Name	Date started	Target end date

GCE A LEVEL — FURTHER MECHANICS B QUESTION PACK

0981-01 (Legacy M2) · New spec A2 Unit 6 Topic 1

REVISE
.wales

FURTHER MATHS — FM B · RECTILINEAR MOTION

Rectilinear Motion — Variable Acceleration & the Chain Rule

Every rectilinear-motion / variable-acceleration question from the legacy WJEC M2 papers (June 2005 – June 2017 + Specimen) and a couple from M3 that map onto the new-spec A2 Unit 6 Topic 1.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 56 minutes

*Derived from the legacy M2/M3 paper's pace of **~1.3 min/mark** (135 marks over 14 questions). The full Unit 6 exam is **1 hour 45 minutes for 80 marks** (25% of the A-level qualification, A2 optional paper alongside Unit 5 Further Statistics B).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every rectilinear motion question from the legacy WJEC M2/M3 papers (2008 modular spec) that maps onto new-spec A2 Unit 6 Topic 1 (2.6.1). Unit 6 (Further Mechanics B) is one of two **80-mark A2 optional papers** (the other being Unit 5 Further Statistics B), each worth 25% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet for Mechanics may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 05 Q2	12	
2	Jun 06 Q3	9	
3	Jun 07 Q1	11	
4	Jun 08 Q3	13	
5	Jun 09 Q1	10	
6	Jun 10 Q1	10	
7	Jun 11 Q1	8	

Q	Source	Max	Mark
8	Jun 13 Q5	7	
9	Jun 14 Q2	10	
10	Jun 15 Q2	10	
11	Jun 16 Q1	9	
12	Jun 17 Q2	9	
13	Spec. Q1	6	
14	Jun 15 Q1	11	
Total		**135**	

Rectilinear Motion — Variable Acceleration & the Chain Rule — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 6: Further Mechanics B · Topic 2.6.1.

Rectilinear motion 2.6.1

- Form and solve simple equations of motion in which acceleration is given as a function of time, displacement or velocity.
- Velocity may be given as a function of time or displacement.
- Use of $a = dv/dt$, $a = v\,dv/dx$, $a = d^2x/dt^2$.
- Resulting first-order differential equations are usually separable.

Acceleration as a function of t 2.6.1

- Easiest case: integrate a once to get v, once more to get x.
- $v(t) = v(0) + \int_0^t a(s)\,ds$.
- $x(t) = x(0) + \int_0^t v(s)\,ds$.
- Initial conditions are essential — track them through every integration.

Acceleration as a function of v 2.6.1

- Use $dv/dt = a(v) \rightarrow t = \int dv/a(v)$.
- For position: $v\,dv/dx = a(v) \rightarrow x = \int v\,dv/a(v)$.
- Choose the form that puts the unknown on the right side and the known v on the left.
- Typical setting: motion against resistance kv or kv^2.

Acceleration as a function of x 2.6.1

- Always use the chain rule $a = v\,dv/dx$.
- Gives separable equation $v\,dv = a(x)\,dx$.
- Integrate both sides $\rightarrow \frac{1}{2}v^2 = \int a(x)\,dx + \text{const}$.
- Common when motion is governed by a position-dependent force such as Hooke or inverse-square.

Crib sheet »

Rectilinear Motion in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Three forms of a

The acceleration of a particle moving on the x-axis can be written as:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$a = v\,\frac{dv}{dx} \quad \text{(chain rule)}$$

Pick the form that matches the variable a is given as a function of.

Acceleration as a fn of t

If $a = a(t)$, integrate directly:

$$v(t) = v_0 + \int_0^t a(s)\,ds$$

$$x(t) = x_0 + \int_0^t v(s)\,ds$$

Always carry initial conditions v_0 and x_0 through.

Acceleration as a fn of v

If $a = a(v)$, separate:

$$\frac{dv}{dt} = a(v) \Rightarrow t = \int \frac{dv}{a(v)}$$

For position, use the chain rule:

$$v\,\frac{dv}{dx} = a(v) \Rightarrow x = \int \frac{v\,dv}{a(v)}$$

Acceleration as a fn of x

If $a = a(x)$, use the chain rule:

$$v\,dv = a(x)\,dx$$

Integrate to get the "energy-like" relation:

$$\tfrac{1}{2}v^2 = \int a(x)\,dx + C$$

Resistance to motion

Common setting: a force minus a resistive force.

If $m\dot{v} = F - kv$: gives a linear DE in v with exponential approach to terminal speed F/k.

If $m\dot{v} = F - kv^2$: separable in v; terminal speed $\sqrt{F/k}$.

Use partial fractions or substitution as appropriate.

Boundary conditions

Each integration introduces a constant. Use the given initial conditions to pin them down:

At $t = 0$: $v(0) = v_0$, $x(0) = x_0$.

For an $a(x)$ problem, v is often given at a specific x (e.g. $v = u$ when $x = 0$).

Substitute the boundary conditions *after* integrating, not before.

Distance vs displacement

Displacement = signed change in position.

Distance travelled $= \int_0^T |v(t)|\,dt$.

If v changes sign at $t = t^*$, split:
distance $= \int_0^{t^*} v\,dt + \int_{t^*}^T (-v)\,dt$.

Common pitfalls

- Confusing $a = dv/dt$ (time) with $a = v\,dv/dx$ (position).
- Forgetting the constant of integration.
- Treating "total distance" as the same as "displacement".
- Confusing terminal velocity (when $\dot{v} = 0$) with stopping.

Strategy

1. Identify which variable a depends on (t, x or v).
2. Pick the matching form (dv/dt, $v\,dv/dx$, or $dv/dt = a(v)$).
3. Separate and integrate; carry constant.
4. Apply initial conditions; check units.

SECTION T1

Rectilinear Motion

Questions 1–14 · 135 marks

2. A particle moves in a straight line such that its acceleration a ms^{-2} is given by

$$a = 4 - 6t \quad \text{for} \quad t \geqslant 0.$$

At time $t = 0$, the particle is at the point O and its velocity is 4 ms^{-1}.

(a) Find an expression for the velocity of the particle at time t s. [3]

(b) Find an expression for the displacement of the particle from O at time t s. [3]

(c) Determine the time when the particle comes to rest instantaneously and the distance of the particle from O at this time. [3]

(d) Calculate the **speed** of the particle when $t = 3$, and determine whether or not the **speed** of the particle is increasing or decreasing at this time. [3]

3. A particle P, of mass 3 kg, moves along the horizontal x-axis under the action of a resultant force F N. Its velocity v ms^{-1} at time t seconds is given by

$$v = 12t - 3t^2.$$

(a) Given that the particle is at the origin O when $t = 1$, find an expression for the displacement of the particle from O at time t s. [4]

(b) Find the acceleration of the particle at time t s. [2]

(c) Find the power of the force F when $t = 1{\cdot}5$. [3]

1. A particle P is projected from the origin O so that it moves along the x-axis. At time t s after projection, the velocity of the particle, $v\,\text{ms}^{-1}$, is given by

$$v = 3t^2 - 24t + 45.$$

(a) Show that P first comes to instantaneous rest when $t = 3$. [2]

(b) Find an expression for the acceleration of P at time t s. [2]

(c) Find an expression for the displacement of P from O at time t s. [3]

(d) Find the distance travelled by the particle in the first 3 seconds of its motion. [2]

(e) Find the distance travelled by the particle in the first 4 seconds of its motion. [2]

3. A particle, of mass 5 kg, moves in a straight line under the action of a single force whose magnitude F N at time t s is given by

$$F = 15t^2 - 60t, \qquad t \geqslant 0.$$

(a) Find the acceleration of the particle when $t = 2$. [2]

(b) The velocity of the particle at time t s is denoted by v ms^{-1}. Given that $v = 35$ when $t = 0$, find an expression for v in terms of t. [4]

(c) Calculate the least value of the speed of the particle. [3]

(d) Determine the distance travelled by the particle between $t = 2$ and $t = 8$. [4]

1. A particle moves along the x-axis and its velocity v ms^{-1} at time t s is given by

$$v = \cos 2t - 3\sin t.$$

(a) Find the acceleration of the body when $t = \pi$. [4]

(b) Given that $x = 4$ when $t = 0$, calculate the distance of the particle from the origin O when $t = \frac{\pi}{4}$. [6]

1. A particle P moves in a straight line so that its acceleration $a\,\text{ms}^{-2}$ at time $t\,\text{s}$, is given by

$$a = 3 - 4t.$$

At time $t = 0$, the particle P passes through the point O and its velocity is $-1\,\text{ms}^{-1}$.

(a) Find an expression for the velocity of P at time $t\,\text{s}$. [4]

(b) Find the values of t when P is instantaneously at rest. [2]

(c) Find the distance between the points at which P is instantaneously at rest. [4]

1. A particle moves along the x-axis and its velocity $v\,\text{ms}^{-1}$ at time $t\,\text{s}$ is given by

$$v = 12\sin 3t - 8\cos 2t.$$

(a) Find an expression for the acceleration of the particle at time $t\,\text{s}$. [3]

(b) Given that at time $t = 0$ the particle is at the origin O, find an expression for the displacement of the particle from O at time $t\,\text{s}$. [5]

5. A particle moves along a straight horizontal line. Its velocity $v\,\text{ms}^{-1}$ at time $t\,\text{s}$, is given by

$$v = 2t(t - 6).$$

(a) Find the set of values of t for which the velocity of the particle is negative. [2]

(b) Find the total distance travelled by the particle in the interval $0 \leqslant t \leqslant 9$. [5]

2. A particle of mass 5 kg moves under the action of a horizontal force given by $F = 30t^{-2} - 30$ N at time t s, where $t > 0$. It also experiences a constant resistance to motion of magnitude 120 N.

(a) Show that the motion of the particle satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 6t^{-2} - 30,$$

where v ms^{-1} is the velocity of the particle at time t s. [2]

(b) Calculate the value of t when the acceleration of the particle is 24 ms^{-2}. [2]

(c) Given that the velocity of the particle is 18 ms^{-1} when $t = \frac{1}{3}$, find an expression for v in terms of t. Hence find the values of t when v = 10. [6]

2. An object of mass 50 kg moves in a straight horizontal line under the action of a constant horizontal force of magnitude 1600 N acting along the line. The resistance to motion of the object is proportional to time t seconds. At time t seconds, the velocity of the object is v ms^{-1} and at time $t = 2$, it is moving with velocity 41 ms^{-1} and acceleration -4 ms^{-2}.

(a) Show that v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 32 - 18t.$$ [4]

(b) Find an expression for v in terms of t and determine the times when the velocity of the object is 28 ms^{-1}. [6]

1. A particle of mass 4 kg moves along the x-axis, starting, when $t = 0$, from the point where $x = 3$. At time t s, its velocity v ms^{-1} is given by

$$v = 12t^2 - 7kt + 1,$$

where k is constant.
When $t = 2$, the displacement of the particle from the origin is 16 m.

(a) Determine the value of k. [5]

(b) Calculate the magnitude of the force acting on the particle when $t = 5$. [4]

2. A particle *P*, of mass 0·8 kg, moves along the x-axis so that its velocity at time t seconds is v ms^{-1}, where $v = 4t^3 - 6t + 7$. Given that the displacement of *P* is 5 m from the origin when $t = 0$, find

(a) the displacement of *P* from the origin when $t = 2$, [5]

(b) the force acting on *P* when $t = 3$. [4]

$$v = 3t^2 + 10t^4.$$

(a) Find the acceleration of the particle at time t s. [2]

(b) Given that when $t = 0$, $x = -3$, find the displacement of the particle at time $t = 2$ s. [4]

1. A particle of mass 400 kg moves along a straight horizontal road under the action of a horizontal force F. The magnitude of the force F may be modelled by $500\left(\frac{x}{v+2}\right)$ N, where v ms^{-1} is the speed of the particle and x m is the distance of the particle from a point O on the road.

(a) Show that the motion of the particle satisfies the differential equation

$$4v(v+2)\frac{\mathrm{d}v}{\mathrm{d}x} = 5x.$$ [2]

(b) When $x = 0$, the particle is at rest.

(i) Find an expression for x in terms of v.

(ii) Find the distance of the particle from O and the acceleration of the particle when its speed is 3 ms^{-1}. [9]

END OF RECTILINEAR MOTION PACK

Source: WJEC M2/M3 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec A2 Unit 6 — Topic 1 (2.6.1)