Name	Date started	Target end date

GCE A LEVEL — FURTHER MECHANICS B QUESTION PACK

0982-01 (Legacy M3) · New spec A2 Unit 6 Topic 3

REVISE
.wales

FURTHER MATHS — FM B · MOMENTS & CENTRE OF MASS

Moments & Centre of Mass — Coplanar Particles, Laminae & Rigid Bodies

The legacy WJEC M2/M3 papers (2008 modular spec) do not contain dedicated centre-of-mass-of-a-lamina questions — that content sat in legacy M1. This pack therefore exercises the principle of moments and the uniform-body COM convention through rigid-body moment problems where the placement of the centre of mass is the load-bearing step. M3 June 2017 Q6(b) is the one legacy question that explicitly asks for the position of the COM of a non-uniform body.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 1 minutes

*Derived from the legacy M3 paper's pace of **~1.3 min/mark** (93 marks over 7 questions). The full Unit 6 exam is **1 hour 45 minutes for 80 marks** (25% of the A-level qualification, A2 optional paper alongside Unit 5 Further Statistics B).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every moments & centre of mass question from the legacy WJEC M3 papers (2008 modular spec) that maps onto new-spec A2 Unit 6 Topic 3 (2.6.3). Unit 6 (Further Mechanics B) is one of two **80-mark A2 optional papers** (the other being Unit 5 Further Statistics B), each worth 25% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet for Mechanics may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 08 Q5	13	
2	Jun 09 Q6	12	
3	Jun 11 Q6	13	
4	Jun 12 Q6	14	
5	Jun 13 Q6	13	
6	Jun 16 Q6	14	
7	Jun 17 Q6	14	
	Total	93	

Moments & Centre of Mass — Coplanar Particles, Laminae & Rigid Bodies — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 6: Further Mechanics B · Topic 2.6.3.

Coplanar particles 2.6.3

- For particles of masses m1, ..., mn at positions r1, ..., rn:
- Centre of mass: weighted sum mi ri divided by total mass sum mi.
- In coordinates: xbar = sum(mi xi) / sum(mi), ybar = sum(mi yi) / sum(mi).
- A uniform rod has its centre of mass at its midpoint (by symmetry).

Uniform laminae 2.6.3

- Triangle — centre of mass at the centroid: one-third of the way along each median from the side.
- Rectangle — centre of mass at the intersection of its diagonals.
- Semicircle of radius r — centre of mass at 4r/(3 pi) from the straight edge along the axis of symmetry.
- Quarter-circle of radius r — centre of mass at 4r/(3 pi) from each bounding edge.

Composite shapes 2.6.3

- Treat each sub-shape as a point mass located at its own centre of mass.
- Apply the same weighted-average formula across the sub-shapes.
- For a shape with a hole, treat the hole as *negative* mass.
- Use of symmetry — if a lamina has a line of symmetry, its COM lies on that line.

Suspended bodies 2.6.3

- A lamina suspended freely from a point hangs so that its centre of mass lies vertically below the point of suspension.
- Angle of inclination is determined by geometry: connect the suspension point to the COM, measure against the desired reference.
- The centre of mass of a uniform rigid body lies on the axis of symmetry (where one exists).
- For non-standard shapes, use integration: xbar = integral(x dm) / integral(dm).

Moments & Centre of Mass in one page

Quick-reference notes — revisit before each section. Don't use during questions.

COM of n particles

For point masses $m_1, m_2, \ldots, m_n$ at positions $\vec{r}_1, \ldots, \vec{r}_n$:

$$\bar{\vec{r}} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

Coordinates: $\bar{x} = \dfrac{\sum m_i x_i}{\sum m_i}$.

Uniform triangle

For a uniform triangular lamina, the centre of mass is at the *centroid*:

One-third of the way along each median, measured from the side.

Equivalent: $\bar{\vec{r}} = \frac{1}{3}(\vec{r}_A + \vec{r}_B + \vec{r}_C)$ for vertices A, B, C.

Uniform semicircle

Semicircle of radius r: COM lies on the axis of symmetry at

distance $= \dfrac{4r}{3\pi}$ from the straight edge

(Quarter-circle: $\dfrac{4r}{3\pi}$ from each bounding straight edge.)

Composite shapes

Treat each sub-shape as a point mass at its own COM.

For a shape with a hole, treat the hole as *negative* mass:

$$\bar{x} = \frac{m_1 x_1 - m_h x_h}{m_1 - m_h}$$

Uniform rod and rectangle

Uniform rod: COM at midpoint.

Uniform rectangle: COM at the intersection of its diagonals.

For a rigid body, the COM lies on every axis of symmetry.

Integration for COM

For a non-standard shape with mass density ρ:

$$\bar{x} = \frac{\int x\,dm}{\int dm}$$

If the lamina has surface density σ: $dm = \sigma\,dA$.

Suspension geometry

When a lamina is suspended freely from a point P, it comes to rest with its COM G vertically below P.

Inclination angle to a side or axis is computed from the geometry of PG.

$\tan\theta = \dfrac{\text{horizontal offset of } G}{\text{vertical drop of } G}$ — relative to the natural reference.

Moments about a pivot

The moment of a force F about a point O is $F \cdot d$, where d is the perpendicular distance from O to the line of action.

For a rigid body in equilibrium, $\sum M_O = 0$ about *any* point.

Pick O to be where unknown forces act — their moments are zero, simplifying the equation.

Strategy

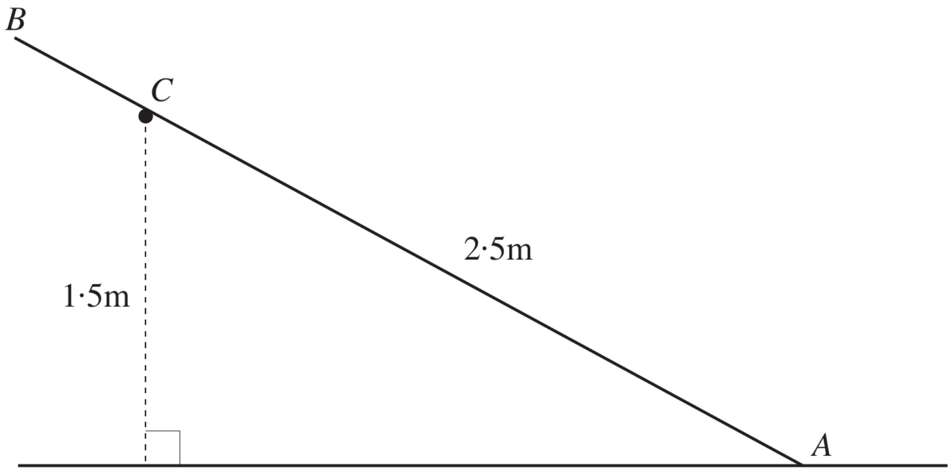
1. For particles or composite: sum mass-weighted positions.
2. Use symmetry to locate COM where possible.
3. For laminae, recall the standard formulas.
4. For suspension, find COM then use geometry.

SECTION T3

Moments & Centre of Mass

Questions 1–7 · 93 marks

5. A small smooth peg C is fixed at a height of 1·5 m above the horizontal ground. A uniform rod AB, of mass 20 kg and length 3 m, rests on C with the end A on the rough ground as shown in the diagram.

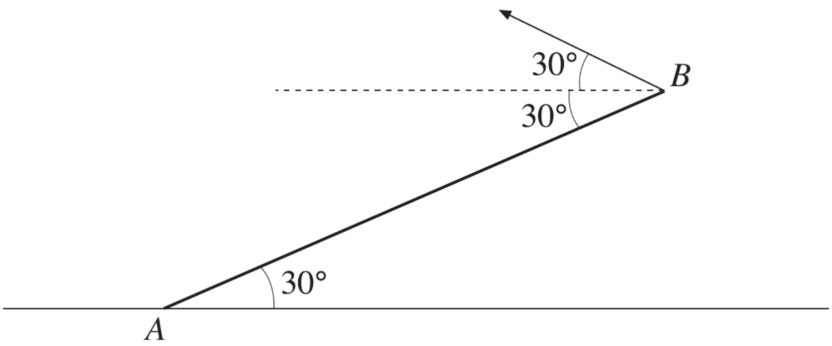


The rod AB is at rest in limiting equilibrium with $AC = 2{\cdot}5$ m.

(a) Calculate the magnitude of the reaction of C on the rod AB. [5]

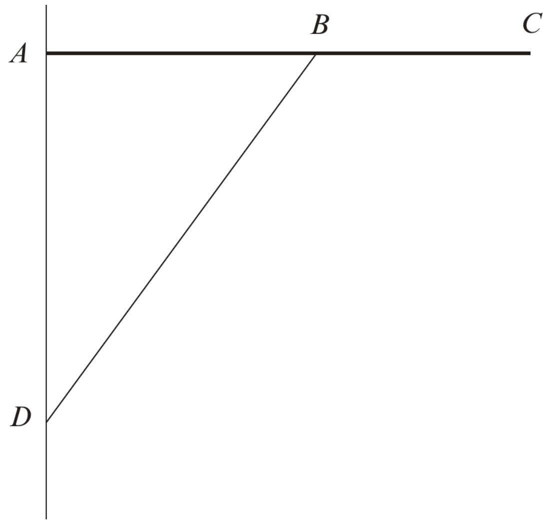
(b) Find the coefficient of friction between the rod AB and the rough ground, giving your answer correct to two decimal places. [8]

6. The diagram shows a uniform rod AB of mass 15 kg with its lower end A resting on a rough horizontal floor. A string is attached to the end B of the rod and applies a force on the rod at B in the direction shown in the diagram. The rod is in equilibrium when it is inclined at an angle of 30° to the floor. The coefficient of friction between the rod and the floor is μ.



Find the least possible value for μ. [12]

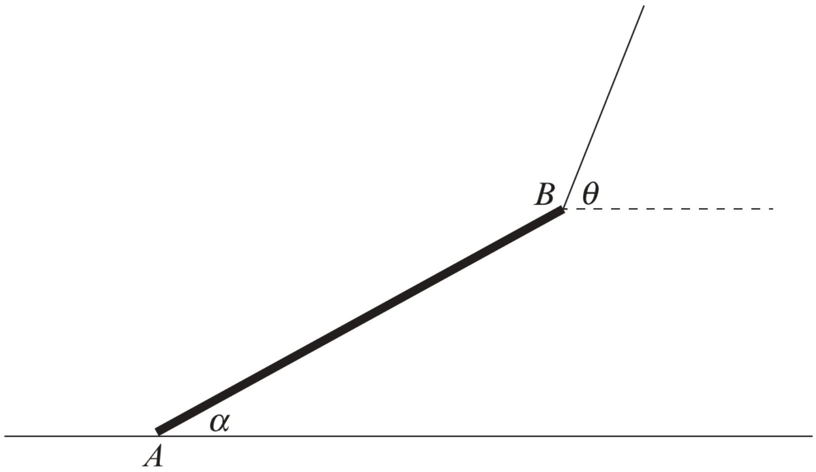
6. The diagram shows a uniform plank *AC*, of mass 15 kg and length 1·2 m, hinged to a vertical wall at *A*. The plank is supported in a horizontal position by a fixed light rod *BD*, where *D* is on the wall and *B* is the midpoint of *AC*. The length *AD* is 0·8 m. A boy leans on the plank at *C* exerting a force of 20 N vertically downwards.



(a) Find the thrust in the rod *BD*. [5]

(b) Calculate the magnitude and direction of the reaction at the hinge *A*. [8]

6. The diagram shows a straight uniform beam AB of weight 2100 N and length 2 m resting in equilibrium with its end A on rough horizontal ground. A light cable, which is attached to the other end B, is holding the beam with the end B off the ground so that the beam makes an angle α with the ground, where $\tan\alpha = \frac{5}{12}$. The cable makes an angle θ with the horizontal.

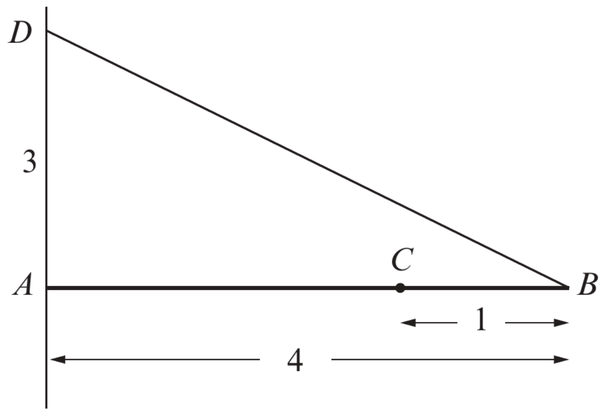


The coefficient of friction between the end A and the ground is $\frac{3}{4}$. Given that the end A of the beam is about to slip,

(a) find the normal reaction of the ground on the beam at A, [6]

(b) calculate the tension in the cable and the value of the angle θ. [8]

6. The diagram shows a uniform rod AB, of mass 6 kg and length 4 m, held in a horizontal position by means of a light inextensible string BD, where D is a point 3 m vertically above A. The end A of the rod rests against a rough vertical wall. A particle of mass 3 kg is attached to the rod at C, where $BC = 1$ m. The rod is in limiting equilibrium in a vertical plane perpendicular to the wall.



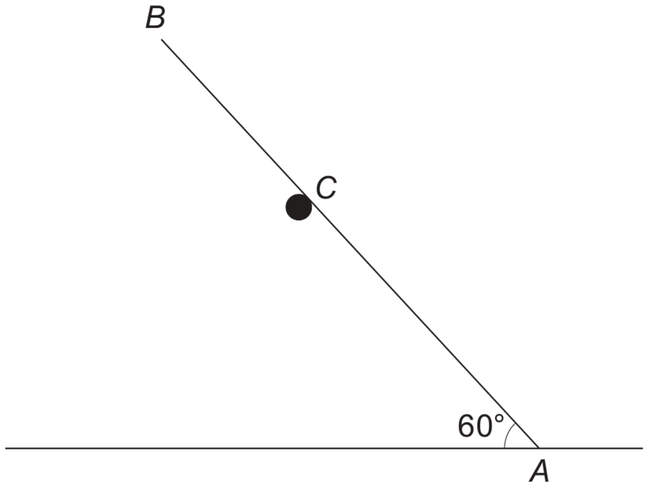
(a) Calculate the tension in the string. [4]

(b) Find the vertical component and the horizontal component of the force exerted by the wall on the rod.
Hence find

(i) the magnitude of the resultant force exerted by the wall on the rod,

(ii) the coefficient of friction between the rod and the wall. [9]

6. The diagram shows a uniform rod AB, of length 10 m and mass 25 kg, in limiting equilibrium with its end A on rough horizontal ground and point C resting against a smooth fixed peg. The rod is inclined at an angle of 60° to the ground.



The distance AC is x m and the coefficient of friction between the rod and the ground is 0·3.

(a) Draw a diagram showing all the forces acting on the rod. Label all points and forces clearly. [2]

(b) Determine the magnitude of the reaction at C and the magnitude of the normal reaction at A. [8]

(c) Find the value of x. [4]

END OF PAPER

6. A ladder *AB*, of length 8 m and weight W N, rests with one end *A* against a vertical wall and the other end *B* on horizontal ground. The ladder makes an angle α with the horizontal where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the ladder and the wall is λ and the coefficient of friction between the ladder and the ground is μ.

(a) Consider the case when the ladder is **uniform**. Given that $\lambda = 0$ and the ladder is on the point of slipping, determine the value of μ in this case. [4]

(b) Consider the case when the ladder is **non-uniform** and its centre of mass is x m from *A*. Given that $\lambda = \mu = 0{\cdot}6$ and the ladder is on the point of slipping, calculate the value of x in this case. [10]

END OF PAPER

END OF MOMENTS & CENTRE OF MASS PACK

Source: WJEC M3 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec A2 Unit 6 — Topic 3 (2.6.3)