Name	Date started	Target end date

GCE A LEVEL — FURTHER PURE MATHEMATICS B QUESTION PACK

0982-01 (Legacy M3) · New spec A2 Unit 4 Topic 8

REVISE
.wales

FURTHER MATHS — FP B · DIFFERENTIAL EQUATIONS

Differential Equations — Integrating Factor & Auxiliary Equation

Every differential-equation question from the legacy WJEC M3 papers (June 2006 – June 2017 + Specimen) covering the exact techniques specified in new-spec A2 Unit 4 Topic 8. Note: legacy FP2/FP3 do not cover differential equations — the technique was housed in legacy M3 under the 2008 modular spec. The mechanics framing is incidental; the auxiliary equation, complementary function, particular integral and integrating-factor methods are identical to those required for U4.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~4 hours 10 minutes

*Derived from the legacy M3 paper's pace of **~1.5 min/mark** (167 marks over 14 questions). The full Unit 4 exam is **2 hours 30 minutes for 120 marks** (35% of the A-level qualification).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every differential equations question from the legacy WJEC M3 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 8 (2.4.8). Unit 4 (Further Pure Mathematics B) is the **120-mark compulsory A2 paper**, worth 35% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 09 Q1	10	
2	Jun 11 Q1	11	
3	Jun 14 Q1	10	
4	Jun 15 Q1	11	
5	Jun 16 Q1	9	
6	Jun 17 Q1	10	
7	Jun 06 Q2	12	
8	Jun 10 Q3	12	
9	Jun 12 Q3	12	
10	Jun 13 Q3	14	
11	Jun 14 Q4	12	
12	Jun 16 Q3	12	
13	Jun 17 Q3	18	
14	Spec. Q3	14	
	Total	**167**	

Differential Equations — Integrating Factor & Auxiliary Equation — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 4: Further Pure Mathematics B · Topic 2.4.8.

First-order linear DEs 2.4.8

- Standard form: $\frac{dy}{dx} + P(x)y = Q(x)$.
- Integrating factor: $\mu(x) = e^{\int P(x)\,dx}$.
- Multiply through and integrate: $\frac{d}{dx}(\mu y) = \mu Q$, so $\mu y = \int \mu Q\,dx + C$.
- Separable case: write as $g(y)\,dy = f(x)\,dx$ and integrate both sides.

Second-order linear DEs — homogeneous 2.4.8

- Equation: $y'' + ay' + by = 0$.
- Auxiliary equation: $\lambda^2 + a\lambda + b = 0$, roots λ_1, λ_2.
- Real distinct $\lambda_1 \neq \lambda_2$: $y = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$.
- Repeated λ: $y = (A + Bx)e^{\lambda x}$.
- Complex $\lambda = p \pm iq$: $y = e^{px}(A\cos qx + B\sin qx)$.

Non-homogeneous: CF + PI 2.4.8

- For $y'' + ay' + by = f(x)$: $y = y_{\text{CF}} + y_{\text{PI}}$.
- Complementary function y_{CF}: solve the homogeneous version.
- Particular integral y_{PI}: guess form based on $f(x)$:
- $f = \text{poly} \Rightarrow$ try y_{PI} poly of same degree. $f = ke^{qx} \Rightarrow$ try Ce^{qx}.
- $f = m\cos\omega x + n\sin\omega x \Rightarrow$ try $C\cos\omega x + D\sin\omega x$.

Working scientifically general

- Apply initial conditions *after* writing the full general solution — not earlier.
- For repeated roots and PI clash: multiply PI guess by x (or x^2).
- Coupled systems: write as a single 2nd-order equation in one variable and apply CF+PI.
- Spec note: contexts in new questions exclude mechanics — but the technique is identical.

Differential Equations in one page

Quick-reference notes — revisit before each section. Don't use during questions.

First-order linear DE

Standard form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Integrating factor:

$$\mu(x) = e^{\int P(x)\,dx}$$

Solving via integrating factor

Multiply through by μ:

$$\frac{d}{dx}(\mu y) = \mu Q$$

Integrate both sides:

$\mu y = \int \mu Q\,dx + C$, so $y = \frac{1}{\mu}\left(\int \mu Q\,dx + C\right)$.

Separable DE

If $\frac{dy}{dx} = f(x)g(y)$:

$$\frac{dy}{g(y)} = f(x)\,dx$$

Integrate both sides; arbitrary constant on one side.

Second-order homogeneous

Equation: $y'' + ay' + by = 0$.

Auxiliary: $\lambda^2 + a\lambda + b = 0 \Rightarrow \lambda_{1,2}$.

Discriminant > 0: $y = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$.

Auxiliary root cases

Real distinct $\lambda_1 \neq \lambda_2$: $y = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$.

Real repeated λ: $y = (A + Bx)e^{\lambda x}$.

Complex $p \pm iq$: $y = e^{px}(A\cos qx + B\sin qx)$.

Particular integral — choice

For $f(x) = A + Bx$ (linear): try $y_{\text{PI}} = a + bx$.

For $f(x) = cx^2 + dx + e$: try $y_{\text{PI}} = ax^2 + bx + c$.

For $f(x) = ke^{qx}$: try $y_{\text{PI}} = Ce^{qx}$.

For $f(x) = m\cos\omega x + n\sin\omega x$: try $y_{\text{PI}} = C\cos\omega x + D\sin\omega x$.

PI clash with CF

If your trial y_{PI} is already in y_{CF}: multiply by x.

If still in y_{CF} (repeated root): multiply by x^2.

Example: $y'' - 2y' + y = e^x$, $\lambda = 1$ repeated $\Rightarrow$ try $y_{\text{PI}} = Cx^2e^x$.

Common pitfalls

- Applying initial conditions before adding $y_{\text{CF}} + y_{\text{PI}}$.
- Wrong form for PI when CF clashes — check repeats.
- Sign error in auxiliary equation — double-check a, b signs.
- Forgetting integrating factor when $P(x)$ is a non-constant function.

Strategy

1. Identify: first-order linear, separable, or second-order linear?

2. First-order linear: compute μ, multiply, integrate.

3. Second-order: auxiliary → CF; trial → PI; combine; apply initial conditions.

4. Always verify the general solution by substituting back.

SECTION T8

Differential Equations

Questions 1–14 · 167 marks

1. A body, of mass 9 kg, is projected along a straight horizontal track with an initial speed of 20 ms^{-1}. At time t s the body experiences a resistance of magnitude $(0{\cdot}2 + 0{\cdot}03v)$ N where v ms^{-1} is its speed.

(a) Show v satisfies the differential equation [3]

$$900\,\frac{\mathrm{d}v}{\mathrm{d}t} = -(20 + 3v).$$

(b) Find an expression for t in terms of v. [5]

(c) Calculate, to the nearest second, the time taken for the body to come to rest. [2]

1. A vehicle P, of mass 800 kg, on a straight horizontal road passes the point O with velocity $5\,\text{ms}^{-1}$. At time t s later its velocity is $v\,\text{ms}^{-1}$ and the vehicle is subject to a resistance given by $(4000 + 1600v)$ N.

(a) Show that v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -(5 + 2v)\ .$$ [2]

(b) (i) Find the time when P is at rest.

(ii) Find an expression for v in terms of t. [9]

1. A car of mass 1200 kg is initially at rest on a straight horizontal road. The car moves under the action of a horizontal tractive force of 500 N. The resistance to motion of the car is $100v$ N, where v ms^{-1} is the speed of the car at time t s.

(a) Show that the motion of the car satisfies the differential equation

$$\frac{dv}{dt} = \frac{5 - v}{12}.$$ [2]

(b) Find an expression for v in terms of t and write down the limiting speed of the car. [6]

(c) Calculate the time taken for the car to reach a speed of 4 ms^{-1}. [2]

1. A particle of mass 400 kg moves along a straight horizontal road under the action of a horizontal force F. The magnitude of the force F may be modelled by $500\left(\frac{x}{v+2}\right)$ N, where v ms^{-1} is the speed of the particle and x m is the distance of the particle from a point *O* on the road.

(a) Show that the motion of the particle satisfies the differential equation

$$4v(v+2)\frac{dv}{dx} = 5x.$$

[2]

(b) When $x = 0$, the particle is at rest.

(i) Find an expression for x in terms of v.

(ii) Find the distance of the particle from *O* and the acceleration of the particle when its speed is 3 ms^{-1}. [9]

1. A particle of mass 60 kg moves along the horizontal x-axis under the action of a horizontal constant force of 1800 N. The magnitude of the resistance to motion of the particle is $120v$ N, where v ms^{-1} is the velocity of the particle. At time $t = 0$ seconds, the particle is moving with velocity 8 ms^{-1}.

(a) Show that v satisfies the differential equation

$$\frac{dv}{dt} = 30 - 2v.$$

[2]

(b) Find an expression for v at time t. Determine the limiting value of v. [7]

1. A particle moves along the x-axis such that its displacement x metres at time t seconds satisfies the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} + x = 2.$$

The particle passes through the origin when $t = 0$.

(a) Find the time when the particle reaches the point $x = 1$, and determine an expression for x at time t. [7]

(b) Hence find an expression for the acceleration of the particle at time t. [3]

2. Find the general solution of the second order differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 5t - 14,$$

such that $x = 4\frac{1}{2}$ and $\frac{dx}{dt} = 3\frac{1}{2}$ when $t = 0$. [12]

3. Find the solution of the differential equation

$$4\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 9x = 18t - 87,$$

such that $x = 5$ and $\frac{dx}{dt} = 10$ when $t = 0$. [12]

3. Find the solution of the second order differential equation

$$2\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 2x = 6t + 5$$

such that $x = 3$ and $\frac{dx}{dt} = 2$ when $t = 0$. [12]

3. *(a)* A particle P, of mass 2 kg, moves along the horizontal x-axis under the action of a force directed towards the origin O. The magnitude of the force is equal to $8x$ N, where x m is the displacement of P from O. The particle is also subjected to a resistive force which is equal to $10v$ N, where $v\,\text{ms}^{-1}$ is the speed of P at time t s. When $t = 0$ s, the particle P is at $x = 2$ m and it is moving away from O with speed $3\,\text{ms}^{-1}$.

(i) Show that the equation of motion of the particle is

$$\frac{d^2x}{dt^2} = -4x - 5\frac{dx}{dt}.$$

(ii) Find an expression for x in terms of t. [10]

(b) Find the general solution of the second order differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 12t - 3.$$ [4]

4. The reading x of the pointer on a set of kitchen scales at time t is modelled by the differential equation

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 6\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 1.$$

(a) Find the general solution of the equation for x. [5]

(b) Determine the limiting value of x. [2]

(c) Given that x = 0·5 and $\frac{\mathrm{d}x}{\mathrm{d}t}$ = 0 when t = 0,

(i) find an expression for x in terms of t,

(ii) calculate the instantaneous reading of the scale when $t = \frac{\pi}{3}$.
Give your answer correct to three significant figures. [5]

3. Solve the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+6\frac{\mathrm{d}x}{\mathrm{d}t}+9x=27t,$$

where $x=\frac{\mathrm{d}x}{\mathrm{d}t}=0$ when t = 0. Hence find the value of x when t = 2. [12]

3. The function x satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 6\frac{\mathrm{d}x}{\mathrm{d}t} + (10-k)x = \frac{1}{50}k(k-5)(12t-26),$$

where k is a constant. When $t = 0$, $x = 8$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 16$. Find x in each of the following cases.

(a) $k = 5$. [5]

(b) $k = 0$. [5]

(c) $k = 10$. [8]

3. A particle moves in a straight line such that at time t s, its displacement x m, from a fixed point O, satisfies the differential equation

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 12x = 12t + 20 .$$

Given that when t = 0, x = 0 and the particle is moving with velocity 3 ms^{-1}, find its displacement at time t = 2 s. [14]

END OF DIFFERENTIAL EQUATIONS PACK

Source: WJEC M3 (2008 modular spec) · 2005–2017
Curated for WJEC FM 2017 spec A2 Unit 4 — Topic 8 (2.4.8)