Name	Date started	Target end date

GCE AS / A LEVEL — PURE MATHEMATICS A QUESTION PACK

0973-01 (Legacy C1) · New spec Unit 1 Topic 1 · AS unit, 25% of A-level, 120 marks, 2h 30min paper

REVISE
.wales

MATHEMATICS — PURE A · ALGEBRA & SURDS

Algebra & Surds

Surd simplification, completing the square and discriminant / range-of-k questions from the legacy WJEC C1 papers (June 2011 - June 2017)

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~4 hours 1 minutes

*Derived from the legacy C1/C2 paper's pace of **~1.25 min/mark** (193 marks over 29 questions).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains questions from the legacy WJEC C1 and C2 papers (2008 modular spec) that maps onto new-spec AS Unit 1 Topic 1 (2.1.1).

Questions are ordered chronologically.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed (except where specified by individual questions). The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 11 Q2	8	
2	Jun 11 Q4	4	
3	Jun 11 Q5	9	
4	Jan 12 Q2	8	
5	Jan 12 Q5	5	
6	Jan 12 Q6	7	
7	Jun 12 Q2	7	
8	Jun 12 Q5	5	
9	Jun 12 Q6	7	
10	Jan 13 Q2	7	
11	Jan 13 Q4	9	
12	Jan 13 Q5	7	
13	Jun 13 Q2	8	
14	Jun 13 Q4	5	
15	Jun 13 Q6	9	

Q	Source	Max	Mark
16	Jan 14 Q2	4	
17	Jan 14 Q4	5	
18	Jan 14 Q6	6	
19	Jun 14 Q2	8	
20	Jun 14 Q5	5	
21	Jun 14 Q6	7	
22	Jun 15 Q2	7	
23	Jun 15 Q4	6	
24	Jun 15 Q5	5	
25	Jun 16 Q2	4	
26	Jun 16 Q5	10	
27	Jun 16 Q6	8	
28	Jun 17 Q2	8	
29	Jun 17 Q4	5	
	Total	**193**	

Algebra & Surds — what the new spec asks

WJEC GCE AS / A Level Mathematics (from 2017) · Unit 1: Pure Mathematics A · Topic 2.1.2.

Indices & surds 2.1.2

- Laws of indices: $a^m a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, $a^{-n} = 1/a^n$, $a^{1/n} = \sqrt[n]{a}$.
- Surd rules: $\sqrt{ab} = \sqrt{a}\sqrt{b}$, $\sqrt{a/b} = \sqrt{a}/\sqrt{b}$.
- Rationalise denominators by multiplying by the conjugate surd.

Completing the square 2.1.2

- $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$.
- Reveals the vertex $(-b/(2a),\ c - b^2/(4a))$ of the parabola.
- Max / min value of $\frac{1}{ax^2+bx+c}$ comes from the same form.

Discriminant 2.1.2

- For $ax^2 + bx + c = 0$: $\Delta = b^2 - 4ac$.
- $\Delta > 0$: two distinct real roots; $\Delta = 0$: one repeated root; $\Delta < 0$: no real roots.
- Use to find a range of values of k for which a parameterised quadratic behaves a given way.

Quadratic inequalities 2.1.2

- Often arise after a discriminant condition produces a quadratic in k.
- Find the roots, sketch the parabola, read off the values where > 0 or < 0.
- If leading coefficient is positive: values outside the roots are positive.

Algebra & Surds in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Surd simplification

$\sqrt{ab} = \sqrt{a}\sqrt{b}$, $\sqrt{a/b} = \sqrt{a}/\sqrt{b}$.

$(\sqrt{a})^2 = a$ for $a \geq 0$.

Collect like surds at the end: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$.

Rationalising the denominator

For $\frac{1}{a + b\sqrt{c}}$, multiply top and bottom by $a - b\sqrt{c}$.

Denominator becomes $a^2 - b^2c$ — rational.

For $\frac{1}{\sqrt{a}}$, multiply by $\sqrt{a}/\sqrt{a}$.

Index laws

$a^m \cdot a^n = a^{m+n}$, $a^m/a^n = a^{m-n}$.

$(a^m)^n = a^{mn}$, $(ab)^n = a^n b^n$.

$a^0 = 1$, $a^{-n} = 1/a^n$, $a^{m/n} = \sqrt[n]{a^m}$.

Completing the square

For $x^2 + bx + c$: $(x + b/2)^2 + (c - b^2/4)$.

For $ax^2 + bx + c$: factor out a first.

$$a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

Vertex: $\big(-b/(2a),\ c - b^2/(4a)\big)$.

Using the completed form

Max / min of the quadratic equals the constant term in the completed form.

Max of $\frac{1}{(\text{positive quadratic})}$ occurs at the quadratic's minimum.

Sketch: vertex + y-intercept tells you the shape.

Discriminant

For $ax^2 + bx + c = 0$: $\Delta = b^2 - 4ac$.

$\Delta > 0$: two distinct real roots.

$\Delta = 0$: one repeated root.

$\Delta < 0$: no real roots.

Discriminant with parameter k

Substitute parameterised a, b, c into $b^2 - 4ac$.

Expand carefully — signs matter when b involves k.

Result is a quadratic in k; apply inequality condition.

Quadratic inequalities in k

From $\Delta < 0$ (or > 0): get $f(k) < 0$ (or > 0).

Find roots of $f(k) = 0$, sketch parabola.

Leading coefficient > 0: values between roots are negative.

Strategy

1. Surds: rationalise first, simplify last.
2. Completing the square: factor out leading coefficient.
3. Discriminant problems: substitute, expand, simplify.
4. Inequality on k: sketch the parabola in k.

SECTION T1

Algebra & Surds

Questions 1–29 · 193 marks

2. Simplify

(a) $\dfrac{9}{\sqrt{3}-1}+\dfrac{7}{\sqrt{3}+1}$, [4]

(b) $\dfrac{90}{\sqrt{3}}-\sqrt{6}\times\sqrt{8}-\left(2\sqrt{3}\right)^{3}$. [4]

4. Express $-x^2 + 6x - 7$ in the form $-(x + a)^2 + b$, where the values of the constants a and b are to be found.
Hence sketch the graph of $y = -x^2 + 6x - 7$, indicating the coordinates of its stationary point.
[4]

5. The curve C has equation

$$y = x^2 + (4k+3)x + 7,$$

and the line L has equation

$$y = x + k,$$

where k is a constant.
Given that L and C intersect at two distinct points,

(a) show that $4k^2 + 5k - 6 > 0$, [6]

(b) find the range of values of k satisfying this inequality. [3]

3

dy

2. Simplify

(a) $\dfrac{9+4\sqrt{2}}{5+3\sqrt{2}}$, [4]

(b) $\left(\sqrt{8}\times\sqrt{10}\right)+\dfrac{\sqrt{90}}{\sqrt{2}}-\dfrac{30}{\sqrt{5}}$. [4]

5. *(a)* Express $3x^2 - 6x + 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants whose values are to be found. [3]

(b) **Use your answer to part *(a)*** to find the greatest value of

$$\frac{1}{3x^2 - 6x + 11}\ .$$ [2]

6. Given that the quadratic equation

$$(k + 6)x^2 + 4x + (k + 3) = 0$$

has no real roots, show that

$$k^2 + 9k + 14 > 0.$$

Find the range of values of k satisfying this inequality. [7]

2. Simplify

(a) $\dfrac{10}{7+2\sqrt{11}}$, [3]

(b) $\left(4\sqrt{3}\right)^2 - \left(\sqrt{8}\times\sqrt{50}\right) - \dfrac{5\sqrt{63}}{\sqrt{7}}$. [4]

5. *(a)* Express $3x^2 - 12x + 29$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Using your answer to part *(a)***, write down the stationary value of $y = 3x^2 - 12x + 29$. State whether this stationary value is a maximum or a minimum. [2]

6. *(a)* Show that the equation

$$x^2 + (2k - 1)x + (k^2 - k + 2) = 0$$

has no real roots, whatever the value of the constant k. [4]

(b) Find the range of values of x satisfying the inequality

$$3x^2 + 16x - 12 > 0.$$ [3]

2. Simplify

(a) $\dfrac{6\sqrt{7} - 11\sqrt{2}}{\sqrt{7} - \sqrt{2}}$, [4]

(b) $\dfrac{3}{2\sqrt{6}} + \left(\dfrac{\sqrt{6}}{2}\right)^3$. [3]

4. *(a)* (i) Express $x^2 + 8x + 5$ in the form $(x + a)^2 + b$, where the values of a, b are to be determined.

(ii) **Use your answers to part (i)** to find the least value of $3x^2 + 24x + 15$ and the corresponding value of x. [4]

(b) Solve the simultaneous equations $y = x^2 - x - 9$ and $y = 2x - 5$ algebraically.
Write down a geometrical interpretation of your results. [5]

5. *(a)* Find the range of values of k for which the quadratic equation

$$5x^2 + 6x - 3k = 0$$

has two distinct real roots. [4]

(b) Solve the inequality $2x^2 - 11x + 15 \leqslant 0$. [3]

2. Simplify

(a) $\dfrac{2+5\sqrt{7}}{4+\sqrt{7}}$, [4]

(b) $\sqrt{360}-\sqrt{2}\times\left(\sqrt{5}\right)^3-\dfrac{\sqrt{30}\times\sqrt{8}}{\sqrt{6}}$. [4]

4. *(a)* Express $2x^2 - 16x - 8$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Using your answer to part *(a)***, find the least value of $x^2 - 8x - 4$ and the corresponding value of x. [2]

6. *(a)* (i) Assuming that the quadratic equation

$$(k + 1)x^2 + (4k + 1)x + (k - 5) = 0$$

has **two equal** roots, show that

$$4k^2 + 8k + 7 = 0.$$

(ii) Hence show that there are **no real** values of k such that the quadratic equation

$$(k + 1)x^2 + (4k + 1)x + (k - 5) = 0$$

has two equal roots. [6]

(b) Find the range of values of x satisfying the inequality

$$4x^2 - 9x - 9 \geqslant 0.$$ [3]

2. Simplify $\dfrac{3\sqrt{3} - 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}}$. [4]

4. Show that $x^2 + 1{\cdot}6x - 24{\cdot}36$ may be expressed in the form $(x + p)^2 - 25$, where p is a constant whose value is to be found.
Hence solve the quadratic equation $x^2 + 1{\cdot}6x - 24{\cdot}36 = 0$. [5]

6. Given that the quadratic equation

$$(2k - 3)x^2 + 8x + (2k + 3) = 0$$

has no real roots, show that k satisfies an inequality of the form

$$m - nk^2 < 0,$$

where m, n are integers whose values are to be found.

Hence find the range of values of k such that the quadratic equation

$$(2k - 3)x^2 + 8x + (2k + 3) = 0$$

has no real roots. [6]

3

2. Simplify

(a) $\dfrac{3\sqrt{3}+1}{5\sqrt{3}-7}$, [4]

(b) $\left(\sqrt{12}\times\sqrt{24}\right)+\dfrac{\sqrt{150}}{\sqrt{3}}-\dfrac{36}{\sqrt{2}}$. [4]

5. *(a)* Express $4x^2 - 8x + 11$ in the form $a(x + b)^2 + c$, where a, b and c are constants whose values are to be found. [3]

(b) **Use your answer to part *(a)*** to find the greatest value of $\dfrac{1}{4x^2 - 8x + 29}$. [2]

6. Given that the quadratic equation

$$(k-1)x^2 + 2kx + (7k-4) = 0$$

has no real roots, show that

$$6k^2 - 11k + 4 > 0.$$

Find the range of values of k satisfying this inequality. [7]

2. Simplify

(a) $\dfrac{4\sqrt{2} - \sqrt{11}}{3\sqrt{2} + \sqrt{11}}$, [4]

(b) $\dfrac{7}{2\sqrt{14}} + \left(\dfrac{\sqrt{14}}{2}\right)^3$. [3]

4. *(a)* Express $4x^2 - 24x - 189$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Using your answer to part *(a)***, solve the equation

$$4x^2 - 24x - 189 = 0.$$ [3]

3

5. *(a)* Find the range of values of k for which the quadratic equation

$$kx^2 + (2k - 5)x + (k - 6) = 0$$

has **no real roots**. [4]

(b) Without carrying out any further calculation, write down the value of k for which the quadratic equation

$$kx^2 + (2k - 5)x + (k - 6) = 0$$

has **two equal roots**. [1]

2. Simplify $\dfrac{5\sqrt{7}+4\sqrt{2}}{3\sqrt{7}+5\sqrt{2}}$. [4]

5. *(a)* Express $x^2 + 4x - 8$ in the form $(x + a)^2 + b$, where a and b are constants whose values are to be found. [2]

(b) Use an algebraic method to solve the simultaneous equations $y = x^2 + 4x - 8$ and $y = 2x + 7$. [4]

(c) Draw a sketch illustrating geometrically the results of both part *(a)* and part *(b)*. [4]

6. *(a)* Find the range of values of k for which the quadratic equation

$$9x^2 + 8x - 2k = 0$$

has **two distinct real roots**. [4]

(b) Solve the inequality $x(5x - 7) \geqslant 6$. [4]

3

2. Simplify

(a) $\dfrac{5\sqrt{5}-9}{3+2\sqrt{5}}$, [4]

(b) $\left(2\sqrt{13}\right)^2-\left(3\sqrt{7}\times\sqrt{28}\right)-\dfrac{5\sqrt{99}}{\sqrt{11}}$. [4]

4. *(a)* Express $-2x^2 - 20x + 35$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Without carrying out any further calculation**, write down the stationary value of $y = -2x^2 - 20x + 35$ and state whether this stationary value is a maximum or a minimum. [2]

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END OF ALGEBRA & SURDS PACK

Source: WJEC C1 + C2 (2008 modular spec) · 2011–2017
Curated for WJEC Maths 2017 spec AS Unit 1 — Topic 1 (2.1.1)