Name	Date started	Target end date

GCE A LEVEL — FURTHER PURE MATHEMATICS B QUESTION PACK

0979-01 (Legacy FP3) · New spec A2 Unit 4 Topic 6

REVISE
.wales

FURTHER MATHS — FP B · POLAR COORDINATES

Polar Coordinates — Curves, Tangents & Area Enclosed

Every polar-coordinate question from the legacy WJEC FP3 papers (June 2006 – June 2017 + Specimen) that maps onto the new-spec A2 Unit 4 Topic 6.

LEGACY 2008 SPECIFICATION

Estimated time for entire question pack: ~3 hours 45 minutes

*Derived from the legacy FP3 paper's pace of **~1.5 min/mark** (150 marks over 12 questions). The full Unit 4 exam is **2 hours 30 minutes for 120 marks** (35% of the A-level qualification).*

*You are advised to **not** attempt to complete all of this in one sitting.*

ABOUT THIS QUESTION PACK

This is a **comprehensive practice question pack**, not a single mock paper. It contains every polar coordinates question from the legacy WJEC FP3 papers (2008 modular spec) that maps onto new-spec A2 Unit 4 Topic 6 (2.4.6). Unit 4 (Further Pure Mathematics B) is the **120-mark compulsory A2 paper**, worth 35% of the A-level qualification.

Questions are ordered roughly by topic / difficulty.

INSTRUCTIONS

Use black ink or black ball-point pen. Show all working — method marks are awarded for clear setup.

A calculator is allowed. The WJEC Formula Booklet may be referred to.

For Examiner's use only

Q	Source	Max	Mark
1	Jun 07 Q4	9	
2	Jun 08 Q7	14	
3	Jun 09 Q5	14	
4	Jun 10 Q6	13	
5	Jun 11 Q6	12	
6	Jun 12 Q4	13	

Q	Source	Max	Mark
7	Jun 13 Q6	14	
8	Jun 14 Q6	13	
9	Jun 15 Q7	13	
10	Jun 16 Q1	7	
11	Jun 17 Q7	15	
12	Spec. Q6	13	
Total		150	

Polar Coordinates — Curves, Tangents & Area Enclosed — what the new spec asks

WJEC GCE A Level Further Mathematics (from 2017) · Unit 4: Further Pure Mathematics B · Topic 2.4.6.

Polar / Cartesian conversion 2.4.6

- Forward: $x = r\cos\theta, y = r\sin\theta$.
- Reverse: $r = \sqrt{x^2+y^2}, \theta = \arctan(y/x)$ (with quadrant check).
- Allowable: $r \geq 0$; θ in $[0, 2\pi)$ or $(-\pi, \pi]$.
- Some questions use signed r — double-check the conventions.

Sketching polar curves 2.4.6

- Simple curves: $r = a(b + c\cos\theta)$ (cardioid / limaçon) and $r = a\cos n\theta$ (rose).
- Find the values of θ where $r = 0$ — these are the angular "pinch" points.
- Symmetry: $r(\theta) = r(-\theta) \Rightarrow$ symmetric about initial line.
- Tangents parallel to initial line: $\frac{dy}{d\theta} = 0$.
- Tangents perpendicular to initial line: $\frac{dx}{d\theta} = 0$.

Area enclosed 2.4.6

- Area of sector swept by $r(\theta)$ from $\theta = \alpha$ to $\theta = \beta$: $A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$.
- Use double-angle identities to integrate r^2 when r contains $\cos\theta, \sin\theta$.
- For the area between two curves, integrate the difference of the two r^2 values.
- Spec: intersection of curves is excluded from area questions.

Working scientifically general

- Always state the range of θ before integrating — many curves are only defined for $0 \leq \theta \leq \pi/2$ etc.
- For tangent direction: convert $r(\theta)$ to (x, y) then use $dy/dx = (dy/d\theta)/(dx/d\theta)$.
- Common pitfall: $r = 0$ gives the pole, not a tangent direction — check $dy/d\theta$ near $r = 0$.
- Double-check using small sample points: $\theta = 0, \pi/4, \pi/2$ etc.

Polar Coordinates in one page

Quick-reference notes — revisit before each section. Don't use during questions.

Polar coordinates

A point (r, θ) where r is distance from pole, θ is angle from initial line.

Conversion to Cartesian: $x = r\cos\theta$, $y = r\sin\theta$.

Reverse: $r = \sqrt{x^2 + y^2}$, $\theta = \arctan(y/x)$ + quadrant correction.

Polar curves

Equation $r = f(\theta)$ defines a curve.

Examples: $r = a$ (circle); $r = 2a\cos\theta$ (off-axis circle); $r = a + b\cos\theta$ (limaçon).

$r = a\cos n\theta$ traces a rose with n (or $2n$) petals.

Sketching tips

Plot r at $\theta = 0, \pi/4, \pi/2, 3\pi/4, \pi, \ldots$.

Find where $r = 0$ (the curve passes through the pole).

Test symmetry: $r(\theta) = r(-\theta) \Rightarrow$ symmetric about initial line.

Tangents parallel to initial line

Tangent **parallel** to initial line: y-coordinate stationary, so:

$$\frac{dy}{d\theta} = 0$$

Here $y = r\sin\theta$, so $\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$.

Tangents perp to initial line

Tangent **perpendicular** to initial line: x -coordinate stationary:

$$\frac{dx}{d\theta} = 0$$

Here $x = r\cos\theta$, so $\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$.

Area enclosed by polar curve

Area swept from $\theta = \alpha$ to $\theta = \beta$:

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$$

Use double-angle identities to integrate r^2 when r contains $\cos, \sin$.

Area between two curves

For r_1 outside r_2 on $[\alpha, \beta]$:

$$A = \frac{1}{2}\int_{\alpha}^{\beta} (r_1^2 - r_2^2) \, d\theta$$

Spec note: intersections of two curves are excluded from the syllabus.

Common pitfalls

- Forgetting the $\frac{1}{2}$ in the area formula.
- Mixing up "parallel to initial line" ($dy/d\theta = 0$) and "perp to initial line" ($dx/d\theta = 0$).
- Wrong range of θ: many curves only sweep a portion of $[0, 2\pi]$.
- Squaring r losing sign — check that $r \geq 0$ on the interval.

Strategy

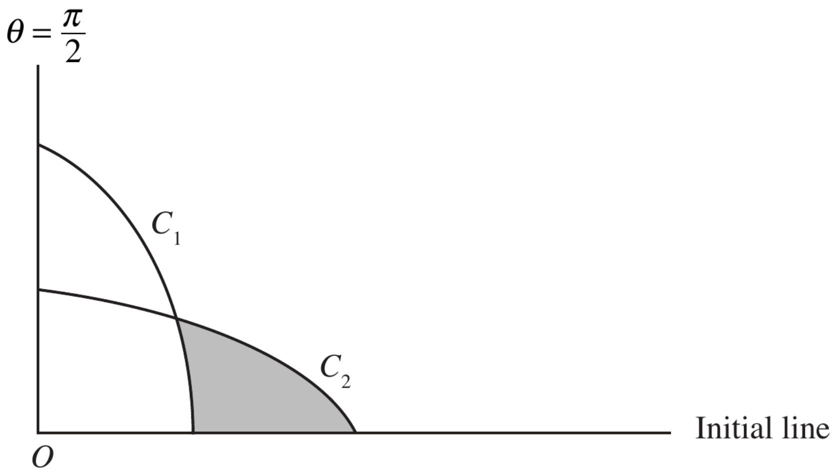
1. Sketch the curve first — identify pinch points ($r = 0$), max r etc.
2. For tangents: compute $dy/d\theta$ or $dx/d\theta$ and set $= 0$.
3. For area: $A = \frac{1}{2}\int r^2 \, d\theta$ over correct range.
4. Always check the θ-range matches the figure.

SECTION T6

Polar Coordinates

Questions 1–12 · 150 marks

4.



The diagram shows the initial line, the line $\theta = \frac{\pi}{2}$ and the curves C_1, C_2 with equations

$C_1 : r = e^{\theta} \left(0 \leqslant \theta \leqslant \frac{\pi}{2}\right)$,

$C_2 : r = 2e^{-\theta} \left(0 \leqslant \theta \leqslant \frac{\pi}{2}\right)$.

(a) Find the polar coordinates of the point of intersection of C_1 and C_2. [4]

(b) Find the area of the shaded region. [5]

7.

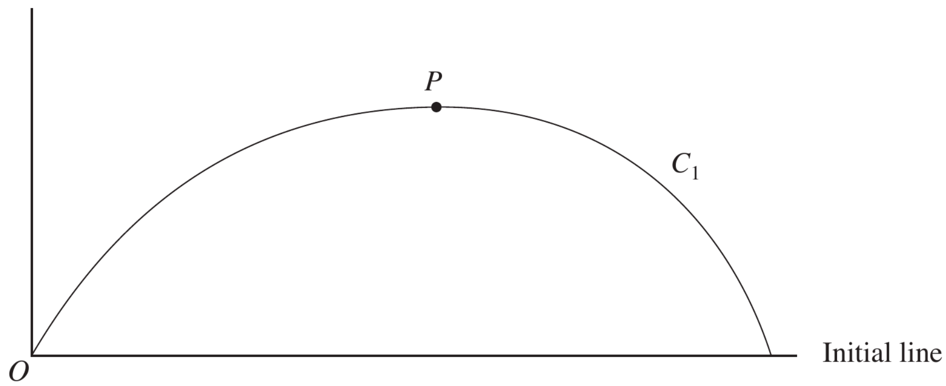


Figure 1

Figure 1 above shows a sketch of the curve C_1 with polar equation

$$r = 1 - \theta, \quad 0 \leqslant \theta \leqslant 1.$$

(a) (i) Given that P is the point on C_1 at which the tangent to C_1 is parallel to the initial line, show that the θ coordinate of P satisfies the equation

$$\theta + \tan\theta = 1.$$

(ii) Show that the area of the region enclosed by C_1 and the initial line is $\frac{1}{6}$. [6]

(b)

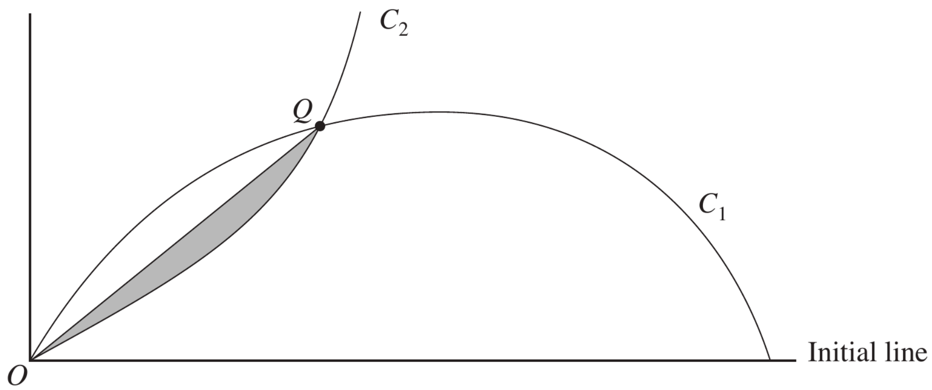


Figure 2

Figure 2 above shows a sketch of the curve C_1 and part of the curve C_2 with polar equation

$$r = 2\theta^2, \quad 0 \leqslant \theta \leqslant 1.$$

(i) Find the polar coordinates of Q, the point of intersection of C_1 and C_2.

(ii) Find the area of the region, shaded in Figure 2, enclosed by C_2 and the straight line OQ. [8]

5. *(a)* Sketch the curve having polar equation

$$r = 2 + \cos\theta \quad (0 \leqslant \theta \leqslant \pi).$$ [1]

(b) Determine the area of the region enclosed between the curve, the initial line and the line

$\theta = \dfrac{\pi}{2}$. [6]

(c) Find the polar coordinates of the point on the curve at which the tangent is parallel to the initial line. [7]

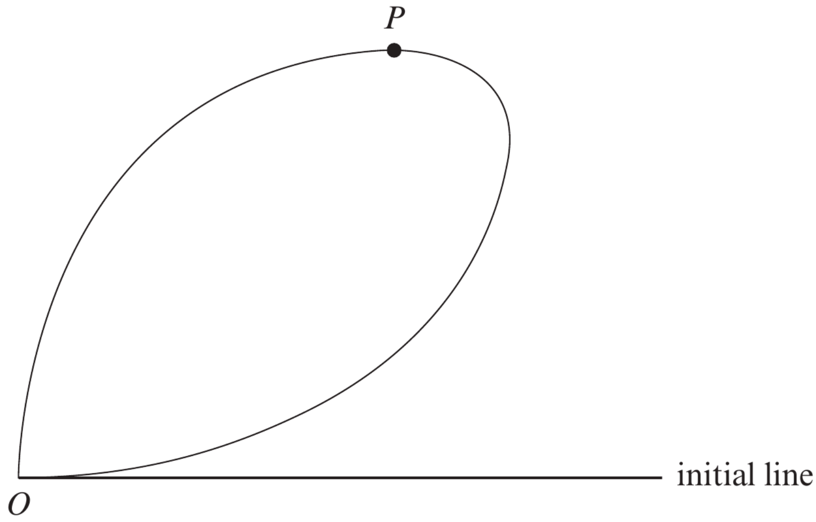
6. The curve C has polar equation

$$r = \cos\theta + 2\sin\theta \quad (0 \leqslant \theta \leqslant \frac{\pi}{2})$$

(a) Find the polar coordinates of the point on C at which the tangent is perpendicular to the initial line. [7]

(b) Determine the area of the region enclosed between C, the initial line and the line $\theta = \frac{\pi}{2}$. [6]

6.



The above diagram shows a sketch of the curve C with polar equation

$$r = \sin 2\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

The point P, marked on the diagram, is the point at which the tangent to C is parallel to the initial line.

(a) Determine the area of the region enclosed by C. [5]

(b) Find the polar coordinates of the point P. [7]

TURN OVER

4. The curve C_1 has polar equation

$$r = 2\cos\theta - \sin\theta \quad (0 \leqslant \theta \leqslant \frac{\pi}{4}).$$

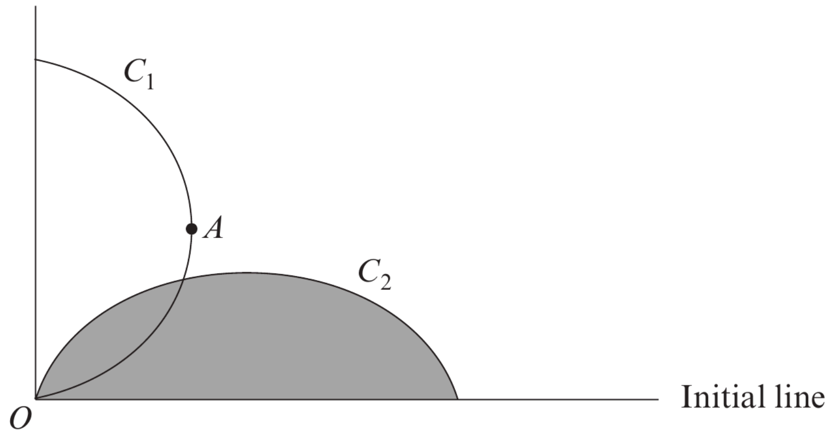
(a) Find the polar coordinates of the point on C_1 at which the tangent is parallel to the initial line. [6]

(b) The curve C_2 has polar equation

$$r = 1 + \sin\theta.$$

Find the polar coordinates of the point of intersection of C_1 and C_2. [7]

6.



The diagram shows sketches, for $0 \leqslant \theta \leqslant \frac{\pi}{2}$, of the curve C_1 having polar equation $r = \sin^2\theta$ and the curve C_2 having polar equation $r = 1 - \sin\theta$.

(a) Find the polar coordinates of the point A on C_1 at which the tangent is perpendicular to the initial line. [8]

(b) Find the area of the shaded region enclosed between C_2 and the initial line. [6]

TURN OVER

6. The curve C has polar equation

$$r = \sin\theta + \cos\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

(a) Find the polar coordinates of the point at which the tangent is parallel to the initial line. [8]

(b) Find the area of the region enclosed between C, the initial line and the line $\theta = \frac{\pi}{2}$. [5]

7.

The above diagram shows the curve C with polar equation

$$r = \tan\left(\frac{\theta}{2}\right), \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

(a) Show that the θ-coordinate of the point A at which the tangent to C is perpendicular to the initial line satisfies the equation

$$2\tan\theta\tan\left(\frac{\theta}{2}\right) = 1 + \tan^2\left(\frac{\theta}{2}\right).$$

Hence find the polar coordinates of A. [9]

(b) Find the area of the shaded region enclosed between C and the line $\theta = \frac{\pi}{2}$. [4]

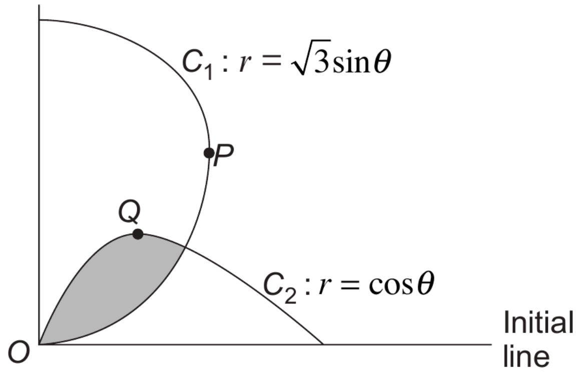
END OF PAPER

1. The curve C has polar equation

$$r = 1 + 2\tan\theta \ , 0 \leqslant \theta \leqslant \frac{\pi}{4} \ .$$

Show that there is no point on C at which the tangent is perpendicular to the initial line. [7]

7.



The diagram shows a sketch of the curve C_1 with polar equation $r = \sqrt{3}\sin\theta$ and a sketch of the curve C_2 with polar equation $r = \cos\theta$, both defined for $0 \leqslant \theta \leqslant \frac{\pi}{2}$.

(a) The point at which the tangent to C_1 is perpendicular to the initial line is denoted by P and the point at which the tangent to C_2 is parallel to the initial line is denoted by Q. Show that the origin O and the points P and Q lie on a straight line. [5]

(b) (i) Show that the polar coordinates of the point of intersection of C_1 and C_2 are $\left(\frac{\sqrt{3}}{2}, \frac{\pi}{6}\right)$

(ii) Find the area of the shaded region enclosed by C_1 and C_2. [10]

END OF PAPER

6. The curves C_1 and C_2 have polar equations as follows:

$$C_1 : r = 1 - \cos\theta \qquad (-\pi \leq \theta \leq \pi)$$

$$C_2 : r = \cos 2\theta \qquad \left(-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}\right)$$

(a) Sketch C_1 and C_2 on the same diagram. [2]

(b) Find the area enclosed by C_1. [5]

(c) Find the polar coordinates of the points of intersection of C_1 and C_2. [6]

END OF POLAR COORDINATES PACK

Source: WJEC FP3 (2008 modular spec) · 2005–2017

Curated for WJEC FM 2017 spec A2 Unit 4 — Topic 6 (2.4.6)