Name	Date started	Target end date

GCE A LEVEL — FURTHER MECHANICS B QUESTION PACK

0982-01 (Legacy M3) · New spec A2 Unit 6 Topic 4

REVISE
.wales

FURTHER MATHS — FM B · EQUILIBRIUM OF RIGID BODIES

Equilibrium of Rigid Bodies — Ladders & Friction at Contacts

Every ladder / rigid-body-equilibrium question from the legacy WJEC M3 papers (June 2006 – June 2017 + Specimen) that maps onto the new-spec A2 Unit 6 Topic 4. Jointed rods and toppling are explicitly excluded from the spec and from this pack.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1 hours 34 minutes

*Derived from the legacy M3 paper's pace of **~1.3 min/mark** (72 marks over 6 questions). The full Unit 6 exam is **1 hour 45 minutes for 80 marks** (25% of the A-level qualification, A2 optional paper alongside Unit 5 Further Statistics B).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every equilibrium of rigid bodies question from the legacy WJEC M3 papers (2008 modular spec) that maps onto new-spec A2 Unit 6 Topic 4 (2.6.4). Unit 6 (Further Mechanics B) is one of two **80-mark A2 optional papers** (the other being Unit 5 Further Statistics B), each worth 25% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet for Mechanics may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 06 Q6	15	
2	Jun 07 Q3	13	
3	Jun 10 Q6	11	
4	Jun 14 Q6	13	
5	Jun 15 Q6	9	
6	Spec. Q4	11	
	Total	**72**	

Equilibrium of Rigid Bodies — Ladders & Friction at Contacts — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 6: Further Mechanics B · Topic 2.6.4.

Equilibrium of a rigid body 2.6.4

- For a single rigid body in equilibrium under coplanar forces:
- Sum of forces in x direction = 0; sum in y direction = 0; sum of moments = 0.
- Three independent scalar equations resolve up to three unknowns.
- Choose pivot wisely — taking moments about a point where unknown forces act eliminates those unknowns.

Friction at contacts 2.6.4

- Friction at a rough contact obeys $F \leq \mu R$, where R is the normal reaction and μ the coefficient of friction.
- *Limiting equilibrium*: about to slip, so $F = \mu R$ exactly.
- Direction of friction opposes the tendency of motion at that contact.
- A smooth contact applies normal reaction only; a rough contact applies normal + tangential friction.

Ladder geometry 2.6.4

- A ladder of length ℓ on smooth or rough ground leaning against a smooth or rough wall.
- Standard forces: weight W at the midpoint (uniform); normal R_g + friction F_g at ground; normal R_w $(+F_w$ if wall is rough) at top.
- Equations: vertical equilibrium gives R_g; horizontal gives R_w; moments (often about the foot) give the missing constraint.
- For a person of mass m_p climbing distance d up the ladder, weight acts at horizontal distance $d \cos \theta$ from the foot.

Strategy 2.6.4

- Draw a clear free-body diagram showing all forces.
- Resolve along two convenient orthogonal directions.
- Take moments about whichever point eliminates the most unknowns.
- Limiting equilibrium → substitute $F = \mu R$ at the contact about to slip.

Equilibrium of Rigid Bodies in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Three equilibrium conditions

For a rigid body in equilibrium under coplanar forces:

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M = 0 \text{ (about any point)}$$

Choice of pivot

Moments can be taken about *any* point.

Pick the point where the most unknown forces act — their moment is zero, killing them from the equation.

For a ladder problem, taking moments about the foot kills both the ground reaction and ground friction.

Friction laws

At any rough contact, the friction F satisfies $F \le \mu R$, where R is the normal reaction.

μ is the coefficient of friction (dimensionless, ≥ 0).

F is whatever value is needed to prevent slipping, up to the limit μR.

Limiting equilibrium

The contact is *about to slip* when $F = \mu R$ exactly.

This is the boundary case — any further perturbation causes motion.

In problems where the body is "just on the point of slipping", set $F = \mu R$ at the slipping contact.

Standard ladder forces

A uniform ladder, length ℓ, mass m, against a wall:

At top (against wall): normal R_w, plus friction F_w if wall is rough.

At foot (on ground): normal R_g, plus friction F_g if ground is rough.

Weight mg acts at the midpoint.

Rough wall + rough ground

Most general case: both contacts rough, two coefficients μ_w and μ_g.

At limiting equilibrium, $F_w = \mu_w R_w$ and $F_g = \mu_g R_g$.

Three equations ($\sum F_x, \sum F_y, \sum M$) in three unknowns ($R_w, R_g$, and one of θ or μ).

Person on a ladder

A person of mass m_p climbs distance d along the ladder.

Their weight $m_p g$ acts at horizontal $d\cos\theta$ from the foot (vertical drop $d\sin\theta$ from the top).

Moments about the foot: $W \cdot \frac{\ell}{2}\cos\theta + m_p g \cdot d\cos\theta = R_w \cdot \ell\sin\theta$.

Common pitfalls

- Forgetting that smooth means "no friction" (only normal reaction).
- Using $F = \mu R$ when the body is *not* at limiting equilibrium — only valid when about to slip.
- Confusing which contact is on the point of slipping.
- Forgetting that the wall friction at the top acts *upwards* on the ladder (opposing tendency to slip down).

Strategy

1. Draw a free-body diagram with every force labelled.
2. Resolve horizontally and vertically.
3. Take moments about the foot (or another convenient point).
4. Apply $F = \mu R$ at any limiting contact.

Questions begin overleaf.

SECTION T4

Equilibrium of Rigid Bodies

Questions 1–6 · 72 marks

6. A uniform ladder, of mass 37·5 kg and length 8 m, rests with its top end against a smooth vertical wall and its bottom end on rough horizontal ground. The ladder is inclined at an angle of 60° to ground. The coefficient of friction between the ladder and the ground is μ. A person, of mass 75 kg, climbs the ladder.

(a) Given that $\mu = 0{\cdot}25$, determine how far the person can ascend before the ladder slips. [10]

(b) Given that the person is able to ascend to the top of the ladder, determine the minimum value of μ. [4]

(c) State **one** modelling assumption you have made in your solution. [1]

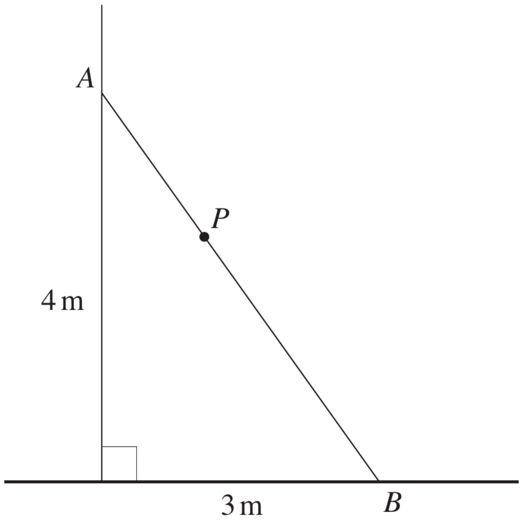
3. A uniform ladder AB, of length 2 m and **weight** 90 N, rests with one end A on rough horizontal ground and the other end B against a smooth vertical wall. The ladder is inclined at an angle θ to the **vertical**, where $\tan\theta = 0{\cdot}8$. The coefficient of friction between the end of the ladder and the ground is 0·6. A force of magnitude P N is applied to the ladder at a point which is $\frac{1}{2}$ m from the end A; the direction of this force is horizontal and towards the wall. The end A is on the point of moving towards the wall.

(a) Draw a diagram and clearly show all the forces acting on the ladder. [2]

(b) Show that $P = 120$. [10]

(c) State **one** modelling assumption you have made about the ladder in your solution. [1]

6. A uniform ladder *AB*, of length 5 m and mass 20 kg, rests with end *A* against a rough vertical wall and end *B* on rough horizontal ground. The vertical distance of *A* from the ground is 4 m, and the horizontal distance of *B* from the wall is 3 m. When a man *P*, of mass 80 kg, stands on the ladder 3 m from the lower end, the frictional force at *A* is limiting. The coefficient of friction between the ladder and the wall is 0·3.



(a) Find the normal reaction at *A*. [6]

(b) Find the least value of the coefficient of friction between the ladder and the ground. Give your answer correct to three significant figures. [5]

6. A uniform ladder *AB*, of length 8 m and mass 12 kg, rests with its top end *A* against a smooth vertical wall and its bottom end *B* on rough horizontal ground. The ladder is inclined at an angle of 75° to the horizontal. A light inextensible rope is attached to the ladder 3 m from the bottom and tied to the wall so that the rope is horizontal. The rope and the ladder are in the same vertical plane. The coefficient of friction between the ladder and the ground is 0·1. A man of mass 70 kg climbs the ladder.

(a) Draw a diagram clearly showing all the forces acting on the ladder. [2]

(b) Write down the magnitude of the normal reaction of the ground on the ladder. [2]

(c) The rope will break when the tension reaches 100 N. Determine how far up the ladder the man can climb before the rope breaks. [8]

(d) State one modelling assumption you have made about the ladder in your solution. [1]

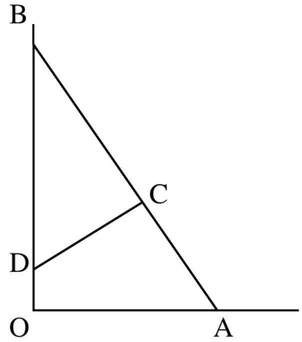
END OF PAPER

6. A uniform ladder of mass 20 kg and length 6 m rests with its top end against a smooth vertical wall and its bottom end on rough horizontal ground. The ladder is inclined at an angle θ to the horizontal. The coefficient of friction between the ladder and the ground is 0·6. A man of mass 80 kg climbs the ladder. When he reaches $\frac{5}{6}$ of the way up, the ladder is in limiting equilibrium.

Calculate the normal reaction at the wall and the value of θ. State one modelling assumption you have made about the ladder in your solution. [9]

END OF PAPER

ground and leans on a smooth vertical wall. It is kept in equilibrium by a light inextensible rope, attached at one end to the ladder at the point C and attached at the end to a point D on the wall. The rope CD is perpendicular to and in the same vertical plane as AB, as shown in the diagram.



The point O is the corner of the wall and the ground. Distances OA = 6 m, OB = 8 m and AC = 4 m.

(a) Calculate the magnitudes of the tension in the rope and the reactions of the wall and the ground on the ladder. [10]

(b) A man, of mass 80 kg, climbs the ladder. The rope will break when the tension exceeds 2000 N.

Determine whether the man will be able to reach the top of the ladder.

(c) State one modelling assumption which you have made in your solution. [1]

END OF EQUILIBRIUM OF RIGID BODIES PACK

Source: WJEC M3 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec A2 Unit 6 — Topic 4 (2.6.4)