Name	Date started	Target end date

GCE AS / A LEVEL — PURE MATHEMATICS A QUESTION PACK

0973-01 (Legacy C1) · New spec Unit 1 Topic 12 · AS unit, 25% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS — PURE A · DIFFERENTIATION

Differentiation

Differentiation from first principles, tangents / normals to curves, stationary points and optimisation from the legacy WJEC C1 papers (June 2011 - June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~3 hours 48 minutes

*Derived from the legacy C1/C2 paper's pace of **~1.25 min/mark** (182 marks over 24 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C1 and C2 papers (2008 modular spec) that maps onto new-spec AS Unit 1 Topic 12 (2.1.7).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q3	5	
2	Jun 11 Q10	7	
3	Jan 12 Q7	9	
4	Jan 12 Q10	6	
5	Jun 12 Q7	7	
6	Jun 12 Q10	9	
7	Jan 13 Q3	5	
8	Jan 13 Q6	7	
9	Jan 13 Q10	10	
10	Jun 13 Q3	7	
11	Jun 13 Q7	7	
12	Jun 13 Q10	8	

Q	Source	Max	Mark
13	Jan 14 Q3	6	
14	Jan 14 Q8	9	
15	Jan 14 Q10	10	
16	Jun 14 Q3	9	
17	Jun 14 Q7	7	
18	Jun 14 Q10	7	
19	Jun 15 Q7	7	
20	Jun 15 Q10	7	
21	Jun 16 Q8	9	
22	Jun 16 Q10	7	
23	Jun 17 Q9	7	
24	Jun 17 Q10	10	
Total		182	

Differentiation — what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 1: Pure Mathematics A · Topic 2.1.7.

Differentiation from first principles 2.1.7

- $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$.
- Expand $f(x+h)$, subtract, divide by h, take the limit.
- For $f(x) = ax^2 + bx + c$: $f'(x) = 2ax + b$.

Power rule & combinations 2.1.7

- $\frac{d}{dx}(x^n) = nx^{n-1}$ for all real n.
- $\frac{d}{dx}(c) = 0$; $(cf)' = cf'$; $(f+g)' = f' + g'$.
- Rewrite $1/x^n$ as x^{-n} and $\sqrt[n]{x}$ as $x^{1/n}$ before differentiating.

Tangents and normals 2.1.7

- Tangent at $x = a$ has gradient $f'(a)$; line: $y - f(a) = f'(a)(x - a)$.
- Normal: gradient $-1/f'(a)$; line: $y - f(a) = (-1/f'(a))(x - a)$.
- Find specific points where tangent has given gradient: solve $f'(x) = m$.

Stationary points & optimisation 2.1.7

- Solve $f'(x) = 0$ to locate stationary points.
- Classify using f'': $f'' > 0$ minimum, $f'' < 0$ maximum, $f'' = 0$ inconclusive.
- Optimisation: express the quantity to be optimised in one variable, differentiate, solve.

Differentiation in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Differentiation from first principles

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Steps: compute $f(x+h)$, subtract $f(x)$, divide by h, simplify, let $h \to 0$.

For $f(x) = ax^2 + bx + c$: $f'(x) = 2ax + b$.

Power rule

$\frac{d}{dx}(x^n) = nx^{n-1}$ for all real n.

$\frac{d}{dx}(c) = 0$; $(cf)' = cf'$; $(f+g)' = f' + g'$.

Differentiating fractional / negative powers

Rewrite first:

$1/x^n = x^{-n}$, $\sqrt[k]{x} = x^{1/k}$.

Then apply the power rule.

Tangent at a point

Gradient at $x = a$: $m = f'(a)$.

Tangent line: $y - f(a) = m(x - a)$.

To find a point where the tangent has a given gradient, solve $f'(x) = m$.

Normal at a point

Normal is perpendicular to the tangent.

Normal gradient: $-1/f'(a)$.

Normal line: $y - f(a) = (-1/f'(a))(x - a)$.

Stationary points

Solve $f'(x) = 0$ to locate stationary points.

Classify with f'':

- $f''(a) > 0$: minimum
- $f''(a) < 0$: maximum
- $f''(a) = 0$: inconclusive (use sign change of f')

Points of inflection

If $f'(a) = 0$ and the sign of f' doesn't change either side, it's a horizontal inflection.

Check sign of f' at $a - \epsilon$ and $a + \epsilon$ to confirm.

Optimisation

1. Write the quantity to be optimised in one variable (use a constraint).
2. Differentiate, set to zero, solve.
3. Check it's a maximum / minimum using f''.
4. Substitute back to find the optimal value.

Strategy

1. For first principles: expand, subtract, divide, take the limit.
2. For tangents / normals: gradient at the point, point-gradient form.
3. For stationary points: $f' = 0$, classify with f''.
4. For optimisation: express in one variable, differentiate, check.

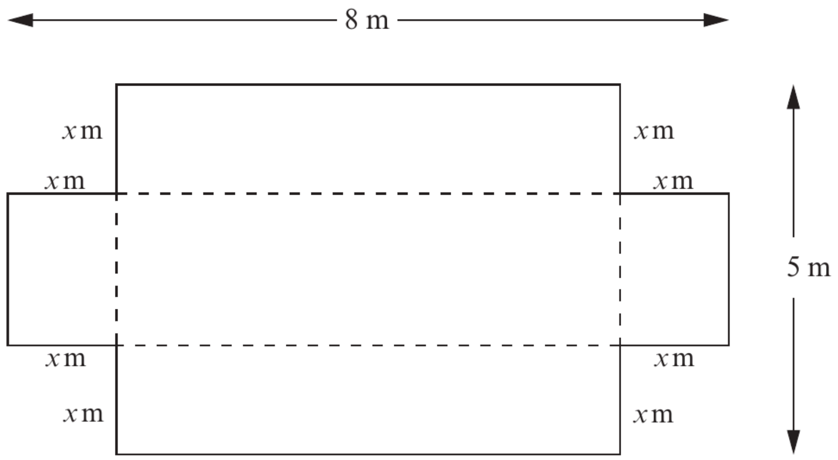
SECTION T12

Differentiation

Questions 1–24 · 182 marks

3. The curve C has equation $y = 3x^2 - 9x + 1$. The point P, whose x-coordinate is 2, lies on the curve C. Find the equation of the tangent to C at P. [5]

10. A rectangular sheet of metal has length 8 m and width 5 m. Four squares, each of side x m, where $x < 2{\cdot}5$, have been cut away from the corners of the rectangular sheet, as shown in the diagram below. The rest of the metal sheet is now bent along the dotted lines to form an open tank in the form of a cuboid.



(a) Show that the volume V m^3 of this tank is given by

$$V = 4x^3 - 26x^2 + 40x\ .$$ [2]

(b) Find the maximum value of V, showing that the value you have found is a maximum value. [5]

7. *(a)* Given that $y = 8x^2 - 5x - 6$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ from first principles. [5]

(b) Given that $y = \frac{a}{x} + 10\sqrt{x}$ and that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3$ when $x = 4$,

find the value of the constant a. [4]

10. The curve C has equation

$$y = x^3 - 6x^2 + 12x - 9.$$

(a) Show that C has only one stationary point. Find the coordinates of this point. [4]

(b) Verify that this stationary point is a point of inflection. [2]

7. *(a)* Given that $y = 3x^2 - 7x + 5$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Differentiate $\frac{2}{3}x^{\frac{1}{4}} + \frac{12}{x^3}$ with respect to x. [2]

10. The curve C has equation

$$y = x^3 + 3x^2 - 1.$$

(a) Find the coordinates and the nature of each of the stationary points of C. [6]

(b) Sketch C, indicating the coordinates of each of the stationary points. [2]

(c) Write down the number of **positive** real roots of the equation

$$x^3 + 3x^2 - 1 = 0.$$ [1]

3. The curve C has equation $y = 3x^2 - 14x + 13$. The point P, whose x-coordinate is 3, lies on the curve C. Find the equation of the **tangent** to C at P. [5]

6. *(a)* Given that $y = -x^2 + 4x - 6$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ from first principles. [5]

(b) Differentiate $5x^{\frac{4}{3}} - \frac{9}{\sqrt{x}}$ with respect to x. [2]

10. The curve C has equation

$$y = x^3 - 3x^2 - 9x + 14.$$

(a) Find the coordinates and the nature of each of the stationary points of C. [6]

(b) Sketch C, indicating the coordinates of each of the stationary points. [2]

(c) Given that the equation

$$x^3 - 3x^2 - 9x + 14 = k$$

has only one real root, find the range of possible values for k. [2]

3. The curve C has equation $y = 2x^2 - 10x + 7$.

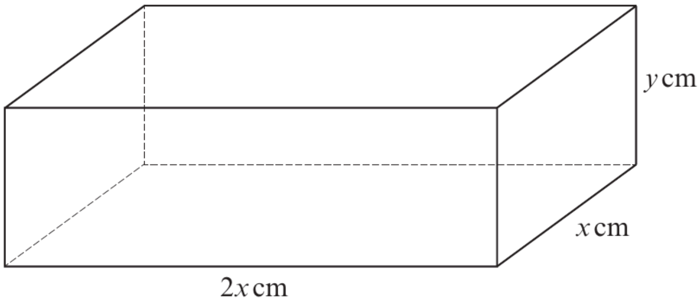
(a) The point P has coordinates $(3, -5)$ and lies on C. Find the equation of the **normal** to C at P. [5]

(b) The point Q lies on C and is such that the **tangent** to C at Q is parallel to the x-axis. Find the x-coordinate of Q. [2]

7. *(a)* Given that $y = 5x^2 + 8x - 11$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Differentiate $6x^{\frac{2}{3}} + \frac{5}{x^2} - 4$ with respect to x. [2]

10. The diagram shows a **closed** box in the form of a cuboid. The length of the box is $2x$ cm, its width is x cm and its height is y cm.



The total surface area of the box is $108\,\text{cm}^2$.

(a) (i) Write down an equation involving x and y and hence show that

$$xy = 18 - \frac{2}{3}x^2 .$$

(ii) Hence show that the volume $V\,\text{cm}^3$ of the box is given by

$$V = 36x - \frac{4}{3}x^3 .$$

[3]

(b) Find the maximum value of V, showing that the value you have found is a maximum value. [5]

3. The curve *C* has equation $y = \frac{20}{x} + 2x^2 - 11$. The point *P* has coordinates (2, 7) and lies on *C*.

Find the equation of the **normal** to *C* at *P*. [6]

8. *(a)* Given that $y = 7x^2 - 6x - 3$, find $\dfrac{dy}{dx}$ from first principles. [5]

(b) Given that $y = ax^{\frac{4}{3}} + 24x^{\frac{1}{2}}$ and that $\dfrac{dy}{dx} = \dfrac{11}{2}$ when $x = 64$,

find the value of the constant a. [4]

10. The curve C has equation

$$y = -2x^3 + 12x^2 - 18x + 5.$$

(a) Find the coordinates and the nature of each of the stationary points of C. [6]

(b) Sketch C, indicating the coordinates of each of the stationary points. [2]

(c) Given that the equation

$$-2x^3 + 12x^2 - 18x + 5 = k$$

has three distinct real roots, find the range of possible values for k. [2]

3. The curve *C* has equation $y = x^2 - 8x + 14$.

(a) The point *P* has coordinates (6, 2) and lies on the curve *C*. Find the equation of the **normal** to *C* at *P*. [5]

(b) The point *Q* lies on *C* and is such that the **tangent** to *C* at *Q* has equation

$$y = 2x + c,$$

where c is a constant. Find the coordinates of Q and the value of c. [4]

7. *(a)* Given that $y = -3x^2 + 8x - 7$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Differentiate $9x^{\frac{5}{4}} - \frac{8}{\sqrt[3]{x}}$ with respect to x. [2]

10. The curve C has equation

$$y = x^3 + 9x^2 + 27x + 31.$$

(a) Show that C has only one stationary point. Find the coordinates of this point. [4]

(b) Verify that this stationary point is a point of inflection. [2]

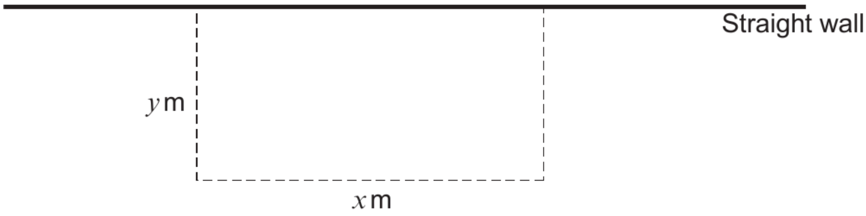
(c) Sketch the graph of C, indicating the coordinates of its stationary point. [1]

END OF PAPER

7. *(a)* Given that $y = 9x^2 - 8x - 3$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Differentiate $\frac{3}{x^6} - 4x^{\frac{5}{3}}$ with respect to x. [2]

10. A sheep farmer wishes to construct a rectangular enclosure for his animals. He decides to use a straight wall as one side of the enclosure and fencing for the other three sides. The area of the enclosure is to be 800 m². The lengths of the sides of the rectangular enclosure are x m and y m, as shown in the diagram, and the total length of the **fencing** is L m.



(a) Show that $L = x + \dfrac{1600}{x}$. [2]

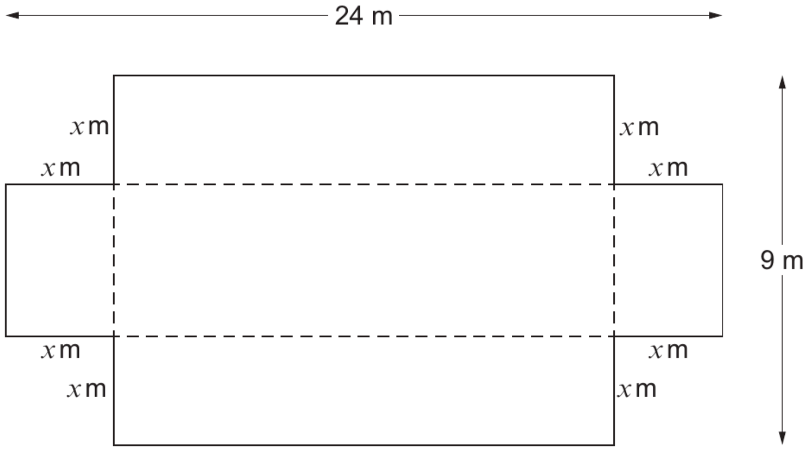
(b) Find the minimum value of L, showing that the value you have found is a minimum value. [5]

END OF PAPER

8. *(a)* Given that $y = 10x^2 - 7x - 13$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Given that $y = 4\sqrt{x} + \frac{45}{x}$, find the value of $\frac{dy}{dx}$ when $x = 9$. [4]

10. A rectangular sheet of metal has length 24 m and width 9 m. Four squares, each of side x m, where $x < 4{\cdot}5$, have been cut away from the corners of the rectangular sheet, as shown in the diagram below. The rest of the metal sheet is now bent along the dotted lines to form an open tank in the form of a cuboid.



(a) Show that the volume V m^3 of this tank is given by

$$V = 4x^3 - 66x^2 + 216x.$$ [2]

(b) Find the maximum value of V, showing that the value you have found is a maximum value. [5]

END OF PAPER

9. *(a)* Given that $y = -5x^2 - 7x + 13$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Differentiate $6x^{\frac{3}{4}} + \frac{5}{x^3} - 9$ with respect to x. [2]

10. The curve C has equation

$$y = x^3 - 9x^2 + 15x + 10.$$

(a) (i) Find the coordinates of each of the stationary points of C.

(ii) Determine the nature of each of these stationary points. [6]

(b) Sketch C, indicating the coordinates of each of the stationary points. [2]

(c) Given that the equation

$$x^3 - 9x^2 + 15x + 10 = k$$

has only one real root, find the range of possible values for k. [2]

END OF PAPER

END OF DIFFERENTIATION PACK

Source: WJEC C1 + C2 (2008 modular spec) · 2011–2017

Curated for WJEC Maths 2017 spec AS Unit 1 — Topic 12 (2.1.7)