Name	Date started	Target end date

GCE A LEVEL — FURTHER MECHANICS B QUESTION PACK

0982-01 (Legacy M3) · New spec A2 Unit 6 Topic 5

REVISE
.wales

FURTHER MATHS — FM B · DIFFERENTIAL EQUATIONS & SHM

Differential Equations & Simple Harmonic Motion — including Damped SHM

Every differential-equation and SHM question from the legacy WJEC M3 papers (June 2006 – June 2017 + Specimen) that maps onto the new-spec A2 Unit 6 Topic 5.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~4 hours 3 minutes

*Derived from the legacy M3 paper's pace of **~1.3 min/mark** (187 marks over 13 questions). The full Unit 6 exam is **1 hour 45 minutes for 80 marks** (25% of the A-level qualification, A2 optional paper alongside Unit 5 Further Statistics B).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every differential equations & shm question from the legacy WJEC M3 papers (2008 modular spec) that maps onto new-spec A2 Unit 6 Topic 5 (2.6.5). Unit 6 (Further Mechanics B) is one of two **80-mark A2 optional papers** (the other being Unit 5 Further Statistics B), each worth 25% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet for Mechanics may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 06 Q2	12	
2	Jun 10 Q3	12	
3	Jun 14 Q4	12	
4	Jun 06 Q4	14	
5	Jun 07 Q2	15	
6	Jun 08 Q3	16	
7	Jun 09 Q2	17	
8	Jun 10 Q2	15	
9	Jun 12 Q2	18	
10	Jun 13 Q2	11	
11	Jun 16 Q2	16	
12	Jun 17 Q5	13	
13	Spec. Q2	16	
	Total	187	

Differential Equations & Simple Harmonic Motion — including Damped SHM — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 6: Further Mechanics B · Topic 2.6.5.

Differential equations in kinematics 2.6.5

- Use first-order separable DEs for $a(v)$ or $a(x)$ problems.
- Second-order linear DEs with constant coefficients model damped or driven oscillations.
- $\ddot{x} + 2\gamma\dot{x} + \omega^2 x = f(t)$.
- Standard solution = complementary function + particular integral.

Simple harmonic motion 2.6.5

- A particle moves with SHM if its acceleration is directed towards a fixed point and proportional to its displacement:
- $\ddot{x} = -\omega^2 x$ (where $\omega > 0$ is the angular frequency).
- Period: $T = 2\pi/\omega$. Frequency: $f = 1/T$.
- Amplitude a is the maximum displacement; max speed ωa; max acceleration $\omega^2 a$.

Solutions and the energy equation 2.6.5

- General solution: $x = A\cos\omega t + B\sin\omega t = a\sin(\omega t + \phi)$.
- Energy form: $v^2 = \omega^2(a^2 - x^2)$ — useful for finding v at a given x without solving the DE.
- Starting from rest at $x = a$: $x = a\cos\omega t$. Starting from $x = 0$ with $v = v_0$: $x = (v_0/\omega)\sin\omega t$.
- Spring-mass on a smooth surface: $\omega = \sqrt{k/m}$ where k is the spring stiffness.

Damped SHM (model refinement) 2.6.5

- Adding linear damping: $\ddot{x} + 2\gamma\dot{x} + \omega^2 x = 0$.
- Three regimes set by discriminant $\gamma^2 - \omega^2$:
- *under-damped* ($\gamma < \omega$) — oscillation with exponential decay; *critically damped* ($\gamma = \omega$); *over-damped* ($\gamma > \omega$) — no oscillation.
- Angular SHM is explicitly **not** assessed under this spec.

Differential Equations & SHM in one page

Quick-reference notes — revisit before each section. Don't use during questions.

SHM equation

A particle moves with SHM about a fixed centre O if

$$\ddot{x} = -\omega^2 x$$

where $\omega > 0$ is the *angular frequency* (radians per second).

Acceleration points towards O and is proportional to displacement.

Period and frequency

Period: $T = \dfrac{2\pi}{\omega}$ (seconds).

Frequency: $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$ (Hz).

Amplitude a is the maximum displacement from O.

Energy equation

Multiply the SHM equation by $2\dot{x}$ and integrate:

$$v^2 = \omega^2(a^2 - x^2)$$

Max speed: ωa at $x = 0$.

$v = 0$ at $x = \pm a$.

General solution

Two equivalent forms:

$$x = A\cos\omega t + B\sin\omega t$$

$$x = a\sin(\omega t + \phi) \text{ or } a\cos(\omega t + \phi)$$

Both have amplitude $a = \sqrt{A^2 + B^2}$.

Initial conditions

Starting from rest at $x = a$: $x = a\cos\omega t$.

Starting from $x = 0$ with speed v_0: $x = \dfrac{v_0}{\omega}\sin\omega t$.

Apply $x(0)$ and $\dot{x}(0)$ to pin down A, B (or a, ϕ).

Spring-mass SHM

Mass m on a spring of stiffness k, smooth surface:

$$m\ddot{x} = -kx \Rightarrow \omega = \sqrt{k/m}$$

For a light elastic string/spring with modulus λ and natural length ℓ, $k = \lambda/\ell$.

Second-order linear DEs

For $\ddot{x} + b\dot{x} + cx = f(t)$:

Solve the homogeneous part (auxiliary equation $\lambda^2 + b\lambda + c = 0$).

Add a particular integral matching $f(t)$ (try same form as f).

General solution = CF + PI.

Damped SHM types

For $\ddot{x} + 2\gamma\dot{x} + \omega^2 x = 0$, discriminant $D = \gamma^2 - \omega^2$:

Under-damped ($D < 0$): oscillation with decaying amplitude $e^{-\gamma t}$.

Critically damped ($D = 0$): returns to equilibrium fastest, no overshoot.

Over-damped ($D > 0$): exponential decay, no oscillation.

Common pitfalls

- Mixing up ω (angular frequency, rad/s) with f (frequency, Hz).
- Wrong sign — SHM requires acceleration *towards* centre, not away from it.
- Forgetting the chain rule when interpreting an energy-style integral $\int v\,dv$.
- Using the wrong reference point for x in spring-mass problems (use equilibrium position).

Strategy

1. Identify SHM by writing the acceleration as $\ddot{x} = -\omega^2 x$.
2. Read off ω and compute $T = 2\pi/\omega$.
3. Apply $v^2 = \omega^2(a^2 - x^2)$ for energy-style questions.
4. Apply initial conditions to pin down A, B if needed.

SECTION T5

Differential Equations & SHM

Questions 1–13 · 187 marks

2. Find the general solution of the second order differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 5t - 14,$$

such that $x = 4\frac{1}{2}$ and $\frac{dx}{dt} = 3\frac{1}{2}$ when $t = 0$. [12]

3. Find the solution of the differential equation

$$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 12\frac{\mathrm{d}x}{\mathrm{d}t} + 9x = 18t - 87,$$

such that $x = 5$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 10$ when $t = 0$. [12]

4. The reading x of the pointer on a set of kitchen scales at time t is modelled by the differential equation

$$2\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 5x = 1.$$

(a) Find the general solution of the equation for x. [5]

(b) Determine the limiting value of x. [2]

(c) Given that $x = 0{\cdot}5$ and $\frac{dx}{dt} = 0$ when $t = 0$,

(i) find an expression for x in terms of t,

(ii) calculate the instantaneous reading of the scale when $t = \frac{\pi}{3}$.

Give your answer correct to three significant figures. [5]

4. A particle P is moving in a straight line with Simple Harmonic Motion about centre O, with period 4 s. The maximum speed of P is 3π ms^{-1}. The point A is 4·8 m from O.

(a) Find the amplitude of the motion. [4]

(b) Find the speed of P when it is at A. [3]

(c) Calculate the time taken by P to move directly from O to A. [3]

(d) Determine the magnitude of the maximum acceleration of P. [2]

(e) Find the distance travelled by the particle P in 12 s. [2]

2. A particle P is moving in a straight line with Simple Harmonic Motion. It starts from rest from a point A, and 2 seconds later, reaches its maximum speed of $3\pi \text{ ms}^{-1}$.

(a) Show that the amplitude of the motion is 12 m. [4]

(b) Calculate the distance from A of the particle $\frac{2}{3}$ s after the start of motion. [4]

(c) Calculate the speed of the particle $\frac{2}{3}$ s after the start of motion. [3]

(d) Points X and Y are equidistant from O, the centre of the motion, and are 10 m apart. Calculate the time taken for P to proceed directly from X to Y. [4]

3. A particle is moving in a straight line with Simple Harmonic Motion with centre O. When the particle is 3 m from O its speed is $5\,\text{ms}^{-1}$ and when it is 4 m from O its speed is $3{\cdot}75\,\text{ms}^{-1}$.

(a) Show that the amplitude of the motion is 5 m and find the period of the motion. [8]

(b) Find, correct to two decimal places, the distance of the particle from O 2 s after the particle passes through O. [3]

(c) How long after passing through O is the speed of the particle two-fifths of its maximum speed? Give your answer correct to two decimal places. [5]

2. At time $t = 0$, a particle P is projected from a point O so that it moves in a straight line with Simple Harmonic Motion with centre O. Two seconds later P comes to rest for the first time at the point A, where $OA = 24$ cm.

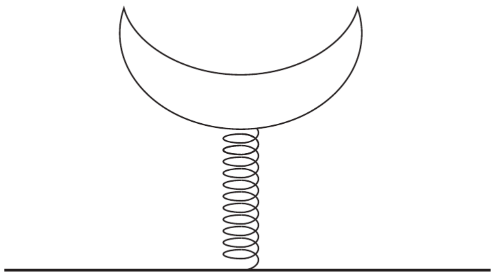
(a) Determine the speed of projection. [6]

(b) The point B is between O and A, such that $OB = 15$ cm. Find the value of t when P is at B for the **third** time. [4]

(c) Calculate the speed of P when $t = 1{\cdot}5$ s. [4]

(d) Find the speed of P when it is at a distance 20 cm from O. [3]

2. The diagram shows a playground ride consisting of a seat, of mass 12 kg, attached to a vertical spring, which is fixed to a horizontal board. When the ride is at rest with nobody on it, the compression of the spring is 0·05 m.



The seat is modelled as a particle P and the spring is modelled as a light spring of natural length 0·75 m and modulus of elasticity λ.

(a) Find the value of λ. [2]

The seat is now pushed vertically downwards a further 0·05 m and is then released from rest.

(b) Show that P makes Simple Harmonic oscillations of period $\frac{\pi}{7}$ and write down the amplitude of the motion. [5]

(c) Find the maximum speed of P. [2]

(d) Calculate the speed of P when it is at a distance 0·03 m from the equilibrium position. [3]

(e) Find the distance of P from the equilibrium position 1·6 s after it is released. [3]

2. The points O, A and B lie, in that order, on a straight line with $OA = 0{\cdot}6\,\text{m}$ and $OB = 0{\cdot}8\,\text{m}$. A particle P performs Simple Harmonic Motion along the line with centre O. The speed of P at A is $0{\cdot}3\sqrt{3}\ \text{ms}^{-1}$ and its speed at B is $0{\cdot}2\sqrt{5}\ \text{ms}^{-1}$.

(a) Show that the amplitude of the motion is $1{\cdot}2\,\text{m}$ and that the period is 4π s. [7]

(b) Determine the magnitude of the acceleration of P at A. [2]

(c) Calculate the time taken for P to move directly from A to B. Give your answer correct to 3 significant figures. [4]

(d) Given that P is at O at time $t = 0$, find the distance of P from O when $t = \dfrac{2\pi}{3}$. [2]

(e) Given that P is at O when $t = 0$, find the speed of P when $t = \dfrac{2\pi}{3}$. [3]

2. A particle P moves in a straight line with Simple Harmonic Motion about a fixed centre O with period 2 s. At time $t = 0$ s, P is at a point A where $OA = 0{\cdot}5$ m and its velocity is zero.

(a) Write down the amplitude of the motion. [1]

(b) Find the maximum magnitude of the acceleration of P and state the positions of P when this occurs. [4]

(c) Find the smallest positive value of time t for which AP is $0{\cdot}75$ m. [3]

(d) Determine the speed of P when it is $0{\cdot}3$ m from O. [3]

2. *(a)* A particle moves along the x-axis such that its position x m after time t seconds is given by

$$x = A\sin\omega t + B\cos\omega t.$$

Show that the motion of the particle is Simple Harmonic. State the value of x at the centre of motion and find the amplitude of the motion. [7]

(b) Another particle moves with Simple Harmonic Motion with centre *O*.
The particle has velocity 13 ms^{-1} when it is 3 m from *O* and 5 ms^{-1} when it is 5 m from *O*.

(i) Find the period and amplitude of the motion.

(ii) Given that the particle is at *O* at time $t = 0$, find the distance of the particle from *O* when $t = 0{\cdot}3$. [9]

5. The speed $v\,\text{ms}^{-1}$ of a particle moving along the x-axis is given by

$$v^2 = -4x^2 + 8x + 21.$$

(a) Show that the motion is simple harmonic and write down the centre of the motion. [5]

(b) Show that the period of the motion is π seconds and determine the amplitude. [4]

(c) Given that when $t = 0$, the particle is at the centre of the motion and moving with positive velocity, write down an expression for x in terms of t and calculate the time taken for the particle to reach $x = 3$ for the first time. [4]

2. A particle moves in a straight line with Simple Harmonic Motion with centre O. The amplitude of the motion is 5 m. When the particle is at O, its speed is 8 ms^{-1}.

(a) Show that the period of the motion is $\frac{5\pi}{4}$, and find the time taken to make 9 complete oscillations. [5]

(b) Determine the speed of the particle when it is at a distance of 4 m from O. [3]

(c) Find the magnitude of the acceleration of the particle when it is at a distance of 3 m from O. [3]

(d) Two points A and B are on different sides of O on the path of the particle. The point A is 2·4 m from O and the point B is 3·6 m from O. Calculate the shortest time for the particle to travel from A to B. [5]

END OF DIFFERENTIAL EQUATIONS & SHM PACK

Source: WJEC M3 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec A2 Unit 6 — Topic 5 (2.6.5)