Name	Date started	Target end date

GCE A LEVEL — FURTHER MECHANICS B QUESTION PACK

0982-01 (Legacy M3) · New spec A2 Unit 6 Topic 6

REVISE
.wales

FURTHER MATHS — FM B · SECOND-ORDER DES APPLIED TO MECHANICS

Second-Order Linear Differential Equations — Complementary Function + Particular Integral

Second-order linear DE questions from the legacy WJEC M3 papers (June 2006, June 2010, June 2014) where the technique — solve auxiliary equation, then add a particular integral — is the load-bearing skill. The M3 June 2014 question is the explicit mechanical model (kitchen scales with damping + steady-state). Pure SHM questions live in T5.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~0 hours 47 minutes

Derived from the legacy M3 paper's pace of ***~1.3 min/mark*** *(36 marks over 3 questions). The full Unit 6 exam is* ***1 hour 45 minutes for 80 marks*** *(25% of the A-level qualification, A2 optional paper alongside Unit 5 Further Statistics B).*

You are advised to ***not*** *attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every second-order des applied to mechanics question from the legacy WJEC M3 papers (2008 modular spec) that maps onto new-spec A2 Unit 6 Topic 6 (2.6.5). Unit 6 (Further Mechanics B) is one of two **80-mark A2 optional papers** (the other being Unit 5 Further Statistics B), each worth 25% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet for Mechanics may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 06 Q2	12	
2	Jun 10 Q3	12	
3	Jun 14 Q4	12	
	Total	**36**	

Second-Order Linear Differential Equations — Complementary Function + Particular Integral — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 6: Further Mechanics B · Topic 2.6.5.

Second-order linear DEs 2.6.5

- Standard form: $\ddot{x} + b\dot{x} + cx = f(t)$.
- General solution: $x = x_{CF} + x_{PI}$ — complementary function + particular integral.
- Complementary function: solves the *homogeneous* equation $\ddot{x} + b\dot{x} + cx = 0$.
- Particular integral: any one solution of the full inhomogeneous equation.

Auxiliary equation & CF 2.6.5

- Substitute $x = e^{\lambda t}$ into the homogeneous equation: $\lambda^2 + b\lambda + c = 0$.
- Distinct real roots λ_1, λ_2: $x_{CF} = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$.
- Repeated root λ: $x_{CF} = (A + Bt)e^{\lambda t}$.
- Complex conjugate roots $\alpha \pm i\beta$: $x_{CF} = e^{\alpha t}(A\cos\beta t + B\sin\beta t)$.

Particular integral & damping regimes 2.6.5

- Try a PI of the same form as $f(t)$: constant $\to$ constant, $f(t) = at + b \to x_{PI} = pt + q$, $f(t) = \sin\omega t \to x_{PI} = p\sin\omega t + q\cos\omega t$.
- Damped SHM: $\ddot{x} + 2\gamma\dot{x} + \omega^2 x = 0$, discriminant $\gamma^2 - \omega^2$.
- *Under-damped* ($\gamma < \omega$): decaying oscillation. *Critically damped* ($\gamma = \omega$): fastest return without overshoot.
- *Over-damped* ($\gamma > \omega$): exponential decay, no oscillation. Angular SHM is explicitly **not** assessed.

Mechanical applications 2.6.5

- A spring-mass system with linear damping models a shock absorber or seismograph.
- The kitchen-scales question (M3 Jun 2014) is the canonical example: PI gives the steady-state reading; CF gives the transient.
- Limiting value (steady state) = $\lim_{t\to\infty} x(t) = x_{PI}$ when the CF decays.
- Initial conditions $x(0), \dot{x}(0)$ pin down the two CF constants — apply *after* writing $x_{CF} + x_{PI}$.

Second-Order DEs Applied to Mechanics in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Standard form

A second-order linear DE with constant coefficients:

$$\ddot{x} + b\dot{x} + cx = f(t)$$

$f(t)$ is the "forcing". If $f(t) = 0$, the equation is *homogeneous*.

CF + PI

General solution: $x(t) = x_{CF}(t) + x_{PI}(t)$.

x_{CF} is the most general solution to the homogeneous equation.

x_{PI} is *any one* solution of the full inhomogeneous equation.

Apply initial conditions to the *sum*, never to x_{CF} alone.

Auxiliary equation

Try $x = e^{\lambda t}$ in $\ddot{x} + b\dot{x} + cx = 0$:

$$\lambda^2 + b\lambda + c = 0$$

Solve the quadratic for λ_1, λ_2 — three cases follow.

Three CF cases

Real distinct ($\lambda_1 \neq \lambda_2$): $x_{CF} = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$.

Repeated ($\lambda_1 = \lambda_2 = \lambda$): $x_{CF} = (A + Bt)e^{\lambda t}$.

Complex ($\alpha \pm i\beta$): $x_{CF} = e^{\alpha t}(A\cos\beta t + B\sin\beta t)$.

Choosing a PI

Try a PI of the *same form* as $f(t)$:

- $f =$ constant $\to x_{PI} = p$.
- $f = at + b \to x_{PI} = pt + q$.
- $f = e^{kt} \to x_{PI} = pe^{kt}$ (unless k is a CF root — then multiply by t).
- $f = \cos\omega t$ or $\sin\omega t \to x_{PI} = p\cos\omega t + q\sin\omega t$.

Damped oscillator

Homogeneous damped SHM: $\ddot{x} + 2\gamma\dot{x} + \omega^2 x = 0$.

Discriminant $\gamma^2 - \omega^2$:

Under-damped ($\gamma < \omega$): decaying oscillation, $e^{-\gamma t}(A\cos\omega_d t + B\sin\omega_d t)$ with $\omega_d = \sqrt{\omega^2 - \gamma^2}$.

Critically / over-damped: no oscillation.

Steady state

For a stable system (CF decays as $t \to \infty$), the long-term behaviour is the PI:

$$\lim_{t\to\infty} x(t) = x_{PI}$$

For a constant $f(t)$: the limit is f/c when $c \neq 0$.

The CF is the *transient* response.

Common pitfalls

- Applying initial conditions to x_{CF} before adding x_{PI}.
- Forgetting to multiply the trial PI by t when the trial is already in the CF.
- Mixing ω in the CF (from auxiliary) with ω in the PI (from $f(t)$).
- Confusing the homogeneous solution with a single particular case.

Strategy

1. Solve auxiliary equation to get x_{CF}.
2. Pick trial x_{PI} matching $f(t)$; substitute and solve for coefficients.
3. Write $x = x_{CF} + x_{PI}$.
4. Apply $x(0), \dot{x}(0)$ to pin down the two CF constants.

SECTION T6

Second-Order Linear Differential Equations — Complementary Function + Particular Integral

Questions 1–3 · 36 marks

2. Find the general solution of the second order differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 5t - 14,$$

such that $x = 4\frac{1}{2}$ and $\frac{dx}{dt} = 3\frac{1}{2}$ when $t = 0$. [12]

3. Find the solution of the differential equation

$$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 12\frac{\mathrm{d}x}{\mathrm{d}t} + 9x = 18t - 87,$$

such that $x = 5$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 10$ when $t = 0$. [12]

4. The reading x of the pointer on a set of kitchen scales at time t is modelled by the differential equation

$$2\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 5x = 1.$$

(a) Find the general solution of the equation for x. [5]

(b) Determine the limiting value of x. [2]

(c) Given that $x = 0{\cdot}5$ and $\frac{dx}{dt} = 0$ when $t = 0$,

(i) find an expression for x in terms of t,

(ii) calculate the instantaneous reading of the scale when $t = \frac{\pi}{3}$.

Give your answer correct to three significant figures. [5]

END OF SECOND-ORDER DES APPLIED TO MECHANICS PACK

Source: WJEC M3 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec A2 Unit 6 — Topic 6 (2.6.5)