Name	Date started	Target end date

GCE A LEVEL — FURTHER STATISTICS B QUESTION PACK

0984-01 (Legacy S2) & 0985-01 (Legacy S3) · New spec A2 Unit 5 Topic 3

REVISE
.wales

FURTHER MATHS — FS B · HYPOTHESIS TESTING

Hypothesis Testing — z-tests, t-tests & Two-sample Comparisons

Every hypothesis-testing question on means from the legacy WJEC S2 and S3 papers that maps onto the new-spec A2 Unit 5 Topic 3 (parts a–c of the spec). Mann-Whitney and Wilcoxon were not assessed in the legacy specification but the technique can be slotted directly into the same five-step framework these questions exercise.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~3 hours 55 minutes

*Derived from the legacy S2/S3 paper's pace of **~1.3 min/mark** (181 marks over 16 questions). The full Unit 5 exam is **1 hour 45 minutes for 80 marks** (25% of the A-level qualification, A2 optional paper alongside Unit 6 Further Mechanics B).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every hypothesis testing question from the legacy WJEC S2/S3 papers (2008 modular spec) that maps onto new-spec A2 Unit 5 Topic 3 (2.5.3). Unit 5 (Further Statistics B) is one of two **80-mark A2 optional papers** (the other being Unit 6 Further Mechanics B), each worth 25% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Statistical Tables and Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 15 Q1	9	
2	Jun 06 Q7	11	
3	Jun 07 Q7	13	
4	Jun 09 Q4	8	
5	Jun 11 Q4	10	
6	Jun 12 Q5	11	
7	Jun 13 Q3	11	
8	Jun 14 Q3	10	

Q	Source	Max	Mark
9	Jun 16 Q4	14	
10	Jun 17 Q4	11	
11	Jun 08 Q3	13	
12	Jun 12 Q2	11	
13	Jun 14 Q2	13	
14	Jun 16 Q2	13	
15	Jun 08 Q5	12	
16	Jun 17 Q3	11	
Total		181	

Hypothesis Testing — z-tests, t-tests & Two-sample Comparisons — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 5: Further Statistics B · Topic 2.5.3.

z-test for mean (known σ) 2.5.3

- Sample $X_1, \dots, X_n$ from $N(\mu, \sigma^2)$ with known σ.
- Test statistic: $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$ under H_0.
- Critical regions depend on H_1: $\mu \neq \mu_0$ (two-sided), $\mu > \mu_0$ or $\mu < \mu_0$ (one-sided).
- Large-sample variant: when n is large, replace σ by s and use CLT.

t-test for mean (unknown σ) 2.5.3

- Sample from $N(\mu, \sigma^2)$ with σ unknown; n small.
- Test statistic: $T = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}$ under H_0.
- Use t-tables with $n-1$ degrees of freedom.
- WJEC: t-test results are interpreted using critical values; p-values are *not* required.

Two-sample z-test 2.5.3

- Independent samples from $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$ with known variances.
- Test statistic: $Z = \dfrac{\bar{X} - \bar{Y} - d_0}{\sqrt{\sigma_X^2/m + \sigma_Y^2/n}} \sim N(0,1)$ under $H_0 : \mu_X - \mu_Y = d_0$.
- The hypothesised difference d_0 may be non-zero.
- For unknown variances with large samples: replace σ_X^2, σ_Y^2 by s_X^2, s_Y^2 and use CLT.

Non-parametric tests 2.5.3

- *Mann-Whitney* U: two independent samples; rank the combined data; compute U_X, U_Y; reject for small $\min(U_X, U_Y)$.
- *Wilcoxon signed-rank*: paired or single sample about a hypothesised median; rank $|d_i|$, sum signed ranks.
- WJEC excludes tied ranks — if the data have ties, the question is not assessable under this technique.
- Choose Mann-Whitney/Wilcoxon when a normal distribution cannot be assumed.

Hypothesis Testing in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Five-step test layout

1. State H_0 and H_1 in terms of parameter.

2. State significance level α.

3. Compute test statistic from data.

4. Compare to critical value or compute p-value.

5. Conclude in context.

z-test for one mean

$X_1, \ldots, X_n \sim N(\mu, \sigma^2)$ with σ known.

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

Under $H_0 : \mu = \mu_0$, $Z \sim N(0,1)$.

Critical values: $|Z| > 1.96$ (5% two-sided); $Z > 1.645$ or $Z < -1.645$ (5% one-sided).

t-test for one mean

$X_1, \ldots, X_n \sim N(\mu, \sigma^2)$ with σ unknown.

$$T = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}$$

Use t-tables with $n-1$ df. WJEC: report against critical value (no p-values for t-test).

Two-sample z-test

Independent samples from $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$ with known variances:

$$Z = \frac{\bar{X} - \bar{Y} - d_0}{\sqrt{\sigma_X^2/m + \sigma_Y^2/n}}$$

d_0 is the hypothesised difference (often 0 but can be non-zero).

Large-sample two-sample test

When variances unknown but m, n large: replace σ^2 with s^2:

$$Z = \frac{\bar{X} - \bar{Y} - d_0}{\sqrt{s_X^2/m + s_Y^2/n}}$$

Approximately $N(0,1)$ under H_0 by CLT.

Mann-Whitney U

Two independent samples sizes m, n. Combine and rank all $m+n$ data.

$$U_X = \text{rank-sum of } X - \frac{m(m+1)}{2};$$

similarly U_Y.

Note: $U_X + U_Y = mn$. Reject H_0 if $\min(U_X, U_Y)$ is too small (table-based).

Wilcoxon signed-rank

For paired data or one sample about hypothesised median m_0:

Compute $d_i = x_i - m_0$ (or $x_i - y_i$ for paired). Rank $|d_i|$ ignoring zero d_i.

$W^+ =$ sum of positive-signed ranks, $W^- =$ sum of negative.

Test statistic: $W = \min(W^+, W^-)$; reject for small W.

Choosing the right test

- Normal pop + known $\sigma \Rightarrow z$-test.
- Normal pop + unknown σ, small $n \Rightarrow t$-test.
- Large n, distribution unknown $\Rightarrow z$-test with s replacing σ (CLT).
- Two independent samples, distributions unknown ⇒ Mann-Whitney.
- Paired or one-sample non-normal ⇒ Wilcoxon signed-rank.

Strategy

1. State H_0, H_1 precisely.

2. Identify distribution and known parameters.

3. Pick appropriate test (z, t, Mann-Whitney, Wilcoxon).

4. Compute test statistic; compare against critical value / compute p-value.

5. State conclusion in terms of the context.

SECTION T3

Hypothesis Testing

Questions 1–16 · 181 marks

1. Jim is a tennis player. His serve has a mean speed of 120 miles per hour (mph). He buys a new racket and he wishes to investigate whether or not using this racket changes the mean speed of his serve. He therefore goes to a tennis centre where he hits 10 serves and the measured speeds are as follows (mph).

121·2 119·1 118·3 120·1 117·9 118·3 119·4 119·6 120·3 117·8

You may assume that this is a random sample from a normal distribution with a standard deviation of 1·2.

(a) State suitable hypotheses for his investigation. [1]

(b) Determine the *p*-value of these results and state your conclusion in context. [8]

7. A Motoring Organisation wished to determine whether or not the fuel consumptions of two car models, A and B, are the same. To do this, six cars of each model were given 10 litres of petrol and driven around a track until they ran out of petrol. The distances travelled (in miles) by the cars were as follows.

Model A	83·1	84·1	84·6	85·1	81·2	82·9
Model B	81·0	82·2	79·9	83·4	81·8	80·7

You may assume that these are random samples from normal distributions with common standard deviation 1·5 miles.

(a) State suitable hypotheses. [1]

(b) Calculate the *p*-value and state your conclusion when the significance level is

(i) 1%,

(ii) 5%. [10]

7. A scientist wishes to determine whether or not there is a difference in the acidity levels of two different liquids. He therefore makes five independent measurements of the acidity level of each liquid with the following results.

Liquid 1	6·31	6·38	6·33	6·34	6·35
Liquid 2	6·28	6·31	6·29	6·35	6·30

You may assume that these are random samples from normal distributions with common standard deviation 0·025.

(a) (i) State suitable hypotheses.

(ii) Calculate the *p*-value of the above measurements and interpret your value in context. [10]

(b) Find a 95% confidence interval for the difference in the acidity levels of the two liquids. [3]

4. A teacher wishes to investigate whether or not boys and girls take the same time, on average, to solve jigsaw puzzles. She therefore gives the same jigsaw puzzle to the 6 girls and the 5 boys in her class. She records the time taken by each girl, x minutes, and the time taken by each boy, y minutes, to complete the puzzle. She finds that

$$\sum x = 94 \cdot 8, \sum y = 81 \cdot 0$$

You may assume that the times are random samples from normal distributions with common standard deviation 0·5 minutes.

(a) State suitable hypotheses for the investigation. [1]

(b) Determine the p-value of these results and state your conclusion in context. [7]

4. A zoologist believes that the mean weights of the adult males and females of a certain species of animal are equal. In order to test this belief, she weighs random samples of males and females with the following results.

Weights of males (kg)	14·3	15·8	13·9	13·4	14·5	15·1	13·6	14·2
Weights of females (kg)	13·2	14·8	13·7	14·7	15·0	13·1	13·5	

You may assume that these are random samples from normal populations with a common standard deviation of 0·5.

(a) State suitable hypotheses for carrying out a two-sided test. [1]

(b) Determine the *p*-value of these results and state whether or not the zoologist's belief is supported at the 5% level of significance. [9]

5. David and Frank are golfers and they wish to determine whether or not there is a difference between the mean distances that they can hit a golf ball. They decide that they should each hit six balls and measure the distances travelled in yards by these balls. The results are shown below.

Distances hit by David	152·1	148·3	150·6	145·4	144·7	149·3
Distances hit by Frank	143·4	147·9	150·8	144·1	145·6	147·2

You may assume that these are random samples from normal populations with a common standard deviation of 1·5.

(a) State suitable hypotheses for testing whether or not there is a difference between the mean distances. [1]

(b) Determine the *p*-value of these results and state your conclusion in context. [10]

3. A teacher in a large college wishes to investigate whether or not boys and girls perform equally well in examinations in practical mathematics. She therefore selects a random sample of 8 boys and 8 girls and gives them an examination.
The marks obtained were as follows.

Boys	52, 47, 62, 75, 51, 69, 56, 70
Girls	48, 39, 56, 69, 71, 45, 43, 59

You may assume that these are random samples from normal populations with a common standard deviation of 7·5.

(a) State suitable hypotheses for this investigation. [1]

(b) Determine the *p*-value of these results and state your conclusion in context. [10]

3. A new species of animal has been found on an uninhabited island. A zoologist wishes to investigate whether or not there is a difference in the mean weights of males and females of the species. She traps some of the animals and weighs them with the following results.

Males (kg)	5·3, 4·6, 5·2, 4·5, 4·3, 5·5, 5·0, 4·8
Females (kg)	4·9, 5·0, 4·1, 4·6, 4·3, 5·3, 4·2, 4·5, 4·8, 4·9

You may assume that these are random samples from normal populations with a common standard deviation of 0·5 kg.

(a) State suitable hypotheses for this investigation. [1]

(b) Determine the *p*-value of these results and state your conclusion in context. [9]

4. The independent random variables X, Y are such that X is $\text{N}(\mu_x, 1{\cdot}5^2)$ and Y is $\text{N}(\mu_y, 2{\cdot}5^2)$.
In order to test the hypotheses

$$H_0 : \mu_x = \mu_y \quad ; \quad H_1 : \mu_x \neq \mu_y$$

a random sample of size 8 is taken from the distribution of X and a random sample of size 12 is taken from the distribution of Y. The means of these two samples are denoted by $\bar{x}$ and $\bar{y}$ respectively. The significance level is to be 10%.

(a) Determine the critical region in the form $|\bar{x} - \bar{y}| > k$, where the value of k is to be found. [5]

(b) (i) If, in fact, $\mu_x - \mu_y = 0{\cdot}5$, find the probability of incorrectly accepting H_0.

(ii) Comment on your result in (i). [9]

4. A motorbike club wished to compare the fuel consumptions of two motorbike models, A and B. To do this, eight motorbikes of each model were given 15 litres of petrol and driven around a track until they ran out of petrol. The distances travelled (in miles) by the motorbikes were as follows.

Model A	168·2	170·5	164·2	169·2	165·8	166·6	162·2	168·5
Model B	161·7	166·3	167·4	164·1	162·7	160·3	165·6	163·1

You may assume that these are random samples from normal distributions with means μ_A, μ_B respectively and common standard deviation 2·5.

(a) Determine a 95% confidence interval for $\mu_A - \mu_B$. [7]

(b) Find the smallest confidence level for which the corresponding confidence interval includes zero. Give your answer as a percentage correct to three significant figures. [4]

3. A farmer claims that the mean weight of apples grown in his orchard is 300 grams. His wife claims that the mean weight is less than this. To investigate these two claims, they agree to weigh a random sample of 12 apples. The results, in grams, are as follows.

286 294 305 277 289 312 301 284 281 315 289 295

You may assume that these weights form a random sample from a normal distribution.

(a) Calculate unbiased estimates of the mean and variance of the weights of apples grown in the orchard. [4]

(b) Stating your hypotheses clearly, test the claims using a significance level of

(i) 5%,

(ii) 10%. [9]

2. The manager of a factory that manufactures a certain type of string claims that its mean breaking strength is 100 Newtons. In order to test this claim, the breaking strengths, in Newtons, of a random sample of ten pieces of this string were determined with the following results.

97·1 100·3 98·6 97·7 101·2 97·6 98·9 101·1 98·5 99·3

You may assume that this is a random sample from a normal distribution with mean μ and variance σ^2.

(a) Calculate unbiased estimates of μ and σ^2. [4]

(b) (i) State suitable hypotheses for testing the manager's claim using a two-sided test.

(ii) Carry out an appropriate test at the 5% significance level. Giving a reason, state your conclusion in context. [7]

2. The mean weight of a certain breed of bird is believed to be 4·5 kg. In order to test this belief, a random sample of 10 birds of the breed was obtained and weighed, with the following results in kg.

4·38 4·18 4·46 4·59 4·16 4·57 4·16 4·26 4·49 4·35

You may assume that the weights of this breed of bird are normally distributed.

(a) State suitable hypotheses for testing the above belief using a two-sided test. [1]

(b) Calculate unbiased estimates of the mean and the variance of the weights of this breed of bird. [5]

(c) Carry out an appropriate test using a 1% significance level and state your conclusion in context, justifying your answer. [7]

2. A car manufacturer claims that the average mileage per gallon for a new model on a motorway journey is 61. However, a motoring organisation claims that the average mileage per gallon is less than this. A trial is therefore set up in which 10 cars of this new model undertake a long motorway journey and the mileage per gallon for each car is recorded as follows.

60·2 59·9 61·2 62·3 58·5 59·7 61·2 60·7 59·4 60·3

You may assume that this is a random sample from a normal distribution with mean μ and variance σ^2.

(a) State suitable hypotheses to test these claims. [1]

(b) Calculate unbiased estimates of μ and σ^2. [5]

(c) Carry out an appropriate hypothesis test at the 5% significance level.
State your conclusion in context, explaining clearly how you reached it. [7]

5. A zoologist wishes to determine whether or not the mean weights of two similar breeds of dog are equal. He obtains random samples of 60 dogs of each breed and he determines the weight, x kg, of each dog of breed A and the weight, y kg, of each dog of breed B. His results are summarised below.

$$\sum x = 1506,\ \ \sum x^2 = 38\,124,\ \ \sum y = 1530,\ \ \sum y^2 = 39\,327.$$

(a) State suitable hypotheses. [1]

(b) Calculate, approximately, the p-value of these results. Interpret its value in context. [11]

3. A zoologist claims that the mean weight of male dogs of a certain breed is 5 kg more than the mean weight of female dogs of the breed. Mair believes that the difference in mean weights is greater than 5 kg. She therefore collects and weighs random samples of 50 male and 50 female dogs of the breed. She defines the following hypotheses,

$$H_0 : \mu_x - \mu_y = 5 ; \qquad H_1 : \mu_x - \mu_y > 5$$

where μ_x, μ_y denote respectively the mean weights, in kg, of the male dogs and female dogs of the breed. The results are summarised below, where x, y denote respectively the weights, in kg, of the male dogs and the female dogs.

$$\sum x = 2055, \ \sum x^2 = 84\,773, \ \sum y = 1745, \ \sum y^2 = 61121$$

Determine an approximate *p*-value for these results and state your conclusion in context. [11]

END OF HYPOTHESIS TESTING PACK

Source: WJEC S2/S3 (2008 modular spec) · 2005–2017

Curated for WJEC FM 2017 spec A2 Unit 5 — Topic 3 (2.5.3)