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GCE AS / A LEVEL – PURE MATHEMATICS A QUESTION PACK

0974-01 (Legacy C2) · New spec Unit 1 Topic 11 · AS unit, 25% of A-level, 120 marks, 2h 30min paper

REVISE
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MATHEMATICS – PURE A · EXPONENTIALS & LOGARITHMS

Exponentials & Logarithms

Log law proofs, exponential equations and reduction-to-linear-form questions from the legacy WJEC C2 papers (June 2011 – June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 46 minutes

Derived from the legacy C1/C2 paper's pace of ~1.25 min/mark (85 marks over 10 questions).

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **single-topic practice question pack**, drilling one narrow new-spec sub-topic. It contains questions from the legacy WJEC C1 and C2 papers (2008 modular spec) that map onto new-spec AS Unit 1 Topic 11 (2.1.6). Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

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| Q | Source | Max | Mark | Q | Source | Max | Mark |
|---|-----------|-----|------|--------------|-----------|-----------|------|
| 1 | Jun 11 Q7 | 7 | | 6 | Jan 14 Q7 | 8 | |
| 2 | Jan 12 Q7 | 11 | | 7 | Jun 14 Q7 | 6 | |
| 3 | Jun 12 Q7 | 10 | | 8 | Jun 15 Q7 | 8 | |
| 4 | Jan 13 Q7 | 6 | | 9 | Jun 16 Q7 | 10 | |
| 5 | Jun 13 Q7 | 10 | | 10 | Jun 17 Q7 | 9 | |
| | | | | Total | | 85 | |

Exponentials & Logarithms – what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 1: Pure Mathematics A · Topic 2.1.6.

Exponential function 2.1.6

- a^x for $a > 0$, $a \neq 1$: passes through $(0, 1)$, monotone.
- Growth if $a > 1$; decay if $0 < a < 1$.
- Modelling: $N(t) = N_0 \cdot a^t$.

Log laws 2.1.6

- $\log_a(xy) = \log_a x + \log_a y$.
- $\log_a(x/y) = \log_a x - \log_a y$.
- $\log_a(x^n) = n \log_a x$; $\log_a a = 1$; $\log_a 1 = 0$.

Solving $a^x = b$ 2.1.6

- Take logs of both sides: $x \log a = \log b$.
- Hence $x = \log b / \log a = \log_a b$.
- Change of base: $\log_a b = \ln b / \ln a$ on any calculator.

Combining logs 2.1.6

- Combine multiple log terms into one before solving.
- If $\log_a A = \log_a B$, deduce $A = B$ (with domain check).
- Domain reminder: the argument of a log must be positive.

Exponentials & Logarithms in one page

Quick-reference notes – revisit before each section. Don't use during questions.

Log laws (product / quotient)

$$\log_a(xy) = \log_a x + \log_a y.$$

$$\log_a(x/y) = \log_a x - \log_a y.$$

$$\log_a a = 1, \log_a 1 = 0.$$

Log laws (power)

$$\log_a(x^n) = n \log_a x.$$

$$\log_a \sqrt[k]{x} = \frac{1}{k} \log_a x.$$

Change of base

$$\log_a x = \frac{\log_b x}{\log_b a}.$$

Most calculators use \log_{10} or \ln – convert with this formula.

Proving log laws

From $a^p = x$ and $a^q = y$: $\log_a x = p$,

$$\log_a y = q.$$

Then $xy = a^{p+q}$, so $\log_a(xy) = p + q$.

Quotient and power laws are derived similarly.

Solving $a^x = b$

Take logs: $x \log a = \log b$, so $x = \log b / \log a$.

Or directly: $x = \log_a b$.

Use \ln or \log_{10} on the calculator.

Equation $\log_a A = \log_a B$

If the bases match, deduce $A = B$ (and check the domain).

Combine multiple log terms into one before equating.

Substitution $u = \log_a x$

An equation like $(\log_a x)^2 - 3 \log_a x + 2 = 0$ is quadratic in $u = \log_a x$.

Solve for u , then back-substitute $x = a^u$.

Exponential modelling

$N(t) = N_0 \cdot a^t$ for growth ($a > 1$) or decay ($0 < a < 1$).

Half-life: $a^t = 1/2$. Doubling time: $a^t = 2$.

Strategy

1. Combine log terms using laws.
2. For $a^x = b$ take logs both sides.
3. For $\log A = \log B$ deduce $A = B$.
4. Argument of log must be positive – reject otherwise.

SECTION T11

Exponentials & Logarithms

Questions 1-10 · 85 marks

7. (a) Given that $x > 0$, $y > 0$, show that

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y. \quad [3]$$

- (b) Express

$$\frac{1}{2} \log_a x^8 - \log_a 4x + 3 \log_a \frac{2}{x}$$

as a single logarithm in its simplest form. [4]

7. (a) Given that $x > 0, y > 0$, show that

$$\log_a xy = \log_a x + \log_a y. \quad [3]$$

- (b) Solve the equation

$$2^{3-5x} = 12.$$

Show your working and give your answer correct to three decimal places. [3]

- (c) (i) Express

$$\log_9(3x - 1) + \log_9(x + 4) - 2\log_9(x + 1)$$

as a single logarithm.

- (ii) Hence solve the equation

$$\log_9(3x - 1) + \log_9(x + 4) - 2\log_9(x + 1) = \frac{1}{2}. \quad [5]$$

7. (a) Given that $x > 0$, show that

$$\log_a x^n = n \log_a x. \quad [3]$$

- (b) Solve the equation

$$9^{\frac{x}{2}-3} = 6.$$

Show your working and give your answer correct to three decimal places. [3]

- (c) Solve the equation

$$\log_a(x-2) + \log_a(4x+1) = 2\log_a(2x-3). \quad [4]$$

7. (a) Given that $x > 0$, $y > 0$, show that

$$\log_a \frac{x}{y} = \log_a x - \log_a y. \quad [3]$$

- (b) Solve the equation

$$6^{2x+5} = 7.$$

Show your working and give your answer correct to three decimal places. [3]

TURN OVER

7. (a) Given that $x > 0$, $y > 0$, show that

$$\log_a xy = \log_a x + \log_a y. \quad [3]$$

- (b) Solve the equation

$$5^{2-3x} = 8.$$

Show your working and give your answer correct to three decimal places. [3]

- (c) Solve the equation

$$\log_a 90x^2 - \log_a \left(\frac{5}{x} \right) = \frac{1}{2} \log_a 144x^8. \quad [4]$$

7. (a) Given that $x > 0$, show that

$$\log_a x^n = n \log_a x. \quad [3]$$

- (b) Solve the equation

$$7^{5-4x} = 11.$$

Show your working and give your answer correct to three decimal places. [3]

- (c) Solve the equation

$$\log_8 x = -\frac{1}{3}. \quad [2]$$

7. (a) Solve the equation

$$3^{\frac{5x}{4}-2} = 7.$$

Show your working and give your answer correct to three decimal places. [3]

- (b) The positive numbers a and b are such that

$$\log_a b = 5.$$

- (i) Express b as a power of a .
- (ii) **Using your answer to part (i)**, evaluate $\log_b a$. [3]

7. (a) Given that $x > 0$, $y > 0$, show that

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y. \quad [3]$$

- (b) Find all values of x satisfying the equation

$$\log_a(6x^2 + 9x + 2) - \log_a x = 4\log_a 2. \quad [5]$$

TURN OVER

7. (a) Given that $x > 0$, show that

$$\log_a x^n = n \log_a x. \quad [3]$$

- (b) Solve the equation

$$4^{3x+1} = 22.$$

Show your working and give your answer correct to two decimal places. [3]

- (c) Given that

$$\log_d z = 2\log_d 6 - \log_d 9 - 1,$$

express z in terms of d , giving your answer in a form **not** involving logarithms. [4]

TURN OVER

7. (a) Given that $x > 0$, $y > 0$, show that

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y. \quad [3]$$

- (b) Express

$$\frac{1}{3} \log_b x^{15} - \log_b 27x + 4 \log_b \frac{3}{x}$$

as a single logarithm in its simplest form. [4]

- (c) Given that $\log_d 5 = \frac{1}{3}$, find the value of d . [2]

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END OF EXPONENTIALS & LOGARITHMS PACK

Source: WJEC C1 + C2 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec AS Unit 1 – Topic 11 (2.1.6)

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