i) Showing your working, find the x-coordinates of A and B.

(ii) Find the area of the shaded region. [9]

TURN OVER

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

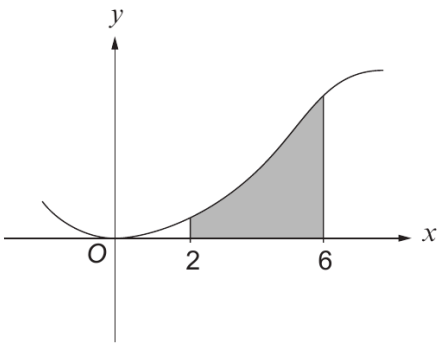
$$\int_2^4 \sqrt{1+\frac{6}{x}}\,\mathrm{d}x.$$

Show your working and give your answer correct to three decimal places. [4]

6. *(a)* Find $\int\left(\frac{5}{x^3} - 2x^{\frac{1}{3}} - 4\right)\mathrm{d}x.$ [3]

(b) The diagram below shows a sketch of the curve with equation $y = 3x^2 - \frac{1}{4}x^3$.

The shaded region is bounded by the curve, the x-axis and the lines $x = 2$, $x = 6$.
Find the area of this shaded region. [4]



TURN OVER

4

1. *(a)* Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_1^3 \log_{10}(3x-1)\,\mathrm{d}x.$$

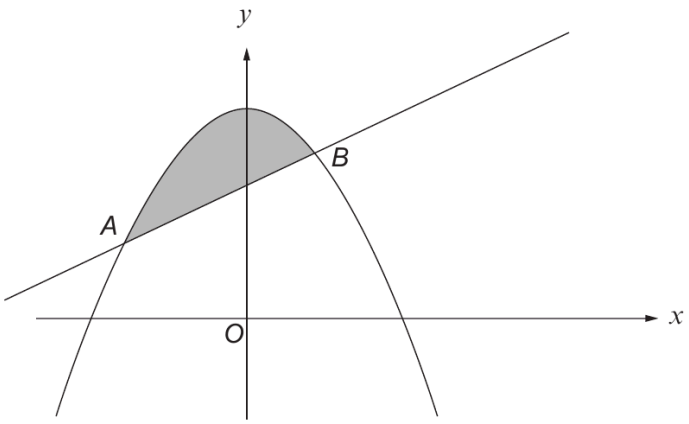
Show your working and give your answer correct to three decimal places. [4]

(b) **Use your answer to part *(a)*** to deduce an approximate value for the integral

$$\int_1^3 \log_{10}(3x-1)^2\,\mathrm{d}x.$$ [1]

6. *(a)* Find $\int \left(\frac{5}{x^{\frac{3}{4}}} - 7\sqrt{x} \right) \mathrm{d}x.$ [2]

(b)



The diagram shows a sketch of the curve $y = 16 - x^2$ and the line $y = x + 10$. The line and the curve intersect at the points *A* and *B*.

(i) Find the coordinates of *A* and *B*.

(ii) Find the area of the shaded region. [10]

TURN OVER

4

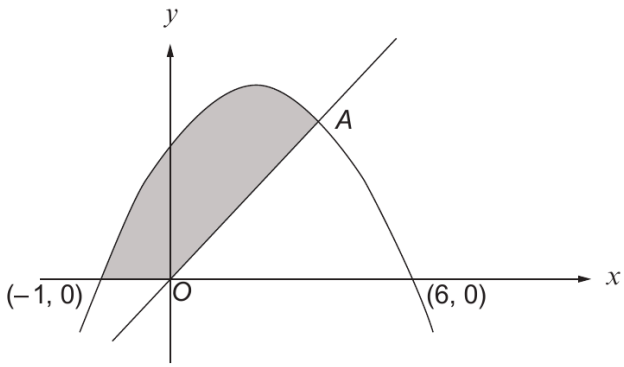
1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_1^3 \frac{x}{10-\sqrt{x}}\,\mathrm{d}x.$$

Show your working and give your answer correct to four decimal places. [4]

6. *(a)* Find $\int\left(\frac{3}{\sqrt{x}} - 6x^{\frac{4}{3}}\right)\mathrm{d}x.$ [2]

(b)



The diagram shows a sketch of the curve $y = 6 + 5x - x^2$ and the line $y = 4x$. The curve and the line intersect at the point *A* in the first quadrant and the curve intersects the x-axis at the points (–1, 0) and (6, 0).

(i) Showing your working, find the x-coordinate of *A*.

(ii) Find the area of the shaded region. [9]

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_3^6 \frac{7-\sqrt{x}}{7+\sqrt{x}}\,\mathrm{d}x\,.$$

Show your working and give your answer correct to three decimal places. [4]

6. *(a)* Find $\int\left(\frac{3}{\sqrt[4]{x}} - 9x^{\frac{5}{2}}\right)\mathrm{d}x$. [2]

(b) The region *R* is bounded by the curve $y = 2x^2 + \frac{6}{x^2}$, the x-axis and the lines $x = 1$, $x = 4$. Find the area of *R*. [5]

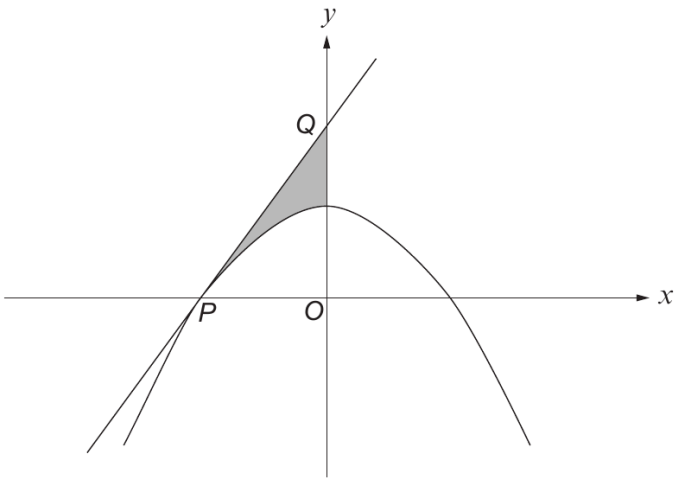
1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_0^2 \sqrt{7 - x^2}\, dx\,.$$

Show your working and give your answer correct to three decimal places. [4]

6. *(a)* Find $\int\left(\frac{2}{x^5} - 6x^{\frac{3}{4}}\right)dx$. [2]

(b)



The diagram shows a sketch of the curve $y = 16 - x^2$ which intersects the negative x-axis at the point $P(a, 0)$.

(i) Write down the value of a.

The tangent to the curve at P intersects the y-axis at the point $Q(0, b)$.

(ii) Show that $b = 32$.

(iii) Find the area of the shaded region. [10]

END OF INTEGRATION PACK

Source: WJEC C1 + C2 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec AS Unit 1 — Topic 13 (2.1.8)