

Name	Date started	Target end date
------	--------------	-----------------

GCE A LEVEL – FURTHER STATISTICS B QUESTION PACK

0984-01 (Legacy S2) & 0985-01 (Legacy S3) · New spec A2 Unit 5 Topic 5

REVISE

.wales

FURTHER MATHS – FS B · CONFIDENCE INTERVALS – POPULATION MEAN

Confidence Intervals for a Single Population Mean – Known σ (z -CI) and Unknown σ (t -CI)

Every confidence-interval question for a single normal mean from the legacy WJEC S2 (z -CI, σ known) and S3 (t -CI, σ unknown) papers that maps onto the new-spec A2 Unit 5 Topic 4.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~2 hours 5 minutes

Derived from the legacy S2/S3 paper's pace of ~1.3 min/mark (96 marks over 12 questions). The full Unit 5 exam is 1 hour 45 minutes for 80 marks (25% of the A-level qualification, A2 optional paper alongside Unit 6 Further Mechanics B).

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every confidence intervals – population mean question from the legacy WJEC S2/S3 papers (2008 modular spec) that maps onto new-spec A2 Unit 5 Topic 5 (2.5.4). Unit 5 (Further Statistics B) is one of two **80-mark A2 optional papers** (the other being Unit 6 Further Mechanics B), each worth 25% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working – method marks are awarded for clear setup.

A calculator is allowed. The WJEC Statistical Tables and Formula Booklet may be referred to.

All question content is © WJEC CBAC Ltd. and reproduced for revision purposes.

For Examiner's use only

Q	Source	Max	Mark	Q	Source	Max	Mark
1	Jun 05 Q1	6		7	Jun 14 Q1	6	
2	Jun 06 Q1	6		8	Jun 16 Q2	10	
3	Jun 08 Q4	8		9	Spec. Q3	8	
4	Jun 09 Q3	9		10	Jun 07 Q2	9	
5	Jun 11 Q2	9		11	Jun 15 Q2	10	
6	Jun 12 Q3	6		12	Jun 17 Q5	9	
Total						96	

Confidence Intervals for a Single Population Mean – Known σ (z -CI) and Unknown σ (t -CI) – what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 5: Further Statistics B · Topic 2.5.4.

CI for μ – known σ 2.5.4

- Sample X_1, \dots, X_n from $N(\mu, \sigma^2)$ with σ known.
- $(1 - \alpha) \cdot 100\%$ CI: $\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$.
- Standard z -values: $z_{0.05} \approx 1.645$, $z_{0.025} \approx 1.96$, $z_{0.005} \approx 2.576$.
- Most legacy S2 CIs are exactly this form – jumps, drive times, train times, light bulbs all give σ explicitly.

CI for μ – unknown σ 2.5.4

- Sample from $N(\mu, \sigma^2)$, σ unknown; small n .
- $(1 - \alpha) \cdot 100\%$ CI uses Student's t -distribution: $\bar{x} \pm t_{n-1, \alpha/2} \cdot \frac{s}{\sqrt{n}}$.
- First compute $s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$ from the data.
- WJEC t -CIs: discuss (S3 June 2007 Q2), crossword (S3 June 2015 Q2), javelin (S3 June 2017 Q5).

Sample-size questions 2.5.4

- Margin of error $ME = z_{\alpha/2} \cdot \sigma / \sqrt{n}$.
- To find n for a target ME: solve $z\sigma / \sqrt{n} \leq ME$, so $n \geq (z\sigma/ME)^2$.
- Halving CI width quadruples n (because \sqrt{n} goes into the denominator).
- Round up to the next integer – you can't take a fractional sample.

Interpretation 2.5.4

- A 95% CI does **not** mean “there is a 95% probability that μ lies in the interval”.
- It means: *if we repeated the sampling many times, about 95% of the resulting CIs would contain the true μ .*
- Once an interval is computed, the parameter either is or isn't in it – there is no probability statement to make about it.
- S2 Specimen Q3 and S3 June 2017 Q4 explicitly test this interpretation.

Confidence Intervals – Population Mean in one page

Quick-reference notes – revisit before each section. Don't use during questions.

CI for μ , known σ

For sample X_1, \dots, X_n from $N(\mu, \sigma^2)$ with σ known:

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

For 95%: $z_{0.025} = 1.96$.

z -values for common levels

90%: $z = 1.645$

95%: $z = 1.96$

98%: $z = 2.326$

99%: $z = 2.576$

These are *two-sided* critical values (i.e. $z_{\alpha/2}$).

CI with t -distribution

For sample from $N(\mu, \sigma^2)$ with σ **unknown**:

$$\bar{x} \pm t_{n-1, \alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Use t -tables with $n - 1$ degrees of freedom.

s is the unbiased sample standard deviation.

Computing \bar{x} , s from data

$$\bar{x} = \frac{1}{n} \sum x_i$$

$$s^2 = \frac{1}{n-1} (\sum x_i^2 - n\bar{x}^2)$$

$s = \sqrt{s^2}$ (the unbiased sample standard deviation).

Margin of error and n

Margin of error $ME = z_{\alpha/2} \cdot \sigma / \sqrt{n}$.

To halve the width: quadruple n .

To find n for a target ME: solve $z\sigma/\sqrt{n} \leq ME$, so $n \geq (z\sigma/ME)^2$, round up.

Interpretation

A 95% CI does **not** mean "there is a 95% probability that μ lies in the interval".

It means: *if we repeated the sampling many times, about 95% of resulting CIs would contain the true μ .*

Once an interval is computed, the parameter either is or isn't in it.

Reading the question

If σ is given numerically $\Rightarrow z$ -CI.

If σ is "unknown" or only the variance estimator is asked first $\Rightarrow t$ -CI.

Width of CI = $2 \cdot z_{\alpha/2}\sigma/\sqrt{n}$ (or with t , s).

Common pitfalls

- Using σ when only s is known – switch to t -distribution.
- Wrong z -value: 1.96 is for 95%, not 99%.
- Forgetting \sqrt{n} in the denominator – using σ/n instead.
- Wrong sign convention (subtract on both sides, not just left).

Strategy

- Decide: is σ known or estimated from data?
- Pick z (known σ / large n) or t -value (unknown σ , small n).
- Compute $\bar{x} \pm$ critical value \cdot SE.
- Interpret in context.

SECTION T5

Confidence Intervals for a Single Population Mean – Known σ (z -CI) and Unknown σ (t -CI)

Questions 1-12 · 96 marks

1. Ann records the times taken (in minutes) to drive to work on twelve consecutive days with the following results.

73.3 74.7 71.2 75.8 70.0 74.0 80.5 70.5 72.6 73.4 69.1 76.9

Assume that these observations form a random sample from a normal distribution with standard deviation 4.0.

- (a) Calculate a 95% confidence interval for the mean time taken by Ann to drive to work. [5]
- (b) Ann believed that the mean time was 75 minutes. State, with a reason, whether or not your result supports this belief. [1]

1. Bill is an athlete specialising in the long-jump. He does 10 jumps at a training session with the following results (in metres).

6.21 6.33 6.02 6.11 6.13 6.40 6.51 6.29 6.16 6.44

You may assume that the above distances form a random sample from a normal distribution with unknown mean μ metres and standard deviation 0.1 metres.

Calculate a 95% confidence interval for μ .

Bill had stated beforehand that the mean length of his jumps was 6.3 metres.

Giving a reason, state whether or not your interval supports this claim.

[6]

4. When a certain instrument is used to measure the concentration of a solution, the reading obtained (in appropriate units) is a normally distributed random variable with mean equal to the actual concentration and standard deviation 0.05. The concentration of a particular solution was measured eight times and the following independent readings were obtained.

6.02 6.10 5.98 6.04 6.07 5.94 6.05 6.12

- (a) Calculate a 99% confidence interval for the concentration of this solution. [5]
- (b) Find the minimum number of measurements required to obtain a 99% confidence interval of width less than 0.04. [3]

3. A gardener wishes to estimate the acidity level of the soil in his garden. He therefore takes ten readings of the acidity level with the following results.

6.62, 6.84, 6.77, 6.52, 6.86, 6.51, 6.82, 6.71, 6.49, 6.66

You may assume that this is a random sample from a normal distribution with standard deviation 0.1

- (a) Calculate a 99% confidence interval for the acidity level of his soil. [5]
- (b) A friend uses the same data to calculate a confidence interval and obtains the following result.

[6.62, 6.74]

Calculate the confidence level of this interval. [4]

2. The random variable X has a normal distribution with unknown mean μ and standard deviation 0.5.
- (a) A random sample of 60 values of X was taken and it was found that $\sum x = 1290$. Calculate a 95% confidence interval for μ , giving the end-points of your interval correct to two decimal places. [5]
- (b) Determine the minimum sample size required for the width of a 95% confidence interval for μ to be less than 0.1. [4]

3. The lifetime, X thousand hours, of a certain type of electric light bulb may be assumed to be normally distributed with unknown mean μ and standard deviation 0.1. The lifetimes of a random sample of 75 of these bulbs were measured and it was found that $\sum x = 69.9$.
- (a) Find a 90% confidence interval for μ . [5]
- (b) Give an interpretation of this confidence interval. [1]

1. The times taken, in minutes, for trains to travel between two stations on a particular day were recorded and are given below.

48.2 49.4 56.2 44.6 47.3 55.2 50.8 53.9

It may be assumed that this is a random sample from a normal distribution with mean μ mins and standard deviation 4 mins. Determine a 90% confidence interval for μ . [6]

2. Sue keeps chickens in her garden. She selects, at random, 10 of the eggs produced and weighs them. The results, in grams, are shown below.

62.5 64.2 61.5 65.2 66.2 63.8 60.1 63.2 64.4 66.1

You may assume that this is a random sample from a normal distribution with a standard deviation of 1.9.

- (a) Determine a 95% confidence interval for the mean weight of eggs produced by Sue's chickens. [6]
- (b) Sue was hoping to obtain a 95% confidence interval of width 1 at most. Calculate the minimum sample size necessary to achieve this. [4]

3. Ann drives to work and she records the times taken over a 10-day period with the following results (in minutes):

72 76 69 77 81 74 71 69 72 74

You may assume that the time taken to drive to work can be modelled by a normally distributed random variable with standard deviation 4 minutes,

- (a) Calculate a 95% confidence interval for the mean time. [6]
- (b) Explain what is meant by a 95% confidence interval. [2]

2. When Bill throws the discus, the distance thrown (in metres) can be assumed to be normally distributed with mean μ and variance σ^2 . He throws the discus 10 times with the following results.

25.3 23.8 24.7 24.9 23.7 25.6 24.6 24.0 25.3 24.1

- (a) Calculate unbiased estimates of μ and σ^2 . [5]
- (b) Calculate a 95% confidence interval for μ . [4]

2. Emlyn solves the crossword in the Daily Bugle every day. He records the time taken to do this on 12 randomly chosen days with the following results (in minutes).

16.3 17.4 14.3 15.6 16.4 13.9 16.9 17.4 17.9 15.3 16.6 14.9

You may assume that these times are normally distributed with mean μ and variance σ^2 .

- (a) Calculate unbiased estimates of μ and σ^2 . [5]
- (b) Determine a 99% confidence interval for μ . [5]

5. When Dawn throws the javelin, the distance thrown (in metres) can be assumed to be normally distributed with mean μ and variance σ^2 . She throws the javelin 9 times with the following results.

33.5 34.6 33.3 34.3 34.6 34.0 33.1 35.0 33.6

- (a) Calculate unbiased estimates of μ and σ^2 . [5]
- (b) Calculate a 95% confidence interval for μ . [4]

END OF CONFIDENCE INTERVALS – POPULATION MEAN PACK

Source: WJEC S2/S3 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec A2 Unit 5 – Topic 5 (2.5.4)

© WJEC CBAC Ltd. Pack layout © revise.wales for revision purposes only.