Name	Date started	Target end date

GCE A LEVEL — FURTHER STATISTICS B QUESTION PACK

0984-01 (Legacy S2) · New spec A2 Unit 5 Topic 2

REVISE
.wales

FURTHER MATHS — FS B · STATISTICAL DISTRIBUTIONS & CLT

Statistical Distributions & the Central Limit Theorem — Linear Combinations of Normals

Every linear-combination / sample-mean / CLT question from the legacy WJEC S2 papers (June 2005 – June 2017 + Specimen) that maps onto the new-spec A2 Unit 5 Topic 2.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~3 hours 15 minutes

*Derived from the legacy S2 paper's pace of **~1.3 min/mark** (150 marks over 12 questions). The full Unit 5 exam is **1 hour 45 minutes for 80 marks** (25% of the A-level qualification, A2 optional paper alongside Unit 6 Further Mechanics B).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every statistical distributions & clt question from the legacy WJEC S2 papers (2008 modular spec) that maps onto new-spec A2 Unit 5 Topic 2 (2.5.2). Unit 5 (Further Statistics B) is one of two **80-mark A2 optional papers** (the other being Unit 6 Further Mechanics B), each worth 25% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Statistical Tables and Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 05 Q3	17	
2	Jun 06 Q8	6	
3	Jun 07 Q3	12	
4	Jun 09 Q2	10	
5	Jun 10 Q1	9	
6	Jun 11 Q1	13	
7	Jun 12 Q2	18	
8	Jun 13 Q1	16	
9	Jun 15 Q5	9	
10	Jun 16 Q3	14	
11	Jun 17 Q3	15	
12	Spec. Q1	11	
	Total	**150**	

Statistical Distributions & the Central Limit Theorem — Linear Combinations of Normals — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 5: Further Statistics B · Topic 2.5.2.

Linear combinations of normals 2.5.2

- If $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$ are independent then for constants a, b:
- $aX + bY \sim N(a\mu_X + b\mu_Y, a^2\sigma_X^2 + b^2\sigma_Y^2)$.
- For n independent normals: sum $\sim N(\sum \mu_i, \sum \sigma_i^2)$.
- Generalised to sums of i.i.d. copies: $X_1 + X_2 + \cdots + X_n \sim N(n\mu, n\sigma^2)$.

Sample mean from a normal 2.5.2

- If $X_1, \ldots, X_n$ are i.i.d. from $N(\mu, \sigma^2)$, then:
- $\bar{X} = \frac{1}{n}\sum X_i \sim N(\mu, \frac{\sigma^2}{n})$.
- Note the variance scales by $1/n$, not by $1/n^2$.
- Standardising: $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$.

Central Limit Theorem 2.5.2

- Let $X_1, \ldots, X_n$ be a random sample from *any* distribution with mean μ and variance σ^2.
- For large n (rule of thumb $n \geq 30$):
- $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ approximately.
- Allows normal-based inference even when the underlying distribution is unknown.

Working scientifically general

- CLT requires *large* n — not applicable to small samples from non-normal populations.
- Always state where CLT is being invoked when justifying a normal approximation.
- For Poisson with large λ, normal approximation $X \approx N(\lambda, \lambda)$ is a CLT instance.
- For binomial with large n and moderate p, normal approximation $X \approx N(np, np(1-p))$ also comes from CLT.

Statistical Distributions & CLT in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Linear combos of normals

If $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$ independent:

$aX + bY \sim N(a\mu_X + b\mu_Y,\ a^2\sigma_X^2 + b^2\sigma_Y^2)$

Sum: $X + Y \sim N(\mu_X + \mu_Y,\ \sigma_X^2 + \sigma_Y^2)$.

Difference: $X - Y \sim N(\mu_X - \mu_Y,\ \sigma_X^2 + \sigma_Y^2)$.

Sums of i.i.d. normals

For $X_1, \dots, X_n$ i.i.d. from $N(\mu, \sigma^2)$:

$$X_1 + \cdots + X_n \sim N(n\mu,\ n\sigma^2)$$

Variance scales by n (*not* n^2) because the X_i's are independent.

Sample mean distribution

For $X_1, \dots, X_n$ i.i.d. from $N(\mu, \sigma^2)$:

$$\bar{X} = \frac{1}{n}\sum X_i \sim N\left(\mu,\ \frac{\sigma^2}{n}\right)$$

So $\mathrm{Var}(\bar{X}) = \sigma^2/n$ and SE $= \sigma/\sqrt{n}$.

Central Limit Theorem

For $X_1, \dots, X_n$ i.i.d. from *any* distribution with finite mean μ and variance σ^2:

$$\bar{X} \overset{\sim}{\to} N\left(\mu,\ \frac{\sigma^2}{n}\right)$$

Rule of thumb: $n \geq 30$ is "large enough". The approximation improves with larger n.

Standardising

Whenever you have $\bar{X} \sim N(\mu, \sigma^2/n)$, transform to Z:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

Look up Z-values in standard normal tables for probabilities.

CLT for sums

For large n, the *sum* $S_n = X_1 + \cdots + X_n$ also approximately normal:

$$S_n \overset{\sim}{\to} N(n\mu,\ n\sigma^2)$$

Used for Poisson with large λ and binomial with large n.

Normal approximations

Binomial: $X \sim B(n, p) \to N(np, np(1-p))$ when n is large and p not too close to 0 or 1.

Poisson: $X \sim \mathrm{Po}(\lambda) \to N(\lambda, \lambda)$ when $\lambda \geq 10$ approximately.

These are CLT applications — in legacy WJEC papers, often examined explicitly.

Common pitfalls

- Confusing $\mathrm{Var}(X + Y) = \sigma_X^2 + \sigma_Y^2$ with $\mathrm{Var}(X - Y)$: both equal $\sigma_X^2 + \sigma_Y^2$ when X, Y independent.
- Forgetting a^2 in $\mathrm{Var}(aX) = a^2\,\mathrm{Var}(X)$, not $|a|\,\mathrm{Var}(X)$.
- Applying CLT to small samples ($n < 30$) from non-normal population.
- Using σ/n instead of $\sigma/\sqrt{n}$ for SE of $\bar{X}$.

Strategy

1. Identify whether X is exactly normal or only approximately (CLT).
2. For sums/differences of independent normals: add the variances.
3. For sample mean: $\bar{X} \sim N(\mu, \sigma^2/n)$.
4. Standardise and look up Z-values.

SECTION T2

Statistical Distributions and CLT

Questions 1–12 · 150 marks

3. The weights, X kg, of male students in a hall of residence are normally distributed with mean 70 kg and standard deviation 6 kg.

(a) Find the probability that the weight of a randomly chosen male student lies between 67 kg and 79 kg. [5]

The weights, Y kg, of female students in the hall of residence are normally distributed with mean 50 kg and standard deviation 5 kg.

(b) Find the mean and variance of the random variable $2Y - X$. Hence find the probability that the weight of a randomly chosen male student is more that twice the weight of a randomly chosen female student. [6]

(c) The hall of residence has a lift installed with a maximum recommended load of 500 kg. On one occasion, there are 3 male students and 6 female students in the lift. Find the probability that their combined weight exceeds the recommended maximum. [6]

8. *(a)* State the Central Limit Theorem. [1]

(b) When a cubical die is thrown, the score obtained has a mean of $\frac{7}{2}$ and a variance of $\frac{35}{12}$. Such a die is thrown 50 times. Find, approximately, the probability that the mean of the 50 scores obtained exceeds 3. [5]

3. The weights of apples may be assumed to be normally distributed with mean 75 grams and standard deviation 5 grams.

(a) (i) Find the probability that a randomly chosen apple weighs less than 80 grams.

(ii) Find the upper quartile of the weights of apples. [6]

(b) The weights of plums may be assumed to be normally distributed with mean 56 grams and standard deviation 4 grams. Calculate the probability that the combined weight of 3 plums exceeds the combined weight of 2 apples. [6]

2. Roger is a tennis player. When he serves, the speed of the ball may be assumed to be an independent normally distributed random variable with mean 140 km/h and standard deviation 8 km/h.

(a) He serves three times. Calculate the probability that

(i) the speed of the first serve exceeds 150 km/h,

(ii) the speed of all three serves exceeds 150 km/h. [5]

(b) Andy is another tennis player. When he serves, the speed of the ball may be assumed to be an independent normally distributed random variable with mean 145 km/h and standard deviation 6 km/h. Andy and Roger each serve once. Calculate the probability that the speed of Roger's serve is greater than the speed of Andy's serve. [5]

1. A large batch of tomatoes is delivered to a packing station. The weights of these tomatoes may be assumed to be independent and normally distributed with mean 106 grams and standard deviation 8 grams.

(a) Find the probability that the weight of a randomly selected tomato exceeds 120 grams. [3]

(b) A pack contains 10 randomly selected tomatoes. Find the probability that the total weight of these 10 tomatoes is less than 1 kilogram. [6]

1. The time taken by Alan to drive to work may be assumed to be normally distributed with mean 28 minutes and standard deviation 2 minutes.

(a) Find the probability that,

(i) on a particular day, he takes more than 30 minutes to drive to work,

(ii) in a particular 5-day week, the mean time taken to drive to work is less than 30 minutes. [8]

(b) The time taken by Brenda to drive to work may be assumed to be normally distributed with mean 25 minutes and standard deviation 3 minutes. Find the probability that, on a particular day, Brenda takes longer to drive to work than Alan. [5]

2. The weights X kg of male birds of a certain species are normally distributed with mean 4·4 kg and standard deviation 0·2 kg.

(a) (i) Find the probability that the weight of a randomly selected male bird exceeds 4·5 kg.

(ii) Determine the 95th percentile of X. [5]

(b) The weights Y kg of female birds of the same species are normally distributed with mean 2·6 kg and standard deviation 0·15 kg.

(i) Find the mean and variance of $2Y - X$.

(ii) Find the probability that the weight of a randomly chosen male bird is more than twice the weight of a randomly chosen female bird.

(iii) Two male birds and three female birds are placed on a weighing machine whose maximum permissible weight is 16 kg. Find the probability that the maximum weight is exceeded. [13]

1. The random variable X is normally distributed with mean 10 and standard deviation 2.

(a) (i) Evaluate $P(X \leqslant 10{\cdot}5)$.

(ii) Given that $P(X \geqslant x) = 0{\cdot}1$, find the value of x. [5]

(b) The independent random variable Y is normally distributed with mean 12 and standard deviation 3.

(i) Evaluate $P(X + 2Y < 36)$.

(ii) Given that X_1, X_2, X_3 is a random sample from the distribution of X and Y_1, Y_2 is a random sample from the distribution of Y, evaluate

$$P(X_1 + X_2 + X_3 < Y_1 + Y_2).$$ [11]

5. A fair dice with faces numbered 1, 2, 3, 4, 5 and 6 respectively is thrown 100 times. Use the Central Limit Theorem to calculate, approximately, the probability that the mean of the 100 scores obtained is at least 3·75. [9]

3. For a certain breed of dog, the weights of the males are normally distributed with mean 40 kg and standard deviation 2·5 kg. The weights of the females are normally distributed with mean 32 kg and standard deviation 1·5 kg.

(a) Calculate the upper quartile of the weights of male dogs of this breed. [2]

(b) A random selection is made of 3 males and 2 females of the breed.
Calculate the probability that

(i) the combined weight of the 5 dogs exceeds 185 kg,

(ii) the combined weight of the 3 males is less than twice the combined weight of the 2 females. [12]

3. A grocer sells apples and pears. The weights of the apples may be assumed to be normally distributed with mean 110 grams and standard deviation 14 grams. The weights of the pears may be assumed to be normally distributed with mean 160 grams and standard deviation 16 grams.

(a) Find the 90th percentile of the weights of the apples. [2]

(b) George buys 10 apples. Find the probability that the total weight of his 10 apples is less than 1000 grams. [6]

(c) Sue buys 3 apples and 2 pears. Find the probability that the combined weight of her 3 apples is more than the combined weight of her 2 pears. [7]

be modelled by a normal random variable with adjustable mean μ ml and standard deviation 5 ml.

(a) The cups used have capacity 200 ml.

(i) What proportion of cups will overflow if μ is set to 195?

(ii) To what value should μ be set to ensure that only 1% of the cups overflow? [6]

(b) A customer wants to fill a bottle of capacity 1000 ml so he decides to make five independent discharges into the bottle. Given that $\mu = 196$, find the probability that the bottle overflows. [5]

END OF STATISTICAL DISTRIBUTIONS & CLT PACK

Source: WJEC S2 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec A2 Unit 5 — Topic 2 (2.5.2)