Name | Date started | Target end date

GCE A LEVEL — FURTHER PURE MATHEMATICS B QUESTION PACK

0982-01 (Legacy M3) · New spec A2 Unit 4 Topic 9

REVISE .wales

Differential Equations — 1st-Order (Integrating Factor & Separation)

Every first-order differential-equation question from the legacy WJEC M3 papers (2009–2017) that maps onto new-spec A2 Unit 4 Topic 8 (2.4.8 first-order linear DEs and separation of variables).

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~1h 32m

Derived from the legacy M3 paper's pace of ~1.5 min/mark (61 marks over 6 questions). The full Unit 4 exam is 2 hours 30 minutes for 120 marks (35% of the A-level qualification). You are advised to not attempt to complete this in one sitting.

ABOUT THIS QUESTION PACK

This is a comprehensive practice question pack focused on first-order differential equations. It contains every first-order DE question from the legacy WJEC M3 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 8 (2.4.8). Questions cover separation of variables and the integrating-factor method for dy/dx + P(x)y = Q(x). The mechanics context (motion under resistance) is incidental — the techniques are identical to those examined in U4.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

FOR EXAMINER'S USE ONLY

Q	Source	Max	Mark
1	Jun 09 Q1	10	
2	Jun 11 Q1	11	
3	Jun 14 Q1	10	
4	Jun 15 Q1	11	
5	Jun 16 Q1	9	
6	Jun 17 Q1	10	
Total		**61**	

1. A body, of mass 9 kg, is projected along a straight horizontal track with an initial speed of $20\,\text{ms}^{-1}$. At time t s the body experiences a resistance of magnitude $(0{\cdot}2 + 0{\cdot}03v)$ N where $v\ \text{ms}^{-1}$ is its speed.

(a) Show v satisfies the differential equation [3]

$$900\,\frac{\mathrm{d}v}{\mathrm{d}t} = -(20 + 3v).$$

(b) Find an expression for t in terms of v. [5]

(c) Calculate, to the nearest second, the time taken for the body to come to rest. [2]

1. A vehicle P, of mass 800 kg, on a straight horizontal road passes the point O with velocity $5\,\text{ms}^{-1}$. At time t s later its velocity is $v\,\text{ms}^{-1}$ and the vehicle is subject to a resistance given by $(4000 + 1600v)$ N.

(a) Show that v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -(5 + 2v) \ .$$ [2]

(b) (i) Find the time when P is at rest.

(ii) Find an expression for v in terms of t. [9]

1. A car of mass 1200 kg is initially at rest on a straight horizontal road. The car moves under the action of a horizontal tractive force of 500 N. The resistance to motion of the car is $100v$ N, where v ms^{-1} is the speed of the car at time t s.

(a) Show that the motion of the car satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{5 - v}{12}. \qquad [2]$$

(b) Find an expression for v in terms of t and write down the limiting speed of the car. [6]

(c) Calculate the time taken for the car to reach a speed of 4 ms^{-1}. [2]

1. A particle of mass 400 kg moves along a straight horizontal road under the action of a horizontal force F. The magnitude of the force F may be modelled by $500\left(\frac{x}{v+2}\right)$ N, where v ms^{-1} is the speed of the particle and x m is the distance of the particle from a point O on the road.

(a) Show that the motion of the particle satisfies the differential equation

$$4v(v+2)\frac{\mathrm{d}v}{\mathrm{d}x} = 5x.$$ [2]

(b) When $x = 0$, the particle is at rest.

(i) Find an expression for x in terms of v.

(ii) Find the distance of the particle from O and the acceleration of the particle when its speed is 3 ms^{-1}. [9]

1. A particle of mass 60 kg moves along the horizontal x-axis under the action of a horizontal constant force of 1800 N. The magnitude of the resistance to motion of the particle is $120v$ N, where v ms^{-1} is the velocity of the particle. At time $t = 0$ seconds, the particle is moving with velocity 8 ms^{-1}.

(a) Show that v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 30 - 2v.$$ [2]

(b) Find an expression for v at time t. Determine the limiting value of v. [7]

1. A particle moves along the x-axis such that its displacement x metres at time t seconds satisfies the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} + x = 2.$$

The particle passes through the origin when $t = 0$.

(a) Find the time when the particle reaches the point $x = 1$, and determine an expression for x at time t. [7]

(b) Hence find an expression for the acceleration of the particle at time t. [3]